PROCEEDINGS OF SCIENCE

Solar modulation of cosmic rays in a semi-analytical framework

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When Galactic cosmic rays enter the heliosphere they encounter the solar wind with its frozen-in magnetic field, which modulates the cosmic ray flux in the heliosphere up to rigidities as high as 40 GeV. In this work, we present a new and straightforward extension to the force-field model that takes into account charge sign dependent modulation due to drifts in the heliospheric magnetic field. We start by validating our semi-analytical approach via a comparison to a fully numerical model. Our model nicely reproduces the AMS-02 measurements and we find the strength of diffusion and drifts to be strongly correlated with the heliospheric tilt angle and magnitude of the magnetic field. We are able to predict the electron and positron fluxes beyond the range for which measurements by AMS-02 have been presented.

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1. Introduction

Upon entering the heliosphere, Galactic cosmic rays encounter the magnetised solar wind and are thus subject to a number of transport processes: advection with the wind, diffusion in the small-scale turbulent magnetic field, drifts due to variations of the large-scale field and adiabatic energy losses in the expanding flow. Together, these effects suppress the fluxes of cosmic rays at Earth compared to the interstellar fluxes. Collectively this process is referred to as solar modulation. (See Ref. [1] for a review.) It is lore that solar modulation is negligible above rigidities of $\sim 10\,\mathrm{GV}$, but with the small statistical and systematic errors of modern experiments this number is likely higher. AMS-02, for instance, has reported time variations of the proton and Helium fluxes which must be attributed to solar activity up to rigidities of $40\,\mathrm{GV}$ [2].

For the modelling of solar modulation, two approaches have been adopted in the literature: Numerical codes solve the transport equation for models of the heliosphere of varying sophistication and complexity [3, 4, 5, 6]. Such models have been successfully applied to time-dependent data, too (e.g. [7]). While such approaches have the potential to reproduce observations elsewhere in the heliosphere and thus provide a more global picture, the complexity comes at the price of a large number of in principle unknown parameters. These parameters need to be determined by fitting the models to various observables. As the input interstellar fluxes depend also on unknown parameters, running such global fits is prohibitively expensive.

Phenomenological studies of Galactic cosmic ray transport on the other hand oftentimes employ the classic force-field model of Gleeson and Axford [8]. The force-field model is conceptually simple and all the complexity of the heliosphere is condensed into only one parameter, which can be fit very easily. In addition, allowing for this electro-static potential to be time-dependent, some degree of correlation between solar activity and this parameter can be found. On the downside, the force-field model assumes a higher degree of symmetry and ignores transport processes that must be important, i.e. drifts. Most importantly, the force-field model has trouble reproducing the measured fluxes. One example is crossing of fluxes, e.g. the fluxes of Bartels rotations 2460 and 2476 cross at $4\,\mathrm{GeV}$ [9]. In the force-field model, fluxes modulated with different potentials differ at all energies and never cross.

A number of authors have tried to allow for more freedom while maintaining the simplicity of the force-field model, for instance by making the force-field potential rigidity-dependent [10, 11]. Here, we follow a different approach and present a systematic extension of the conventional force-field model. We start from the general transport equation in 2D that includes drifts and then simplify it–under a limited number of assumptions–to a force-field like structure. Our semi-analytical model allows modulating the fluxes at a very moderate computational cost. In order to justify our approximations and to check our result, we use our own finite-difference code. Our model contains two free parameters per time interval which we determine by fitting to the AMS-02 data. We further investigate temporal correlations with solar wind parameters and replace the free parameters by a linear model of tilt angle and field strength. Combining the semi-analytical model with the solar wind correlations allows reproducing the AMS-02 data and predicting electron and positron fluxes, for example. We have made an example script for the semi-analytical model available in both Python and C++ at https://git.rwth-aachen.de/kuhlenmarco/effmod-code.

2. Semi-analytical method

The propagation of cosmic rays in the heliosphere is generally described in terms of a transport equation,

$$\frac{\partial f}{\partial t} + \nabla \cdot (C\mathbf{V} f - K \cdot \nabla f) + \frac{1}{3p^2} \frac{\partial}{\partial p} \left(p^3 \mathbf{V} \cdot \nabla f \right) = q \, , \tag{2.1}$$

where f is the cosmic ray phase space density with momenta p measured in the fixed frame, C is the Compton-Getting factor, $\mathbf{V}$ denotes the solar wind velocity, K is the diffusion tensor, the third term on the left hand side describes the adiabatic losses in the expanding solar wind and q is a source term.

We start by assuming a steady state and follow the conventional force-field approach insofar as we ignore the adiabatic energy losses in the fixed frame [12]. In the absence of sources the streaming $(C\mathbf{V} f - K \cdot \nabla f)$ is thus divergence-free and its integral over an arbitrary surface must vanish. After some manipulations (see Ref. [13]) this leads to the partial differential equation for $\tilde{f} = \tilde{f}(r, p)$

$$\frac{\partial \tilde{f}}{\partial r} + \frac{p\tilde{V}}{3\tilde{K}_{rr}} \frac{\partial \tilde{f}}{\partial p} = -\frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}} \tilde{f} \, , \tag{2.2}$$

where tildes denote polar angle averages and $\tilde{v}_{gc,r}$ is the radial part of the average of the gradient curvature drift. The boundary condition is $\tilde{f}(R, p) = f_{\mathrm{LIS}}(p)$, R being the radius of the heliosphere. Note that the absorbing the non-trivial polar angle dependencies into the averaged quantities has significantly reduced the complexity.

We can solve eq. (2.2) using the method of characteristics,

$$\tilde{f}(r, p) = f_{\mathrm{LIS}}(p_{\mathrm{LIS}}(r, p)) \exp \left[-\int_0^r \mathrm{d}r' \, \frac{\tilde{v}_{gc,r}(r', p_{\mathrm{LIS}}(r', p))}{K_{rr}(r', p_{\mathrm{LIS}}(r', p)} \right] , \tag{2.3}$$

where $p_{\mathrm{LIS}}(r, p)$ is a solution of the initial value problem with $p_{\mathrm{LIS}}(R, p) = p$,

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \frac{p\tilde{V}}{3\tilde{K}_{rr}} \, . \tag{2.4}$$

3. Validation

In order to verify the validity of the approximations made in deriving Eqs. (2.3) and (2.4), we have solved the transport eq. (2.1) directly using our own finite-difference code. The particular heliospheric transport model adopted is closely emulating a model that has been successfully applied to data from the PAMELA experiment [7] with some simplifications.

We assume a radial solar wind between $\sim 400\,\mathrm{km\,s^{-1}}$ in the equatorial plane and $\sim 800\,\mathrm{km\,s^{-1}}$ in the polar region with the transition depending on the tilt angle α. For the magnetic field we adopt a simple Parker spiral [14] with a modification in the polar region [15, 16]. The diffusion coefficient is modeled as a softly broken power law going from a constant at low energies, motivated by theoretical calculations of the mean free path of electrons [17], to a power law index of 1.55 at high energies. For simplicity, the diffusion coefficients perpendicular to the magnetic field have the same energy dependence and are only rescaled by a relative factor $f_{\perp} = 0.02$. The drift coefficient is slightly modified with respect to the weak scattering case as needed to account for the small latitudinal gradients observed by Ulysses [18] as shown with numerical simulations [19].

As a first check of our semi-analytical model, we confirm that the adiabatic term always contributes less than 10% to the transport equation. Next, we check whether the fluxes modulated according to eqs. (2.3) and (2.4) agree with the results from the fully numerical code. For the parameters of the semi-analytical method (see Sec. 2), we adopt fiducial parametrisations that reflect those used in the numerical code. The diffusion coefficient is parametrised again as a softly broken power law in rigidity R (understood to be measured in GV),

$$\tilde{K}_{rr} = K_0 R^a \left(\frac{R^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c} , \tag{3.1}$$

with power law indices $a = 0$ and $b = 1.55$, $c = 3.5$ and a break rigidity $R_k = 0.28$ fixed to values obtained in previous studies [7]. The radial drift velocity is parametrised as

$$\tilde{v}_{gc,r} = \kappa_0 \frac{R}{3B_0} \frac{10\,R^2}{1 + 10\,R^2} . \tag{3.2}$$

The averaged radial solar wind obtains a momentum dependence due to the non-uniform distribution of cosmic rays in the heliosphere in the different polarity cycles. Motivated by results of our numerical simulation it will be modeled as a step function in momentum

$$\tilde{V} = V_0 (1 + \Delta V\, \theta(R - R_b)) . \tag{3.3}$$

While the rigidity R_b and the height of the step ΔV have to be fitted to data for different products of charge sign and magnetic field polarity, the normalization V_0 is degenerate with K_0 and κ_0 and has thus been fixed to $620\,\mathrm{km\,s^{-1}}$.

In Fig. 1 we show the results of fitting our semi-analytical model (eqs. (2.3) and (2.4)) to the results from the numerical code. The fit agrees with the numerical solution to within less than 10%. We find significantly better agreement than with the conventional force-field model.

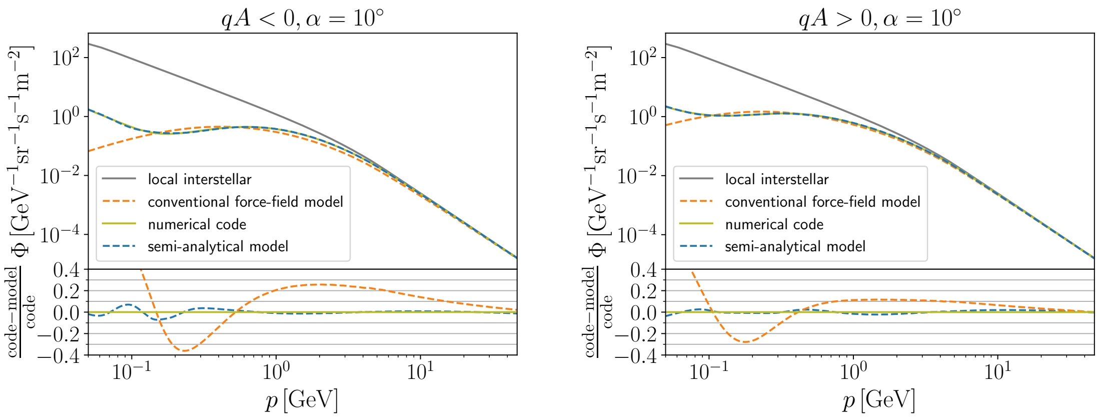

Figure 1: Fit of our two dimensional extension of the force field model to the fully numerical solution of the transport equation for a tilt angle of $\alpha = 10°$, for $qA < 0$ (left panel) and $qA > 0$ (right panel). In the lower panels the relative deviation from the numerical solution is shown.

4. Application to experimental data

In a first step, we determine for each time bin separately the free parameters that describe diffusion and drifts, K_0 and κ_0. The resulting modulated fluxes for one exemplary Bartels rotation is shown in Fig. 2 and compared to AMS-02 data. We also show the results from the conventional force-field model. In particular for the case of electrons the agreement with data is much better for the semi-analytical model than for the force field model.

One possible reason for remaining deviations of the experimental data from the model are features of the heliosphere on short time scales that were not taken into account such as coronal mass ejections that have been shown to influence the flux of cosmic rays in the heliosphere.

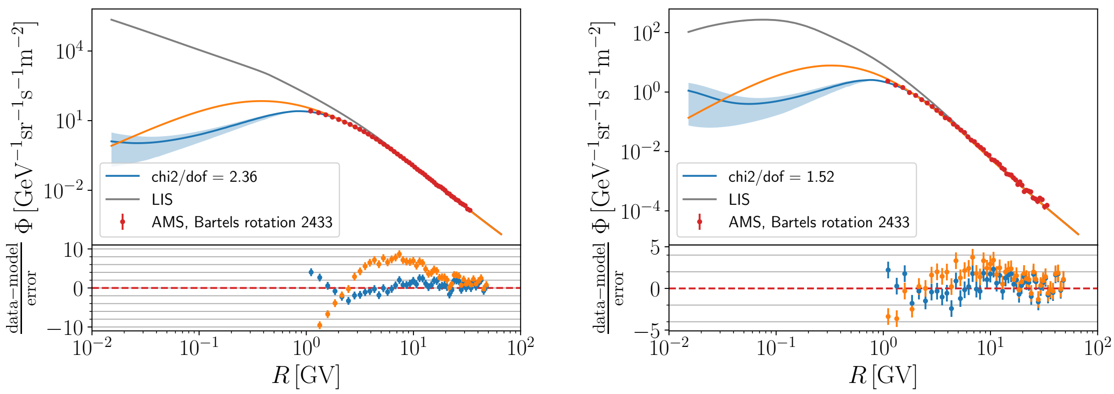

Figure 2: Fit of our semi-analytical model (blue line) to AMS-02 data [20] and the conventional force-field fit (orange line) for Bartels rotation 2433 for electrons (left panel) and positrons (right panel). In both cases the local interstellar spectrum is taken from Ref. [21]. The lower part of the plots shows the pull distribution for the fit. For the blue line the step in the solar wind is fixed while for the blue band it is varied between $\Delta V = 0$ and 0.4.

In order to predict modulated fluxes, we need to model the parameters K_0 and κ_0 as a function of time. Physically, we would expect them to be (anti-)correlated with solar wind parameters. For example, K_0, which parametrises the strength of diffusion, could be correlated with a proxy for solar activity, e.g. the tilt angle α while κ_0, which parametrises the relative strength of drifts, could be anti-correlated with the strength of the magnetic field, B. We stress that both the tilt angle as measured in the solar corona and the field strength as measured by ACE are relatively local observables and due to the particles spending a finite time in the heliosphere, we would expect the fitted parameters to be affected only with a certain delay and after averaging over time.

We have therefore looked for linear correlations between the K_0 and κ_0 fitted to AMS-02 data and the title angle and magnetic field strength using a moving average of width ΔT, $\langle\alpha\rangle_{\Delta T}$ and $\langle B\rangle_{\Delta T}$. We find the largest correlation (Pearson correlation coefficient of ~ 0.9) between K_0 and $\langle\alpha\rangle_{\Delta T}$ for widths of 25 and 45 Bartels rotations for electrons and positrons, respectively. For κ_0, we find an anti-correlation (~ -0.75) for widths of about 5 Bartels rotations. We have noticed that after the solar maximum κ_0 needs to rise faster than the field strength falls. Therefore we also propose a model that is split at the solar maximum and consists of one linear model before the polarity change and one linear model with different coefficients after.

As shown in Fig. 3, the electron fluxes measured by AMS-02 are nicely reproduced for all energies in both the simple linear model and the two interval linear model. We find that we can also reproduce the electron fluxes measured by PAMELA [22] that do not overlap with any of the AMS-02 measurements. In addition we can use this parametrization to predict the electron spectrum beyond the range AMS-02 data has been published so far.

In Fig. 3 we also show the positron-electron ratio from our model and compare it to measurements by AMS-02 and PAMELA. While the overall trend is reproduced, the modelled positron ratio falls too sharply around early 2016 in the one interval linear model. This can be traced back to the fact that the one interval linear model for κ_0 does not work as well for positrons as it does for electrons. In the two interval linear model the AMS-02 data is reproduced well.

5. Conclusion

We have presented a two dimensional semi-analytical model that incorporates charge sign dependent modulation effects due to gradient curvature drifts in the solar magnetic field. We have validated our model by comparing to the results of our finite-difference code and found that it is significantly more accurate than the conventional force field model. Our model has four free parameters, two of which encode the strength of diffusion and drifts. Two additional parameters are necessary to capture the momentum dependence of the averaged solar wind (see Sec. 3). We note that the latter two are only relevant below $\sim 0.3\,\mathrm{GeV}$ while fits to experimental data, even from PAMELA, are not very sensitive to this energy range.

It is remarkable that allowing the two free parameters to vary per time interval we can fit the time-dependent AMS-02 electron and positron fluxes rather accurately. Introducing in addition a linear model relating these two parameters to solar wind parameters, we can reproduce not only the AMS-02 measurements, but also the PAMELA electrons fluxes which do not overlap with the AMS-02 electron fluxes. We can also predict the electron and positron fluxes beyond the range for which data has been published.

We stress that our method is significantly faster than fully numerical methods and more accurate than the standard force field model. It can thus be applied to large datasets with fine time binning like the current AMS-02 data.

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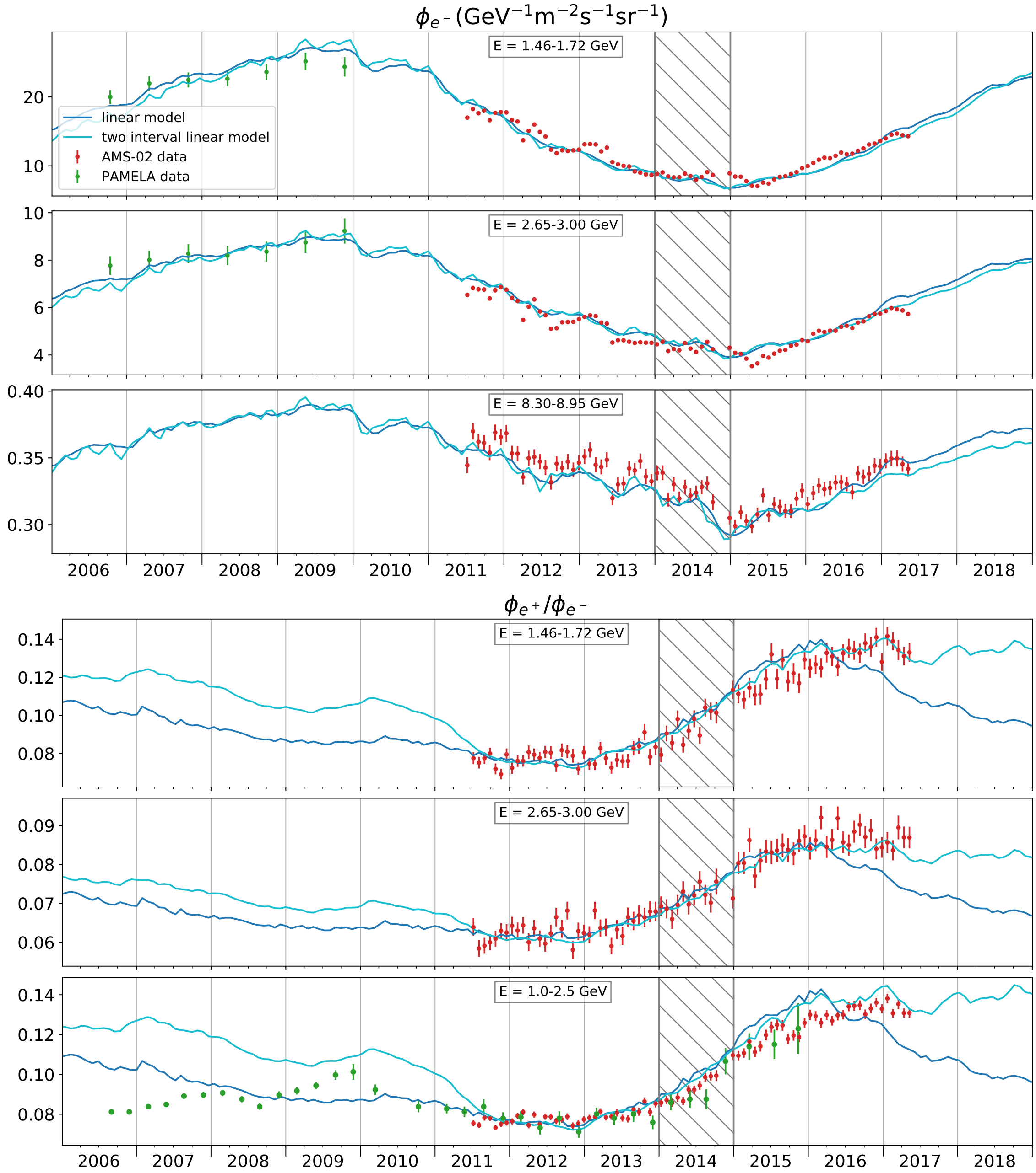

Figure 3: The upper plot shows the time dependent electron flux calculated both in our linear model (blue line) and in the two interval linear model (light blue line). The lower plot shows the positron ratio as a function of time compared to AMS-02 data (red dots). The last panel shows an average over multiple bins in order to compare to PAMELA data [23] (green dots) with wider energy bins. The grey lines show the transition region around the solar maximum between the two different linear models.

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