

$G^{\epsilon\epsilon}$ Lab's Equivalence Principle Experiment

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Baron Roland von Eötvös performed amazing experiments on the equivalence of inertial and gravitational mass. Since his work experiments have become progressively more refined. The $G^{\epsilon\epsilon}$ Lab at Washington University in St. Louis has built a new experiment in the hopes of refining these tests even further. We have operated a prototype of this experiment by continuously monitoring the angular orientation of a torsion balance for over 115 days and the results we have obtained are promising. The experience we have gained from this experiment suggest the need for improved thermal and magnetic shielding; it also gives us confidence that long-period torsion balances have the ability to significantly improve the bounds on violation of the Equivalence Principle. Here we describe our instrument and how these experiences will be used to improve our next generation torsion balance.

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1. Equivalence Principle Status & Motivation

In 1922 the culmination of Baron Roland von Eötvös's work on the Equivalence Principle (EP) was published in the famous "EPF" paper [1]. The experiments described in that paper were truly revolutionary and marked the first modern tests of the EP. From the time of Eötvös's experiments, searchers for EP violation have become progressively more refined, with the most sensitive test to date coming from the MicroSCOPE satellite experiment [2]. The work presented here describes a new experiment testing the EP, developed by the *G^{EE}* Lab at Washington University in St. Louis, MO, USA.

Since Eötvös's experiments, tests of the EP have evolved to explore all the different versions of the EP. Eötvös's experiments, and similar ones that are the most sensitive to date, have tested the "Weak Equivalence Principle" or universality of free fall. This version of the EP states that all non-gravitational energy falls at the same rate in a gravitational field, it is mute on the dynamics of bodies possessing a significant amount of gravitational self-energy. The "Strong Equivalence Principle" (SEP) states that even all masses possessing large amounts of gravitational self-energy fall at the same rate. The most systematic test to date of the Strong Equivalence Principle has been conducted by the "Lunar Laser Ranging" (LLR) experiment [3], first proposed by James Faller [4]. Recent measurements of a triple star system have complemented those of the LLR experiment. The system has two white dwarfs and one pulsar. A.M. Archibald et al. [5] analyzed the timing observations of the pulses coming from the neutron star over a six-year period to show that the relative accelerations of the white dwarfs and the neutron star varied by no more than a fraction $\sim 2.6 \times 10^{-6}$ of their mean accelerations or $\eta_{\text{SEP}} \sim 1.7 \times 10^{-5}$. Despite the myriad technical challenges, tests of the Strong Equivalence Principle are extremely interesting. However, in this manuscript there isn't space to discuss these experiments, we will focus on the Weak Equivalence Principle which is a field rich with its own set of well-motivated experiments; with this in mind going forward when referencing the Equivalence Principle (EP), we will be referring to the Weak Equivalence Principle.

1.1 Types of WEP tests

It is an exciting time to search for violations of the EP. In addition to the torsion-balance tests pioneered by Eötvös, there are a plethora of new approaches to searching for EP violations. The first experiment to discuss is the MicroSCOPE satellite which tracked the differential acceleration of two test bodies, one titanium and one platinum, as they freely fell in Earth's gravitational field; they found that the acceleration of the two bodies differed at most by a part in 10^{15} of their mean acceleration $\sim 7.9 \text{ m s}^{-2}$. While this experiment is exquisitely sensitive, there is motivation to test the EP on shorter length scales. To this end, a number of tests have been conducted using atomic interferometers [6, 7, 8, 9]. While these tests have not quite reached the sensitivity of MicroSCOPE or torsion balance tests, they are intriguing as they will be able to probe the quantum aspects of the EP. Two good examples of this are the AEGIS experiment [10] and CERN experiments on anti-hydrogen [11] which will explore whether or not antimatter obeys the EP.

Finally, there are a number of modern torsion balance experiments that are building on Eötvös's legacy with increasingly sensitive limits on EP violation. The experiment most closely following Eötvös's work is the one being led by the group at the Wigner Institute in Hungary, which is a

modern recreation of Eötvös's experiment using a replica of an instrument that Eötvös himself used [12, 13]. Currently the most sensitive torsion balance experiment has been conducted by the Eöt-Wash group at the University of Washington, USA. They used a rotating torsion balance to study the differential acceleration of various materials in the Earth's gravitational field; their most sensitive constraint on EP violation came from tests using beryllium and titanium test bodies [14]. Their work is complemented by the work of a group at Huazhong University of Science and Technology, China who also used a rotating torsion balance [15]. There is yet another set of torsion balance EP experiments which operate in the "Dicke/Braginsky" mode, where the differential acceleration of tests masses toward the Sun is measured. One of these is located at Tata Institute for Fundamental Research (TIFR) in India [16] and the other is the primary subject of this manuscript.

Having discussed the types of EP experiments, we can move on to discuss how their results are parameterized and how their sensitivity has progressed since Eötvös's pioneering work. Violations of the EP are typically characterized in two different ways. The first can be thought of as the parameterization of a new force with coupling strength α_{12} and range λ ,

$$V(r) = \frac{Gm_1m_2}{r} \left(1 + \alpha_{12}e^{-r/\lambda}\right). \quad (1.1)$$

The second pertains to the level at which General Relativity is violated, it has been aptly named the Eötvös parameter,

$$\eta_{1,2} = 2 \frac{a_1 - a_2}{a_1 + a_2} = 2 \frac{(m_g/m_i)_1 - (m_g/m_i)_2}{(m_g/m_1)_1 + (m_g/m_1)_2}. \quad (1.2)$$

These parameterizations are consistent with one another, so given the context of this proceedings we will use the Eötvös parameter.

Since the publication of the EPF paper in 1922, bounds on violation of the EP have progressively become more stringent. Table 1 shows this progress as a function of time.

Eötvös - 1922 [1]	$\eta < 4 \times 10^{-9}$
Princeton - 1964 [17]	$\eta = [1.3 \pm 1.0] \times 10^{-11}$
Moscow - 1972 [18]	$\eta = [0.3 \pm 0.9] \times 10^{-12}$
Eöt-Wash - 2012 [14]	$\eta = [-0.7 \pm 1.3] \times 10^{-13}$
Huazhong - 2018 [15]	$\eta = [-1.2 \pm 2.8(\text{stat}) \pm 3.0(\text{syst})] \times 10^{-13}$
MicroSCOPE - 2017 [2]	$\eta = [-1 \pm 9(\text{stat}) \pm 9(\text{syst})] \times 10^{-15}$

Table 1: The improving bounds on violation of Einstein's Equivalence Principle are shown.

With this in mind, G^{EE} Lab hopes to push forward and test the EP at the level of $\eta \sim 5 \times 10^{-14}$.

1.2 Motivation for going forward

Since Fischbach's reanalysis of the data in the EPF paper [19], there has been a strong interest in improving the sensitivity of EP tests. This motivated a number of the experiments that did produce the more stringent results discussed above. Since then Damour showed that any quantum theory of gravity will violate the EP, and the level of this violation might be just beyond that of modern tests [20]. In addition to these, a number of recent cosmological observations are currently

unexplained. The ‘‘Dark Energy’’ that comprises $\sim 75\%$ of the energy budget of the universe is still a major mystery. Its existence was deduced by measuring the distances to high-redshift type IA supernova [21] and it was quantified by fitting the CMB power spectrum with Λ -CDM model [22]. While these two types of measurement have agreed when it comes to Dark Energy, they have recently run into significant tension when it comes to measurements of Hubble’s constant, H_0 . At present the most accepted explanations of Dark Energy lie with models that predict one or more EP violating fields (e.g. [23]) and the model that best relieves the tension in H_0 measurements is one that predicts a violation of the EP [24]. These cosmological conundrums, coupled with Damour’s postulate provide significant excitement for pursuing more advanced tests of the EP.

2. G^{EE} Lab’s Instrument

Our instrument is a Dicke/Braginsky-style torsion balance which is passively isolated from spurious environmental signals. Figure 1 shows a line drawing of the instrument with most of its subsystems labeled. These systems are described in the following sections.

2.1 Torsion Balance

Figure 2 displays the design of the torsion balance which follows the classic design concepts developed by Dicke [17] and Braginsky [18]. The bob of the balance has a four-fold azimuthal mass symmetry, with one 14.33 g mass attached to each of the four tines radiating from a central point forming a cross. Two of the masses are circular mirrors, essentially made of quartz, mostly SiO_2 . Two aluminum discs of equal mass attached at the opposite ends counterbalance the quartz mirrors. Because the density of quartz is slightly smaller than that of aluminum, the quartz masses have a smaller diameter (in construction a special jig is used to ensure that the center of mass of each disc is the same distance from the center of the cross). Thus the balance bob acts as a compositional dipole with its axis directed along a line bisecting the angle between the two tines holding the quartz masses. With a tine length of 25 cm, the moment of inertia of the entire assembly is $3.75 \times 10^{-3} \text{ kg m}^2$. The suspension fiber is made of tungsten, which has a low internal dissipation or equivalently a high Q . With a diameter of $\sim 18 \mu\text{m}$, the bob assembly weighing 72 g loads it to about 70% of its limiting strength. The quartz and aluminum masses together weigh 57.33 g and thus $\sim 75\%$ of the mass of the bob is active and couples to any torques arising from the violation of the EP. The length of the suspension fiber is about 1.6 m leading to a torsional constant $k_f \sim 9 \times 10^{-10} \text{ N m rad}^{-1}$ and a period of torsional oscillations of $\sim 1.27 \times 10^4 \text{ s}$, or slightly more than three and a half hours.

The torsion balance is suspended near the Earth’s surface. In such a configuration it is subject to the gravitational fields of the Sun and the Galactic Dark Matter. For such an instrument an EP violation due to the interaction between the balance and a source S would produce a torque on the

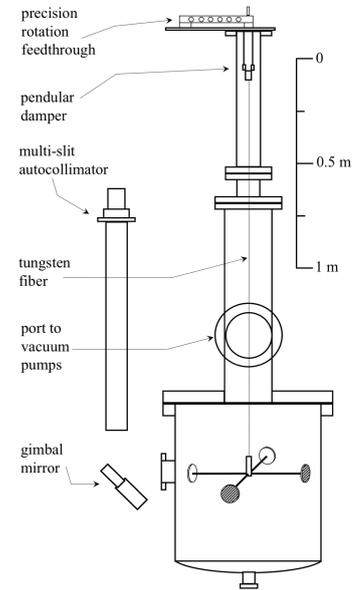


Figure 1: A sketch of the long-period torsion balance with various components labeled is shown.

balance given by

$$\tau(t) = \left[\left(\frac{m_g}{m_i} \right)_1 - \left(\frac{m_g}{m_i} \right)_2 \right] \frac{GM_s m_b}{R^2} r \cos \theta(t) \cos \phi(t), \quad (2.1)$$

where θ and ϕ are the angles in Figure 2, R is the distance from the balance to the source, M_s is the mass of the Sun or any other distant source, m_b is the mass of one of the four masses attached to the end of the tines, and r is $\sqrt{2}$ times the length of the balance's tines.

This design scheme has been chosen primarily to reduce spurious gravitational effects. The mass symmetry of the torsion balance significantly reduces its coupling to gradients in the gravitational field; these gradients are currently the limiting factor in the Eöt-Wash [14] experiments. Moreover, because the gravitational source is the sun and the torsion balance is not moving, any gravitational gradients present in the lab would be stationary. These stationary gradients cause mean position of the torsion balance to change, but they do not induce a rotation at the frequency of the source, meaning that they do not cause a spurious signal. It has been pointed out that the gravitational field of the environment could undergo gradual changes. These changes are at frequencies well below that the signal, and thus can be filtered out in the data analysis procedure.

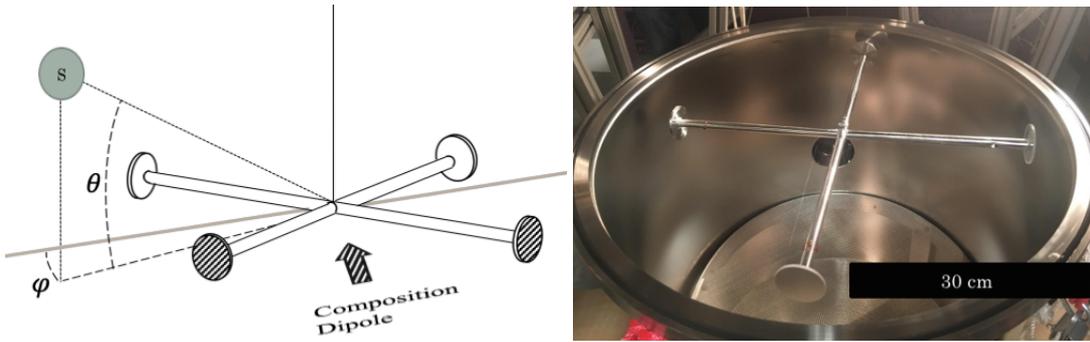


Figure 2: The left panel shows a sketch of the long-period torsion balance and a label indicating the direction of the composition dipole. In the gravitational field of a source, S the balance will experience torque given by Equation 2.1. The right panel is a photo of the balance after it has been suspended, before the vacuum chamber is closed.

2.2 Subsystems

Soon after suspension the torsion balance will oscillate with a large amplitude. We have developed a couple of mechanisms to damp the amplitude of this motion into a regime that is suitable for doing the experiment.

2.2.1 Torsion Fiber Connections

For the first stage of damping we have designed and fabricated a picomotor rotary feedthrough system to which the tungsten fiber suspending the balance is attached. By operating the picomotor the mean position of the balance may be adjusted to center it in the view of the autocollimator. By a sequence of to and fro movement the torsional amplitude of the balance bob can be reduced to $\sim 10^{-6}$ rad as follows. A lever arm, driven by a picomotor, is attached to a steel rod which feeds into the vacuum chamber and suspends the fiber. Gross manual adjustments may be made with

this feedthrough when the torsional amplitude is large. The picomotor is then used to control the angular orientation of the balance down to $\sim 10^{-7}$ rad for fine adjustments when the amplitude is sufficiently small. Well timed adjustments are made when the balance is at a turning point in its torsional mode and most of the balance's energy is stored in the fiber as potential energy. Twisting the steel rod either manually or with the picomotor in the direction of the balance's rotation removes energy from the system and reduces the amplitude.

From cross couplings in the various modes of the balance bob, pendular oscillations are capable of feeding energy into the torsional modes. To address this, the feedthrough also features an assembly similar to those used in [25, 15] which damps pendular oscillations. A copper plate attached to the fiber is suspended between two ceramic ring magnets such that as the balance bob swings the copper plate will follow a similar motion. From this motion eddy currents are induced in the plate and remove energy from the system as they dissipate, damping the pendular oscillations.

2.2.2 Magnetic Induction Control System

The second and final stage of damping is accomplished with a magnetic induction control system. The system consists of two pairs of Helmholtz coils which are perpendicular to each other, each placed around the torsion balance. The current in the two coils are out of phase by $\pi/2$, creating time-varying magnetic fields. Coil A produces a changing magnetic flux through the metallic test mass which induces a voltage, which in turn drives a current. This current interacting with the magnetic field of coil B generates a torque that drives the torsion balance. If rotation in the opposite direction is desired, the phase relationship between the two Helmholtz coils is reversed, and a torque on the balance in the opposite direction is produced. By manipulating these phase reversals appropriately the balance oscillation amplitude may be reduced to very low values.

2.3 Autocollimating Optical Lever

Monitoring the angular orientation of the torsion balance is a challenge because the amplitude of individual balance oscillations can be quite small, yet over the long period which data is taken the mean position of the torsion balance can drift significantly. To address this, a multi-slit autocollimating optical lever (AOL) was developed specifically for this experiment [26]. The AOL views one of the mirrors of the balance via a gimbal-mounted mirror secured to the vacuum chamber. Previous tests of the AOL show that if isolated from environmental disturbances, it is sensitive to angular deflections on the order of

$$\Delta\theta = 2 \times 10^{-10} (0.01\text{Hz}/\nu) \text{rad Hz}^{-1/2}, \text{ for } \nu < 10^{-2} \text{ Hz} \quad (2.2)$$

while having a dynamic range of $\sim 10^6$, making it adequate for this experiment.

2.4 Isolation

Executing a sensitive search for EP violation requires that the experiment be carried out in an environment free of external disturbance. While there are many examples of this, perhaps the state of the art is the remote lab at Gauribidanaur that houses the TIFR EP experiment [27, 16]. Like other experiments, G^{EE} Lab's is operated in a remote lab which is located within Tyson Research Center, a protected area 35 km outside St. Louis, Missouri. The instrument is located inside an old

ammunition bunker (with thick walls of concrete) that is partially buried underground. The bunker isolates the instrument reasonably from the variations in temperature, winds and other disturbances. This is illustrated in Figure 3 that shows the low noise level present on the floor of the bunker lab.

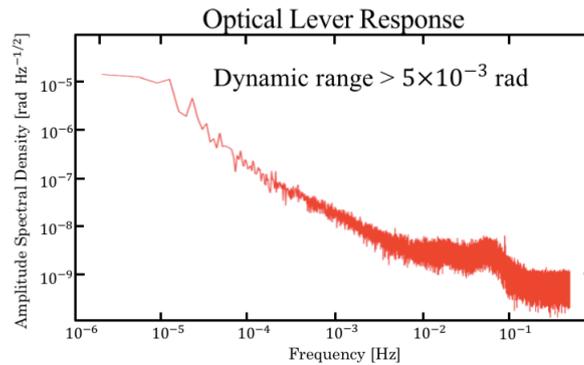


Figure 3: The angular spectral density of the signal obtained from observing a stationary mirror with the autocollimator on the floor of the Tyson bunker. An amplitude of 10^{-5} rad $\text{Hz}^{-1/2}$ at the diurnal signal frequency $f_s = 1.157 \times 10^{-5}$ sets the limiting resolution of the instrument in terms of a measurement of an equivalence principle violation. Adapted from [28].

The large thermal mass of the bunker walls creates a passive thermal control system that significantly damps the temperature fluctuations. Figure 4 shows the temperatures outside and inside the lab. While the temperature outside the lab fluctuates as much as 10 K, the magnitude of the diurnal temperature fluctuation inside the lab is at the mK level.

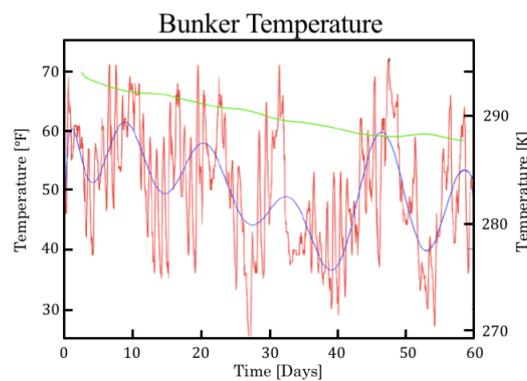


Figure 4: Temperature during the Fall of 2015. The temperature reported at Lambert-Saint Louis International airport is plotted in red, while a high-order polynomial fit to this data is plotted in blue to show low frequency trends. The temperature inside the Tyson bunker is shown in green. Adapted from [28].

To further isolate the instrument, it is suspended inside a vacuum chamber maintained by a vibration-free ion pump at $\sim 5 \times 10^{-9}$ Torr. This assembly is enclosed by a 4-inch layer of Styrofoam, which in turn is surrounded by another similar layer. The second layer is not in contact with the chamber or AOL, thus reduces the effects of air currents within the bunker, besides providing further thermal insulation. The temperature is monitored by six thermistors, Figure 5 shows the average diurnal wave is below ~ 5 millidegree Celsius before (green) and after (blue) the installation of the styrofoam.

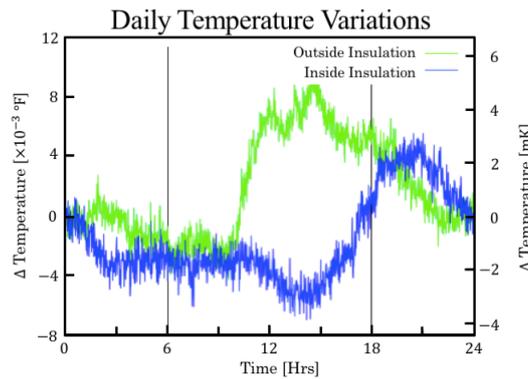


Figure 5: Averaged daily temperature variation with a linear drift removed plotted for 60 days worth of data in the fall of 2015 (in green) and the winter 2016 (in blue). While the general trends for each of these data runs are similar, the thermal insulation in place for the winter 2016 data run seems to introduce a phase shift of about 6 hours. This coincides closely with the times of expected signal maxima from a potential equivalence principle violation, which are marked by vertical lines at $t = 6$ and 18 hours. Adapted from [28].

3. Initial Data

With the previously described systems in place, we commenced a long period of data acquisition on December 22, 2017. With the AOL we sampled the angular position of the balance twice every second until June 10, 2018. Not all these data were useful. It took about three weeks for the balance to settle down, despite continual damping of the torsional oscillations with the rotary feedthrough from which it was suspended; there were also other disturbances such as seismic events and thunderstorms that excited the balance. Accordingly, only the 115 days of data acquired at the end of this period were used. The initial result of these data is shown in the power spectrum of Figure 6.

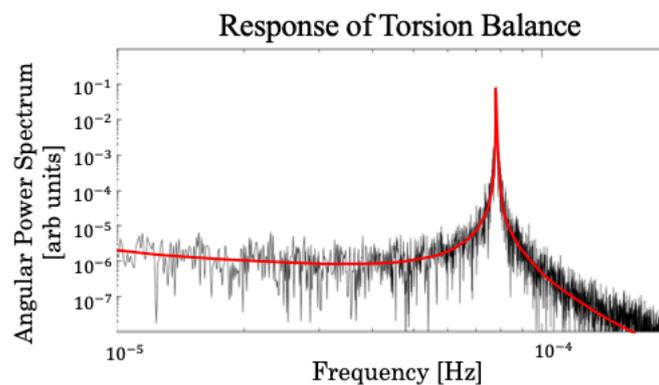


Figure 6: An angular power spectrum of the full data set is displayed. The response function with a quality factor of $Q \approx 700$ is overlaid on the data.

The observations were fit well with the response function of a damped harmonic oscillator of $Q \approx 700$. This value was determined by choosing the parameters which best fit the data in the region immediately around the natural frequency of the torsion balance. The very long, continuous

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data series allowed us to sample these low frequencies with much higher fidelity than in the other experiments. The Q value quoted here is significantly lower than the values commonly quoted in the literature (see for example [14]). However, we note that we are operating at frequencies 15 times lower than those in most of the other experiments. Thus even with about 120 days of data (10^7 s) the observational bandwidth is 10^{-7} Hz, while the natural frequency of the balance is $\sim 7 \times 10^{-5}$ Hz. Accordingly, this value should be considered a lower bound on the Q of this system; this data set is consistent with much higher Q values.

4. Looking Forward

In order to push our results further, we have investigated various aspects of our balance and the cross correlations with a variety of environmental parameters. First, we note that the eddy-current based pendular damper did not function very efficiently. Among the cross correlations the strongest was the correlation with large seismic events and a very weak correlation with ambient temperature and pressure variations. The set of six thermistors mounted on the instrument did not show a pronounced peak at the diurnal frequency down to ~ 1 millidegree Celsius at $\sim 10^{-5}$ Hz; the thermal insulation was sufficient for this experiment. The atmospheric pressure variations showed a broad feature of low amplitude around $\sim 10^{-5}$ Hz, and the cross correlation amplitudes were very low. As the frequency of the EP-violating sidereal torque is highly stable to $\sim 2 \times 10^{-4}$, its effects can be extracted from a broad distribution of background induced by pressure and other variations. These small correlations, coupled with the fact that Dicke/Braginsky torsion balances have minimal coupling to gravitational gradients (the limiting factor in rotating torsion balance experiments), leads us to believe that long-period torsion balances of this type are promising. Keeping in mind that the instrument is a prototype, its preliminary operation achieved an exciting data set, and that improvements to the experimental environment are underway, we are confident that our coming experiments will improve the sensitivity of EP violation searches.

Acknowledgements

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