PROCEEDINGS OF SCIENCE

New Einstein-Hilbert type action for space-time and matter -Nonlinear-supersymmetric general relativity theory-

Kazunari Shima*Saitama Institute of Technology

E-mail: `shima@sit.ac.jp`

We can perform the geometric argument of general relativity principle on (unstable) Riemann space-time just inspired by nonlinear representation of supersymmetry(NLSUSY), whose tangent space is specified by Grassmann degrees of freedom ψ for SL(2,C) besides the ordinary Minkowski one x^a for SO(1,3) and obtain straightforwardly new Einstein-Hilbert(EH)-type action with global NLSUSY invariance (NLSUSYGR)) equipped with the cosmological term. Due to the NLSUSY nature of space-time NLSUSYGR would breaks down(Big Collapse) spontaneously to ordinary E-H action of graviton, NLSUSY action of Nambu-Goldstone fermion ψ and their gravitational interaction. Simultaneously the universal attractive gravitational force would constitute the NG fermion-composites corresponding to the eigenstates of liner-SUSY(LSUSY) super-Poincare space-time symmetry, which gives a new paradigm for the unification of space-time and matter. By linearizing NLSUSY we show that the standard model(SM) of the low energy particle physics can emerge in the true vacuum of NLSUSYGR as the NG fermion-composite massless eigenstates of LSUSY super-Poincare algebra of space-time symmetry, which can be understood as the ignition of the Big Bang and continues naturally to the standard Big Bang model of the universe. NLSUSYGR can bridge naturally the cosmology and the low energy particle physics and provides new insights into unsolved problems of cosmology, SM and mysterious relations between them, e.g. the space-time dimension four, the origin of SUSY breaking, the dark energy and dark matter, the dark energy density$\sim$(neutrino mass)^4, the tiny neutrino mass, the three-generations structure of quarks and leptons, the rapid expansion of space-time, the magnitude of bare gauge coupling constant, etc..

*European Physical Society Conference on High Energy Physics - EPS-HEP2019 -
10-17 July, 2019
Ghent, Belgium*

*Speaker.

Three-generations strucure

Supersymmetry (SUSY) related naturally to space-time symmetry is promissing for the unification of general relativity and the low enegy SM in *one* single irreducible representation of the symmetry group. We have found by group theoretical arguments that among all $SO(N)$ super-Poincaré (sP) groups the $SO(10)$ sP group decomposed as $N = \underline{10} = \underline{5} + \underline{5}^*$ under $SO(10) \supset SU(5)$ may be a unique and minimal group which accomodates all observed particles including graviton in *a single* irreducible representation of N *linear(L)* SUSY. In this case 10 supercharges $Q^I, (I = 1, 2, \cdots .10)$ are embedded as follows: $\underline{10}_{SO(10)} = \underline{5}_{SU(5)} + \underline{5}^*{}_{SU(5)}$, $\underline{5}_{SU(5)} = [\ \underline{3}^{*c}, \underline{1}^{ew}, (\frac{e}{3}, \frac{e}{3}, \frac{e}{3}) : Q_a(a = 1, 2, 3)\ ] + [\ \underline{1}^{c}, \underline{2}^{ew}, (-e, 0) : Q_m(m = 4, 5)\ ]$, . i.e., $\underline{5}_{SU(5)GUT}$ represents [Q_a:d-type, Q_m:Lepton-type] supercharges. The massless helicity state $|h>$ of gravity multiplet of SO(10) sP with CPT conjugate are specified by the helicity $h = (2 - \frac{n}{2})$ and the dimension $\underline{d}_{[n]} = \frac{10!}{n!(10-n)!} (n = 0, 1, , 10)$ as tabulated below. To see low energy massive states we assume a *maximal* $SU(3) \times SU(2) \times U(1)$

$\vert h\vert$	3	$\frac{5}{2}$	2	$\frac{3}{2}$	1	$\frac{1}{2}$	0
$\underline{d}_{[n]}$	$\underline{1}_{[10]}$	$\underline{10}_{[9]}$	$\underline{1}_{[0]}$ $\underline{45}_{[8]}$	$\underline{10}_{[1]}$ $\underline{120}_{[7]}$	$\underline{45}_{[2]}$ $\underline{210}_{[6]}$	$\underline{120}_{[3]}$ $\underline{252}_{[5]}$	$\underline{210}_{[4]}$ $\underline{210}_{[4]}$

invariant superHiggs-like mechanism among helicity states, i.e., all redundant high helicity states for SM become massive by absorbing lower helicity states (and decoupled) in *SM invariant way*. The results are interesting: Spin $\frac{1}{2}$ state survivours after superHiggs-like mechanism are shown in the table (tentatively as Dirac particles).

In the fermion sector, just three generations of quark and lepton states survive as shown in the table. In the bosonic sector, gauge fields of SM in vector states and one Higgs field of SM in

$SU(3)$	Q_e	$SU(2) \otimes U(1)$
$\underline{1}$	0 -1 -2	$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$ ($Q_e = 0, -1$) (E) ($Q_e = -2$)
$\underline{3}$	$5/3$ $2/3$ $-1/3$ $-4/3$	$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$ ($Q_e = 2/3, -1/3$) $\begin{pmatrix} h \\ o \end{pmatrix}$ ($Q_e = -1/3, -4/3$) $\begin{pmatrix} a \\ f \end{pmatrix} \begin{pmatrix} g \\ m \end{pmatrix}$ ($Q_e = 5/3, 2/3$) $\begin{pmatrix} r \\ i \\ n \end{pmatrix}$ ($Q_e = 2/3, -1/3, -4/3$)
$\underline{6}$	$4/3$ $1/3$ $-2/3$	$\begin{pmatrix} P \\ Q \\ R \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$
$\underline{8}$	0 -1	$\begin{pmatrix} N_1 \\ E_1 \end{pmatrix} \begin{pmatrix} N_2 \\ E_2 \end{pmatrix}$

the scalar states survive. Besides those observed states, one color-singlet neutral vector state and one double-charge color-singlet spin $\frac{1}{2}$ state are survived, wich can be tested experimentally. We will show in the next section that no-go theorem for constructing non-trivial $SO(N>8)$SUGRA can be circumvented by adopting the *nonliner (NL)* representation of SUSY, i.e. by introducing *the degeneracy of space-time* through NLSUSY degrees of freedom.

Nonlinear-Supersymmetric General Relativity Theory(NLSUSYGR)

For simplicity we discuss $N=1$ without the loss of the generality.

The fundamental action *nonlinear supersymmetric general relativity theory (NLSUSYGR)* has been constructed by extending the geometric arguments of Einstein general relativity (EGR) on Riemann space-time to new space-time inspired by NLSUSY, where tangent space-time is specified not only by the Minkowski coodinate x_a for $SO(1,3)$ but also by the Grassmann coordinate ψ_α for $SL(2,C)$ related to NLSUSY. They are coordinates of the coset space $\frac{superGL(4,R)}{GL(4,R)}$ and can be interpreted as NG fermions associated with the spontaneous breaking of *super*-$GL(4,R)$ down to $GL(4,R)$. The NLSUSYGR action, is given by

$$L_{NLSUSYGR}(w) = -\frac{c^4}{16\pi G}|w|\{\Omega(w)+\Lambda\}, \tag{1}$$

$$|w| = \det w^a{}_\mu = \det\{e^a{}_\mu + t^a{}_\mu(\psi)\},$$

$$t^a{}_\mu(\psi) = \frac{\kappa^2}{2i}(\bar\psi\gamma^a\partial_\mu\psi - \partial_\mu\bar\psi\gamma^a\psi), \tag{2}$$

where G is the Newton gravitational constant, Λ is a (*small*) cosmological term and κ is an arbitrary constant of NLSUSY with the dimemsion (mass)$^{-2}$. $w^a{}_\mu(x) = e^a{}_\mu + t^a{}_\mu(\psi)$ and $w^\mu{}_a = e^\mu{}_a - t^\mu{}_a + t^\mu{}_\rho t^\rho{}_a - t^\mu{}_\sigma t^\sigma{}_\rho t^\rho{}_a + t^\mu{}_\kappa t^\kappa{}_\sigma t^\sigma{}_\rho t^\rho{}_a$ which terminate at $O(t^4)$ for $N=1$ are the invertible *unified vierbeins* of new space-time. $e^a{}_\mu$ is the ordinary vierbein of EGR for the local $SO(1,3)$ and $t^a{}_\mu(\psi)$ is the mimic vierbein analogue (actually the stress-energy-momentum tensor) of NG fermion $\psi(x)$ for the local $SL(2,C)$. (We call $\psi(x)$ *superon* as the hypothetical fundamental spin $\frac{1}{2}$ particle carrying the supercharge of the supercurrent of the global NLSUSY.) $\Omega(w)$ is the the unified scalar curvature of new space-time computed in terms of the *unified vierbein* $w^a{}_\mu(x)$. Interestingly Grassmann degrees of freedom induce the imaginary part of the unified vierbein $w^a{}_\mu(x)$, which represents straightforwardly the fermionic matter contribution. Note that $e^a{}_\mu$ and $t^a{}_\mu(\psi)$ contribute equally to the curvature of spac-time, which may be regarded as the Mach's principle in ultimate space-time. (The second index of mimic vierbein t, e.g. μ of $t^a{}_\mu$, means the derivative ∂_μ.) $s_{\mu\nu} \equiv w^a{}_\mu\eta_{ab}w^b{}_\nu$ and $s^{\mu\nu}(x) \equiv w^\mu{}_a(x)w^{\nu a}(x)$ are *unified metric tensors* of new spacetime. $L_{NLSUSYGR}(w)$ (2) is invariant under the following NLSUSY transformations:

$$\delta^{NL}\psi = \frac{1}{\kappa^2}\zeta + i\kappa^2(\bar\zeta\gamma^\rho\psi)\partial_\rho\psi, \quad \delta^{NL}e^a{}_\mu = i\kappa^2(\bar\zeta\gamma^\rho\psi)\partial_{[\rho}e^a{}_{\mu]}, \tag{3}$$

where ζ is a constant spinor parameter and $\partial_{[\rho}e^a{}_{\mu]} = \partial_\rho e^a{}_\mu - \partial_\mu e^a{}_\rho$, which close on $GL(4,R)$, i.e. new NLSUSY (3) is the square-root of $GL(4,R)$;

$$[\delta_{\zeta_1},\delta_{\zeta_2}]\psi = \Xi^\mu\partial_\mu\psi, \quad [\delta_{\zeta_1},\delta_{\zeta_2}]e^a{}_\mu = \Xi^\rho\partial_\rho e^a{}_\mu + e^a{}_\rho\partial_\mu\Xi^\rho, \tag{4}$$

where $\Xi^\mu = 2i\kappa(\bar\zeta_2\gamma^\mu\zeta_1) - \xi_1^\rho\xi_2^\sigma e_a{}^\mu(\partial_{[\rho}e^a{}_{\sigma]})$. and induce the following GL(4,R) transformations on the unified vierbein $w^a{}_\mu$ and the metric tensor $s_{\mu\nu}$

$$\delta_\zeta w^a{}_\mu = \xi^\nu\partial_\nu w^a{}_\mu + \partial_\mu\xi^\nu w^a{}_\nu, \quad \delta_\zeta s_{\mu\nu} = \xi^\kappa\partial_\kappa s_{\mu\nu} + \partial_\mu\xi^\kappa s_{\kappa\nu} + \partial_\nu\xi^\kappa s_{\mu\kappa}, \tag{5}$$

where $\xi^{\rho} = i\kappa^{2}(\bar{\zeta}\gamma^{\rho}\psi)$,
NLSUSY GR action (1) possesses promissing large symmetries isomorphic to $SO(N)$ ($SO(10)$) SP group; namely, $L_{NLSYSYGR}(w)$ is invariant under spacetime symmetries: [new NLSUSY] $\otimes$ [local GL$(4,\mathrm{R})$] $\otimes$ [local Lorentz] and under internal symmetries : [global$SO(N)$] $\otimes$ [local$U(1)^{N}$] in case of N superons $\psi^{i}, i = 1,2,\cdots,N$ Note that the no-go theorem is overcome (circumvented) in a sense that the nontivial $N(N > 8)$-extended SUSY with gravity has been constructed in the NLSUSY invariant way.

Big Collapse(BC) of ultimate space-time(NLSUSYGR)

New *(empty)* space-time described by NLSUSYGR action $L_{NLSUSYGR}(w)$ is unstable due to NLSUSY structure of tangent space-time and collapses (called *Big Collapse*) spontaneously to ordinary Riemann space-time with the cosmological term and fermionic matter *superon* (called *superon-graviton model (SGM)*). $L_{SGM}(e,\psi)$ can be recasted formally as the following famlliar form

$$L_{NLSUSYGR}(w) = L_{SGM}(e,\psi) = -\frac{c^{4}}{16\pi G}|w|\{R(e) + \Lambda + \tilde{T}(e,\psi)\}, \tag{6}$$

where $R(e)$ is the Ricci scalar curvature of ordinary EH action and $\tilde{T}(e,\psi)$ represents the kinetic term and the gravitational interaction of superons. Remarkably the first term should reduces to NLSUSY action in Riemann-flat $e_{a}{}^{\mu}(x) \to \delta_{a}{}^{\mu}$ space-time, i.e. the arbitrary constant κ of NLSUSY is fixed to

$$\kappa^{-2} = \frac{c^{4}}{8\pi G}\Lambda. \tag{7}$$

Note that Big Collapse induces the rapid expansion of space-time due to the Pauli principle for fermion *superon*; $ds^{2} = s_{\mu\nu}dx^{\mu}dx^{\nu} = [g_{\mu\nu}(e) + h_{\mu\nu}(e,\psi)]dx^{\mu}dx^{\nu}$ and simultaneously constitutes all possible gravitational composite-eigenstates of space-time symmetey. SGM action, whose gravitational evolution ignites Big Bang of the present observed universe. NLSUSY scenario predicts the dimension of space-time is *four*, for space-time supersymmetry for $SO(1,D-1)$ and $SL(d,C)$ requires

$$\frac{D(D-1)}{2} = 2(d^{2}-1), \tag{8}$$

which holds only for $D = 4, d = 2$.

Evolution of NLSUSYGR/SGM

NLSUSYGR(SGM) with $\Lambda > 0$ evolves toward the true vacuum. The gravirty is the universal attractive force and creates all possible gravitational composites of superons, which is the all possible products of supercharges and corresponds to (massless) helicity-eigenstates of SO(10) *linear(L)*SUSY sP algebra of asymptotic space-time symmetry. (Note that the leading term of supercharge is the superon field.) This means that all component fields of LSUSY supermultiplet are expressed as such composites of superons (called *SUSY compositeness*) as LSUSY transformation of LSUSY supermultiplet are reproduced under the NLSUSY transformations of the constituent superons. Simultaneously the equivalence of NLSUSY action and the LSUSY action holds (called *NL/L SUSY relation*) in the sence that LSUSY action reduces to NLSUSY action when SUSY compositeness is inserted in LSUSY component fields. To see the low energy (vacuum) behavior of $N = 2$ SGM (NLSUSYGR) we consider SGM in asymptotic Riemann-flat space-time, where $N = 2$ SGM reduces to essentially $N = 2$ NLSUSY action. We will show the equivalence of $N = 2$

NLSUSY action to $N = 2$ LSUSY QED action(called *NL/L SUSY relation.*), i.e.

$$L_{\rm NLSUSYGR}(w^a{}_\mu) = L_{\rm SGM}(e^a{}_\mu.\psi) \to L_{\rm NLSUSY}(\psi) + [\text{suface terms}] = f_{\xi^i} L_{\rm LSUSY}(v^a,\ D.\cdots) \quad (9)$$

where f_{ξ^i} is the function of vacuum values ξ^i of auxiliary fields. NL/L SUSY relation is shown explicitly by substituting the following SUSY compositeness into the LSUSY QED theory. For example, the SUSY compositeness for the *minimal gauge* vector supermultiplet $(v^a, \lambda^i, A, \phi, D)$ are

$$v^a = -\frac{i}{2}\xi\kappa\epsilon^{ij}\bar\psi^i\gamma^a\psi^j|w|,\ \ \lambda^i = \xi\left[\psi^i|w| - \frac{i}{2}\kappa^2\partial_a\{\gamma^a\psi^i\bar\psi^j\psi^j(1 - i\kappa^2\bar\psi^k\partial\!\!\!/\psi^k)\}\right],$$

$$A = \frac{1}{2}\xi\kappa\bar\psi^i\psi^i|w|, \phi = -\frac{1}{2}\xi\kappa\epsilon^{ij}\bar\psi^i\gamma_5\psi^j|w|, D = \frac{\xi}{\kappa}|w| - \frac{1}{8}\xi\kappa^3\partial_a\partial^a(\bar\psi^i\psi^i\bar\psi^j\psi^j).\ \cdots \quad (10)$$

and similar results for the scalar supermultiplet. NL/L SUSY relation shows the equivalence(relation) of two theories irrespective of the renormalizability. For the *non-minimal most general* gauge vector and scalar supermultiplet NL/L SUSY relation $f_{\xi^i} = 1$ predicts the magnitude of the bare gauge coupling constant.

$$f(\xi, \xi^i, \xi_c) = \xi^2 - (\xi^i)^2 e^{-4e\xi_c} = 1, \quad i.e., \quad e = \frac{\ln(\frac{\xi^{i2}}{\xi^2 - 1})}{4\xi_c}, \quad (11)$$

Broken LSUSY(QED) gauge theory is encoded in the vacuum of NLSUSY theory as composites of NG fermion. And SM may be an low energy effective theory of SGM.

Low energy particle physics of NLSUSYGR/SGM

NL/L SUSY relation(equivalence) gives:
$L_{N=2\rm SGM} \longrightarrow L_{N=2\rm NLSUSY} + [\text{suface terms}] = L_{N=2\rm SUSYQED}$ in Riemann-flat space-time. The vacuum is given by the minimum of the potential $V(A, \phi, B^i, D)$ of $L_{N=2\rm LSUSYQED}$,

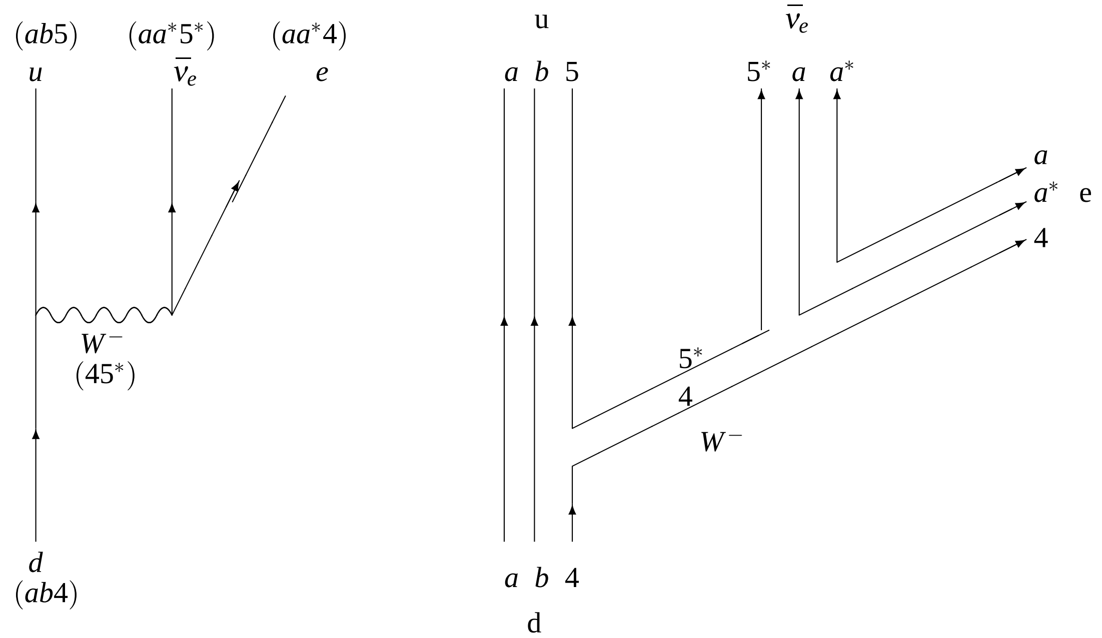
$$V(A, \phi, B^i, D) = -\frac{1}{2}D^2 + \left\{\frac{\xi}{\kappa} - f(A^2 - \phi^2) + \frac{1}{2}e|B^i|^2\right\}D + \frac{e^2}{2}(A^2 + \phi^2)|B^i|^2, \quad (12)$$

which havs mass spectra:

$$m^2_{\hat A} = m^2_{\lambda^i} = 4f^2k^2 = \frac{4\xi f}{\kappa},\ \ m^2_{\hat B^1} = m^2_{\hat B^2} = m^2_\chi = m^2_\nu = e^2k^2 = \frac{\xi e^2}{\kappa f},\ \ m_{v_a} = m_{\hat\phi} = 0, \quad (13)$$

which produces *mass hierarchy* by the factor $\frac{e}{f}$, The vacuum describes qualitatively lepton-Higgs-U(1) sector analogue of the SM: one massive charged Dirac fermion ($\psi_D{}^c \sim \chi + i\nu$), one massive neutral Dirac fermion ($\lambda_D{}^0 \sim \lambda^1 - i\lambda^2$), one massless vector (a photon) (v_a), one charged scalar ($\hat B^1 + i\hat B^2$), one neutral complex scalar ($\hat A + i\hat\phi$), which are composites of superons.

Revisiting SM and GUT in the SQM (superon-quintet composite model) picture may give new insight into the unsolved problems of the SM. One simple assignment of observed particles is: (e, ν_e): $\delta^{ab}Q_aQ^*{}_bQ_m$, (μ, ν_μ): $\delta^{ab}Q_aQ^*{}_b\epsilon^{lm}Q_lQ_mQ^*_n$, (τ, ν_τ):$\epsilon^{abc}Q_bQ_c\epsilon^{ade}Q^*{}_dQ^*_eQ_m$, (u, d): $\epsilon^{abc}Q_bQ_cQ_m$, (c, s): $\epsilon^{lm}Q_lQ_m\epsilon^{abc}Q_bQ_cQ^*{}_n$, (t, b): $\epsilon^{abc}Q_aQ_bQ_cQ^*{}_dQ_m$, *Gauge Boson*: $Q_aQ^*{}_b$, *Higgs Boson*: $\delta^{ab}Q_aQ^*{}_bQ_lQ^*{}_m$, $\cdots$. and SQM diagram, e.g., for β-decay:



Cosmological implications of NLSUSYGR/SGM

The variation of SGM action $L_{\rm SGM}(e,\psi)$ with respect to $e^a{}_\mu$ yields
Einstein equation equipping with matter and cosmological term:

$$R_{\mu\nu}(e)-\frac{1}{2}g_{\mu\nu}R(e)=\frac{8\pi G}{c^4}\{\tilde{T}_{\mu\nu}(e,\psi)-g_{\mu\nu}\frac{c^4\Lambda}{8\pi G}\}. \tag{14}$$

where $\tilde{T}_{\mu\nu}(e,\psi)$ abbreviates the stress-energy-momentum of superon(NG fermion) including the gravitational interaction. Note that the cosmological term $-\frac{c^4\Lambda}{8\pi G}$ can be interpreted as
the negative energy density of space-time, i.e. the dark energy density ρ_D.
Big collapse(BC) may induce 3 dimensional expansion of space-time by Pauli principle:

$$ds^2=s_{\mu\nu}(x)dx^\mu dx^\nu=\{g_{\mu\nu}(e)+h_{\mu\nu}(e,\psi)\}dx^\mu dx^\nu.$$

BC produces composite (massless) eigenstates of SO(N) sP algebra due to the universal attractive force of graviton, which is the ignition of the Big Bang(BB) SM scenario. As shown in the toy model, the vacuum of the composite SGM scenario may explain naturally observed mysterious (numerical) relations: *dark energy density* $\rho_D \sim O(\kappa^{-2}) \sim {m_\nu}^4 \sim (10^{-12}GeV)^4 \sim {g_{s\nu}}^2$, provided ${\lambda_D}^0$ is identified with neutrino and $f\xi \sim O(1)$.

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