Logarithmic corrections to a^2 scaling in lattice Yang Mills theory

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We analyse the leading logarithmic corrections to the a^2 scaling of lattice artefacts in QCD, following the seminal work of Balog, Niedermayer and Weisz in the O(n) non-linear sigma model. Restricting our attention to contributions from the action, the leading logarithmic corrections can be determined by the anomalous dimensions of a minimal on-shell basis of mass-dimension 6 operators. We present results for the SU(N) pure gauge theory. In this theory the logarithmic corrections reduce the cutoff effects. These computations are the first step towards a study of full lattice QCD at O(a^2), which is in progress.

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1. Introduction

Symanzik effective field theory [1, 2, 3, 4] has been used extensively for so called on-shell improvement. This removes the lattice artifacts at leading order in the lattice spacing a [5, 6, 7, 8]. We follow here a related path and compute the leading logarithmic corrections to classical a^2 scaling in pure gauge theory originating from the action as a first step towards full lattice QCD. This fills a gap in the theoretical control over continuum extrapolations, where one usually assumes pure a^2 scaling, although for an asymptotically free theory leading corrections have the form $a^2[\bar{g}^2(1/a)]^{\hat{\gamma}_1}$ with coupling $\bar{g}^2(1/a) \sim -1/\ln(a\Lambda)$ for small a. The parameter $\hat{\gamma}_1$ is proportional to a 1-loop anomalous dimension. Therefore, whether a continuum extrapolation with classical a^2 scaling is accurate, depends on $\hat{\gamma}_1$ and the available lattice spacings.

For example in the 2d non-linear sigma-model, classical a^2 scaling turned out far from realistic. There a pioneering analysis has been performed [9, 10] solving a long-standing puzzle of lattice artifacts, which seemed to behave more like O(a) than the classically expected O(a^2). The issue was solved by large logarithmic corrections, i.e. distinctly negative $\hat{\gamma}_1$.

2. Symanzik effective theory

To describe lattice artifacts impacting expectation values obtained on lattices with finite lattice spacing we use a continuum Symanzik effective theory [1, 2, 3, 4]. There are different sources of lattice artifacts. Firstly, the discretised action described by the effective Lagrangian

$$\mathscr{L}_{\text{eff}} = \mathscr{L} + a^2 \delta\mathscr{L} + \text{O}(a^4), \tag{2.1}$$

with the Lagrangian of the targeted continuum theory

$$\mathscr{L} = -\frac{1}{2g_0^2}\text{tr}(F_{\mu\nu}F_{\mu\nu}), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \tag{2.2}$$

and secondly any local field Φ_R involved in correlation functions of interest[1]

$$\Phi_{\text{eff};R}(x) = \Phi_R(x) + a^2 \delta\Phi(x) + \text{O}(a^4). \tag{2.3}$$

Thirdly, the renormalisation condition employed on the lattice. For now we will assume the use of lattice perturbation theory in the MS lattice (lat) renormalisation scheme [11] and postpone the inclusion of the third contribution to the end of this section.

Consequently, to parametrise all possible lattice artifacts in $\delta\mathscr{L}$ and $\delta\Phi$ we also need two distinct minimal on-shell bases of operators $\mathscr{O}_i$ and Υ_i

$$\delta\mathscr{L} = c_i(a\mu, \bar{g})Z_{ij}^{\mathscr{O}}\mathscr{O}_j, \qquad\qquad \delta\Phi = d_i(a\mu, \bar{g})Z^\Phi Z_{ij}^\Upsilon \Upsilon_j \tag{2.4}$$

with free coefficients c_i and d_i, renormalised coupling $\bar{g} = \bar{g}(\mu)$ and renormalisation scale μ. Furthermore we introduced the mixing matrices $Z^{\mathscr{O}}$ and Z^Υ satisfying

$$\mathscr{O}_{i;R} = Z_{ij}^{\mathscr{O}}\mathscr{O}_j, \quad \Upsilon_{i;R} = Z^\Phi Z_{ij}^\Upsilon \Upsilon_j, \quad \Phi_R = Z^\Phi \Phi, \tag{2.5}$$

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[1]Renormalised local fields have to be used to match the lattice theory and the effective theory.

where, for later use, Z^{Υ} is defined with the renormalisation factor Z^{Φ} of the continuum operator Φ divided out, assuming, for simplicity, no mixing for Φ itself.

Which operators are actually allowed to contribute is determined by the symmetries of the lattice quantity, i.e. for the lattice Yang Mills action we only allow operators complying with

- Local SU(N) gauge symmetry,
- C-, P- and T-symmetry,
- discrete rotation and translation invariance (at least in manifolds without boundaries), i.e. the O(4) symmetry of $\mathscr{L}$ is partially broken and reduced to hypercubic symmetry.

With these restrictions and the requirement of linear independence only the following operators are left [8]

$$\mathscr{O}_1 = \frac{1}{g_0^2}\mathrm{tr}(D_\mu F_{\nu\rho} D_\mu F_{\nu\rho}), \quad \mathscr{O}_2 = \frac{1}{g_0^2}\sum_\mu \mathrm{tr}(D_\mu F_{\mu\rho} D_\mu F_{\mu\rho}), \quad \mathscr{O}_3 = \frac{1}{g_0^2}\mathrm{tr}(D_\mu F_{\mu\nu} D_\rho F_{\rho\nu}) \overset{\text{EOM}}{=} 0. \quad (2.6)$$

As indicated, the operator $\mathscr{O}_3$ can be dropped from the basis as well since we are only interested in a minimal on-shell basis and thus can use equations of motion (EOM) [8, 12], i.e. $D_\mu F_{\mu\nu} = 0$, for the reduction of the basis.

In analogy to [9, 10] we can write, as an example, the lattice artifacts of a renormalised connected 2-point function as

$$\frac{\langle \Phi_{\text{lat}}(x)\Phi_{\text{lat}}(y)\rangle^{\text{con}}_{\text{lattice}}}{\langle \Phi_{\text{lat}}(x)\Phi_{\text{lat}}(y)\rangle^{\text{con}}_{\text{cont}}} = 1 + a^2\left(\sum_i d_i(1,\bar{g})\delta_i^{\Upsilon}(x,y;1/a) - \sum_{j=1}^{2} c_j(1,\bar{g})\delta_j^{\mathscr{O}}(x,y;1/a)\right) + \mathrm{O}(a^4), \quad (2.7)$$

where $\langle\ldots\rangle^{\text{con}}_{\text{lattice}}$ and $\langle\ldots\rangle^{\text{con}}_{\text{cont}}$ are the expectation values in the lattice theory at lattice spacing a and zero respectively. In the continuum theory we introduce the shorthand

$$\delta_i^{\Upsilon}(x,y;\mu) = \left.\frac{\langle \Phi_{\text{R}}(x)\Upsilon_{i;\text{R}}(y)\rangle_{\text{con}} + \langle \Upsilon_{i;\text{R}}(x)\Phi_{\text{R}}(y)\rangle_{\text{con}}}{\langle \Phi_{\text{R}}(x)\Phi_{\text{R}}(y)\rangle_{\text{con}}}\right|_\mu, \quad (2.8)$$

$$\delta_j^{\mathscr{O}}(x,y;\mu) = \left.\int \mathrm{d}^4z \, \frac{\langle \Phi_{\text{R}}(x)\Phi_{\text{R}}(y)\mathscr{O}_{j;\text{R}}(z)\rangle_{\text{con}}}{\langle \Phi_{\text{R}}(x)\Phi_{\text{R}}(y)\rangle_{\text{con}}}\right|_\mu \quad (2.9)$$

and choose the renormalisation scale as $\mu = 1/a$ when matching our Symanzik effective theory to the lattice theory. As an example we choose on the lattice the regulator independent (RI) renormalisation condition (with scale $\mu_{\text{RI}}^{-1} = |x_{\text{RI}}|$) [13]

$$\langle \Phi_{\text{RI}}(x_{\text{RI}})\Phi_{\text{RI}}(0)\rangle^{\text{con}}_{\text{lattice}} = \langle \Phi(x_{\text{RI}})\Phi(0)\rangle^{\text{con}}_{\text{lattice}}\bigg|_{\substack{g_0=0\\a=0}} = \Xi_{\text{RI,lat}}(\bar{g}_{\text{lat}}(1/a);a)\langle \Phi_{\text{lat}}(x_{\text{RI}})\Phi_{\text{lat}}(0)\rangle^{\text{con}}_{\text{lattice}}. \quad (2.10)$$

The equation defines $\Xi_{\text{RI,lat}}$ which relates the MS lattice to our RI scheme in terms of the correlator at a specific distance. For this choice we find for the leading lattice artifacts as $a \searrow 0$

$$\frac{\langle \Phi_{\text{RI}}(x)\Phi_{\text{RI}}(y)\rangle^{\text{con}}_{\text{lattice}}}{\langle \Phi_{\text{RI}}(x)\Phi_{\text{RI}}(y)\rangle^{\text{con}}_{\text{cont}}} = 1 + a^2\left(\sum_i d_i^{(0)}\left\{\delta_i^{\Upsilon}(x,y;1/a) - \delta_i^{\Upsilon}(x_{\text{RI}},0;1/a)\right\}\right. \quad (2.11)$$

$$\left. - \sum_{j=1}^{2} c_j^{(0)}\left\{\delta_j^{\mathscr{O}}(x,y;1/a) - \delta_j^{\mathscr{O}}(x_{\text{RI}},0;1/a)\right\}\right) \times [1 + \mathrm{O}(\bar{g}^2(1/a))] + \mathrm{O}(a^4),$$

where $d_i^{(0)}$ and $c_j^{(0)}$ are the tree-level values of the coefficients and can be obtained from a simple Taylor-expansion in a in the classical theory. Note that if a coefficient vanishes at tree-level the 1-loop coefficient might be needed to obtain the leading powers in the coupling.

3. Lattice artifacts and the Renormalisation Group

In order to understand in which way the $\delta_i^{\Upsilon}(x,y;1/a)$ and $\delta_j^{\mathcal{O}}(x,y;1/a)$ have an impact on pure a^2 scaling we need to understand how they react to a change in the lattice spacing or equivalently to a change of the renormalisation scale μ. This is governed by the Renormalisation Group equations

$$\mu \frac{\mathrm{d}\delta_i^{\chi}(x,y;\mu)}{\mathrm{d}\mu} = \gamma_{ik}^{\chi} \delta_k^{\chi}(x,y;\mu), \quad \chi \in \{\mathcal{O},\Upsilon\}, \tag{3.1}$$

with the anomalous dimension matrices

$$\gamma_{ik}^{\chi} = \mu \frac{\mathrm{d}Z_{ij}^{\chi}}{\mathrm{d}\mu}(Z^{\chi})_{jk}^{-1} = -(\gamma_{\chi}^{(0)})_{ik}\bar{g}^2(\mu) + \mathrm{O}(\bar{g}^4(\mu)) \tag{3.2}$$

and the corresponding mixing matrices Z^{χ}. We choose now the operator bases such that $\gamma_{\chi}^{(0)} = \mathrm{diag}\{\gamma_{\chi,1}^{(0)},\ldots,\gamma_{\chi,n}^{(0)}\}$ and introduce the Renormalisation Group Invariants (RGI)

$$D_j^{\chi}(x,y) = \lim_{\mu\to\infty} \left[2b_0\bar{g}^2(\mu)\right]^{-\hat{\gamma}_j^{\chi}} \delta_j^{\chi}(x,y;\mu), \quad \hat{\gamma}_j^{\chi} = \frac{\gamma_{\chi,j}^{(0)}}{2b_0}, \tag{3.3}$$

where b_0 is the 1-loop coefficient of the β-function. This allows to rewrite

$$\delta_i^{\chi}(x,y;\mu) = \left[2b_0\bar{g}^2(\mu)\right]^{\hat{\gamma}_i^{\chi}} \mathrm{Pexp}\left[\int_0^{\bar{g}(\mu)} \mathrm{d}u \left\{\frac{\gamma^{\chi}(u)}{\beta(u)} - \frac{2\hat{\gamma}^{\chi}}{u}\right\}\right]_{ij} D_j^{\chi}(x,y) \tag{3.4}$$

$$= \left[2b_0\bar{g}^2(\mu)\right]^{\hat{\gamma}_i^{\chi}} D_i^{\chi}(x,y) + \mathrm{O}\left(\left[\bar{g}^2(\mu)\right]^{1+\hat{\gamma}_i^{\chi}}\right), \tag{3.5}$$

where Pexp denotes the path ordered exponential with decreasing $\bar{g}$ from the left to the right. As a result all dependence on the renormalisation scale is absorbed into the prefactor of the RGI. Inserting this into equation (2.11) yields the desired form

$$\frac{\langle\Phi_{\mathrm{RI}}(x)\Phi_{\mathrm{RI}}(y)\rangle_{\substack{\mathrm{con}\\\mathrm{lattice}}}}{\langle\Phi_{\mathrm{RI}}(x)\Phi_{\mathrm{RI}}(y)\rangle_{\substack{\mathrm{con}\\\mathrm{cont}}}} = 1 + a^2\left(\sum_i d_i^{(0)}[2b_0\bar{g}^2(1/a)]^{\hat{\gamma}_i^{\Upsilon}}\left\{D_i^{\Upsilon}(x,y) - D_i^{\Upsilon}(x_{\mathrm{RI}},0)\right\}\right. \tag{3.6}$$

$$\left. - \sum_{j=1}^2 c_j^{(0)}[2b_0\bar{g}^2(1/a)]^{\hat{\gamma}_j^{\mathcal{O}}}\left\{D_j^{\mathcal{O}}(x,y) - D_j^{\mathcal{O}}(x_{\mathrm{RI}},0)\right\}\right) \times [1 + \mathrm{O}(\bar{g}^2(1/a))] + \mathrm{O}(a^4).$$

So far we included contributions of the field Φ to the leading lattice artifacts. Since they depend on the fields of interest, we now restrict ourselves to the contributions from the action. This is sufficient for spectral quantities, such as masses and energies. Such quantities depend only on the quantum numbers of the fields from which they are extracted.

4. Computing 1-loop anomalous dimensions

In equation (3.6) we need the leading anomalous dimensions of the operators $\mathscr{O}_i$. We employed two strategies extracting these anomalous dimensions from different types of Euclidean Green's functions:

(i) connected on-shell Green's functions of fundamental gauge fields A with insertion of an operator $\tilde{\mathscr{O}}[A](q)$ carrying an overall momentum $q \neq 0$, see e.g. [14],

$$G_i^{ab}(p_1, p_2; q) = \left\langle \eta_1 \cdot \tilde{A}_R^a(p_1) \, \eta_2 \cdot \tilde{A}_R^b(p_2) \, \tilde{\mathscr{O}}_i[A](q) \right\rangle_{\text{con}} \tag{4.1}$$

$$G_i^{abc}(p_1, p_2, p_3; q) = \left\langle \eta_1 \cdot \tilde{A}_R^a(p_1) \, \eta_2 \cdot \tilde{A}_R^b(p_2) \, \eta_3 \cdot \tilde{A}_R^c(p_3) \, \tilde{\mathscr{O}}_i[A](q) \right\rangle_{\text{con}} \tag{4.2}$$

with external momenta p_j and polarisation vectors η_j fulfilling the (Minkowskian) on-shell condition

$$(p_j)_0^2 = -\vec{p}_j^2, \quad p_j \cdot \eta_j = 0. \tag{4.3}$$

(ii) 1PI off-shell Green's functions of classical background fields B with insertion of an operator $\tilde{\mathscr{O}}[B+A](q)$ using the background field (BGF) method [15, 16]

$$\Gamma_i^{ab}(p_1, p_2; q) = \left\langle \eta_1 \cdot \tilde{B}^a(p_1) \, \eta_2 \cdot \tilde{B}^b(p_2) \, \tilde{\mathscr{O}}_i[B+A](q) \right\rangle_{\text{1PI}} \tag{4.4}$$

$$\Gamma_i^{abc}(p_1, p_2, p_3; q) = \left\langle \eta_1 \cdot \tilde{B}^a(p_1) \, \eta_2 \cdot \tilde{B}^b(p_2) \, \eta_3 \cdot \tilde{B}^c(p_3) \, \tilde{\mathscr{O}}_i[B+A](q) \right\rangle_{\text{1PI}} \tag{4.5}$$

with $q = 0$ but no additional constraints on the external momenta p_j.

Both strategies allow us to restrict considerations to 2- and 3-point functions as well as 1-loop computations. However this results in an enlarged set of operators during renormalisation by introducing additional redundant operators namely total divergence operators

$$\not{\mathscr{O}}_1 = \frac{1}{g_0^2} \partial_\mu \text{tr}(F_{\nu\rho} D_\mu F_{\nu\rho}), \quad \not{\mathscr{O}}_2 = \frac{1}{g_0^2} \sum_\mu \partial_\mu \text{tr}(F_{\mu\rho} D_\mu F_{\mu\rho}), \tag{4.6}$$

for case (i) and the previously discarded EOM vanishing operator $\mathscr{E}_1 = \mathscr{O}_3$ for case (ii).

In the presence of either set of redundant operators we can extract the desired on-shell mixing matrix $Z^{\mathscr{O}}$ from

$$\begin{pmatrix} \mathscr{O}_i \\ Q_k \end{pmatrix}_{\text{R}} = \begin{pmatrix} Z_{ij}^{\mathscr{O}} & Z_{il}^{\mathscr{O}Q} \\ 0 & Z_{kl}^{Q} \end{pmatrix} \begin{pmatrix} \mathscr{O}_j \\ Q_l \end{pmatrix}, \quad Q \in \{\not{\mathscr{O}}, \mathscr{E}\}, \tag{4.7}$$

where the triangular mixing matrix ensures that the renormalised Q vanish for operator momentum $q = 0$ ($Q = \not{\mathscr{O}}$) and on-shell quantities ($Q = \mathscr{E}$). Due to this structure we only need tree-level contributions from the redundant operators to determine $Z^{\mathscr{O}Q}$ in order to extract $Z^{\mathscr{O}}$ at 1-loop.

Being only interested in 1-loop anomalous dimensions, another simplification can be achieved by use of the identity, see e.g. [17, 18],

$$\frac{1}{(k+p)^2} = \frac{1}{k^2 + \Omega} - \frac{2kp + p^2 - \Omega}{[k^2 + \Omega](k+p)^2}, \tag{4.8}$$

where k is the loop momentum, p a constant vector and $\Omega > 0$. Since the second term on the right hand side is one power less UV divergent we can iterate this step until the terms of the

4

form $[k^2 + \Omega]^{-n}$ carry all the UV divergences. The leftover terms are finite or at most IR divergent and can be dropped because only UV divergences contribute to anomalous dimensions in mass-independent renormalisation schemes such as $\overline{\text{MS}}$ that we employ in $D = 4 - 2\varepsilon$ dimensions. Consequently only the basic integrals of dimensional regularisation $\int d^D k\, (k^2)^m/[k^2 + \Omega]^n$ are left to evaluate.

The Feynman graphs contributing to the 2- and 3-point functions were obtained using QGRAF [19, 20]. The generation of Feynman rules for the operators and the computational steps of dimensional regularisation were then performed in FORM [21].

Independently of the strategy we find[2] for the 1-loop mixing matrix $Z^{\mathscr{O}}$ in the $\overline{\text{MS}}$ scheme

$$
\begin{pmatrix} \mathscr{O}_1 \\ \mathscr{O}_2 \end{pmatrix}_{\text{R}} = \begin{pmatrix} 1 + \frac{7C_A}{3\varepsilon} \frac{\bar{g}^2}{(4\pi)^2} & 0 \\ -\frac{7C_A}{15\varepsilon} \frac{\bar{g}^2}{(4\pi)^2} & 1 + \frac{21C_A}{5\varepsilon} \frac{\bar{g}^2}{(4\pi)^2} \end{pmatrix} \begin{pmatrix} \mathscr{O}_1 \\ \mathscr{O}_2 \end{pmatrix}.
\tag{4.9}
$$

Since this basis is not diagonal we switch to the diagonal basis

$$
\mathscr{B}_1 = \mathscr{O}_1, \quad \hat{\gamma}_1^{\mathscr{B}} = \frac{7}{11} \approx 0.636 \qquad \mathscr{B}_2 = \mathscr{O}_2 - \frac{1}{4}\mathscr{O}_1, \quad \hat{\gamma}_2^{\mathscr{B}} = \frac{63}{55} \approx 1.145.
\tag{4.10}
$$

Note that the first anomalous dimension has been known for quite some time [23].

5. Conclusion

Coming back to our example of the 2-point function we find for the lowest contributing mass m^Φ, i.e. the lightest glueball mass m^Φ with the quantum numbers of Φ,

$$
m_{\text{lattice}}^\Phi = -\lim_{x_0 \to \infty} \frac{1}{a} \ln \frac{\sum_{\mathbf{x}} \langle \Phi_{\text{RI}}(x_0 + a\hat{0}, \mathbf{x})\Phi_{\text{RI}}(0)\rangle_{\substack{\text{con} \\ \text{lattice}}}}{\sum_{\mathbf{x}} \langle \Phi_{\text{RI}}(x_0, \mathbf{x})\Phi_{\text{RI}}(0)\rangle_{\substack{\text{con} \\ \text{lattice}}}}
\tag{5.1}
$$

$$
\approx m_{\text{cont}}^\Phi - \frac{a^2}{2}\left(\bar{c}_1^{(0)}[2b_0\bar{g}^2(1/a)]^{0.636}\Delta_1 + \bar{c}_2^{(0)}[2b_0\bar{g}^2(1/a)]^{1.145}\Delta_2 + \dots \right) + \text{O}(a^4)
$$

where $\Delta_i = \langle \Phi_0 | \mathscr{B}_{i;\text{RGI}}(0) | \Phi_0 \rangle$ with zero-momentum ground state $|\Phi_0\rangle$ and $\bar{c}_i^{(0)}$ are the tree-level coefficients of the diagonalised basis, e.g. $\bar{c}_1^{(0)} = 1/48$, $\bar{c}_2^{(0)} = 1/12$ for the plaquette action [8]. As expected no dependence on the definition of Φ or the chosen renormalisation condition remains for the spectral quantity.

More generally, since $\bar{g}^2(1/a) \sim -1/\ln(a\Lambda)$ and $\hat{\gamma}_i^{\mathscr{B}} > 0$, the leading logarithmic corrections originating from the action accelerate the convergence of the continuum limit. With $\hat{\gamma}_1^{\mathscr{B}} < \hat{\gamma}_2^{\mathscr{B}}$ the leading contribution comes from the O(4) symmetric operator $\mathscr{B}_1$. In case of Symanzik improved actions one replaces $\bar{c}_i^{(0)} \to \bar{c}_i^{(n)}[\bar{g}^2(1/a)]^n$, where $n = 1$ means tree-level improvement.

This information suffices to back up the usually employed naive a^2 extrapolations in lattice Yang Mills theory for all spectral quantities. However, other quantities need additional information namely the matching coefficients and leading anomalous dimensions for the Υ_i. Especially the new sector of Gradient flow observables with flow time t, requires the inclusion of an additional operator, located at the boundary $t = 0$, in the Symanzik effective action of the 5-dimensional formulation [24].

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[2]For more details see [22].

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