Global fits of the SM parameters

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I present a global survey of weak mixing angle measurements and other precision experiments and discuss the issue of correlations of the theoretical uncertainties in precision observables. Hadronic vacuum polarization effects complicate global fits in various ways and are also covered alongside some representative fit results.

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1. Introduction

I will start by presenting a survey of measurements of the weak mixing angle, $\sin^2\theta_W$, as its accurate determination is becoming an ever more global endeavor. But why are we pushing the precision further and further? One can compute and measure $\sin^2\theta_W$ and relate it to the W boson mass, M_W. Therefore, one has three ways of obtaining it, yielding a doubly over-constrained system at sub-per mille precision. And since this system involves relations between couplings and masses of the Standard Model (SM) particles, this is *the* key test of the electroweak symmetry breaking sector. Moreover, comparisons of measurements at different scales or between different initial or final states will provide a window to physics beyond the SM that would remain closed if one would have only one kind of determination, even if that one would be extremely precise. Thus, a global analysis is important.

There are many approaches to measure $\sin^2\theta_W$. One is to tune to the Z resonance, where one can measure forward-backward (FB) or left-right (LR) asymmetries (the latter if one has at least one polarized beam) in e^+e^- annihilation around the Z boson mass, M_Z. Or one can reverse initial and final states and measure the FB asymmetry in pp or $p\bar{p}$ Drell-Yan annihilation in a larger window around M_Z.

A very different route is to move to lower energies [1], and to consider purely weak processes. Using neutrinos which do not know about any other interaction, one can measure the weak mixing angle cleanly in νe scattering, even though the cross sections are small and therefore statistical uncertainties rather large [2]. One can move to the deep inelastic scattering (DIS) regime, where scattering occurs to first approximation off individual quarks, as has been done in experiments such as NuTeV [3] at Fermilab. And very recently the process called Coherent Elastic Neutrino Nucleus Scattering (CEνNS) as has been observed for the first time by the COHERENT Collaboration [4] at Oak Ridge National Laboratory (ORNL).

An alternative strategy to eliminate the electromagnetic interaction is to perform experiments in polarized and therefore parity-violating electron scattering (PVES) [5], measuring tiny cross section asymmetries between left-handed and right-handed polarized initial states,

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \ . \tag{1.1}$$

Just as for the neutrino case, one may consider a purely leptonic process, specifically polarized Møller scattering, $\vec{e}^-e^- \to e^-e^-$ [6, 7]. And again one can scatter deep inelastically (here sometimes called eDIS), but there is an important difference to νDIS mentioned in the previous paragraph. Because of the small cross sections in neutrino scattering one needs large nuclei, which leads to complications from nuclear physics interfering with the unambiguous interpretation of such experiments (see *e.g.* Ref. [8]), while in eDIS it is possible to take a target nucleus as small and simple as the deuteron. Using the 6 GeV CEBAF electron beam, this has been done by the PVDIS Collaboration [9] at Jefferson Laboratory (JLab). In fact, polarized eDIS was the key process to establish the SM in 1979 [10], and a high-precision measurement will be possible with the SoLID detector [11] at the upgraded CEBAF to 12 GeV. The PVES analog of CEνNS on a proton target has been completed very recently by JLab's Qweak Collaboration [12], using the nominal 6 GeV CEBAF beam at a lower energy of $E_e = 1.165$ GeV. This provided the first direct measurement of the weak charge of the proton [13], $Q_W(p)$. The future P2 experiment [14] at the MESA

facility under construction at the Johannes Gutenberg University (JGU) at Mainz, will reduce the error in $Q_W(p)$ by a factor of three. In addition, P2 may also run using a ^{12}C target which is a very interesting nucleus, because it is spherical and iso-scalar and has therefore only one nuclear form factor. Thus, $Q_W(^{12}C)$ would be easier to interpret, especially if form factor effects can be constrained by additional run time at larger momentum transfer Q^2. With this at hand, one would be able to disentangle the weak charges of the proton and the neutron, and consequently the effective vector couplings of the up and down quarks to the Z boson, from PVES alone.

Another new player in this context are isotope ratios in atomic parity violation (APV). Now, APV in a single isotope is a traditional way to address the neutral-current weak interaction, and has been studied successfully in heavy alkali atoms [15]. But one faces atomic physics complications, because one needs to understand the atomic structure in heavy nuclei from sophisticated many-body calculations [16, 17] to a few per mille accuracy. If, on the other hand, one considers isotope ratios, then most of the atomic physics effects cancel. The first measurement of APV as a function of isotope number has been achieved by the group of Dima Budker at the JGU Mainz [18] just a few months ago, where the weak charges of Ytterbium showed the expected isotope dependence.

2. The weak mixing angle

Figure 1 displays all the $\sin^2 \theta_W$ determination with sub-percent precision. The first part are the LEP and SLC measurements in $e^+ e^-$ annihilation near M_Z [19], yielding the combined result,

$$\sin^2 \theta_W (\text{LEP}) = 0.23153 \pm 0.00016 . \tag{2.1}$$

Note, that there has been a change in the $\sin^2 \theta_W$ extraction from the FB asymmetry for $b\bar{b}$ final states by the LEP Collaborations, as two years ago the two-loop QCD correction necessary to extract the pole asymmetry has been obtained with the finite b quark mass dependence [20], reducing the largest LEP discrepancy with the SM by about a quarter of a standard deviation.

Another change affected the extraction from the APV measurement in ^{133}Cs [15], for which the Stark vector transition polarizability needs to be known. The latter has been re-measured [21] very recently, shifting $|Q_W(^{133}\text{Cs})|$ which was 1.4 σ lower than what was expected from the SM much closer to the prediction (see the lower part of Figure 1).

As for hadron colliders, the leptonic FB asymmetry measurements at the Tevatron combine to the value [22],

$$\sin^2 \theta_W (\text{Tevatron}) = 0.23148 \pm 0.00033 . \tag{2.2}$$

We averaged the measurements at the LHC by the ALTAS [23], CMS [24], and LHCb Collaborations [25],

$$\sin^2 \theta_W (\text{LHC}) = 0.23131 \pm 0.00033 , \tag{2.3}$$

by assuming that the smallest theory uncertainty (± 0.00025 for ATLAS) is common to all three detectors [26]. This treatment should be understood as conservative, because the more forward directed kinematics at the LHCb on the one hand, and the more central kinematics at the general purpose detectors, ATLAS and CMS, on the other hand, may in fact provide valuable complementary information on parton distribution functions (PDFs) which when taken into account is likely to eventually reduce the uncertainty in the combination. Since rather different aspects of the PDFs are
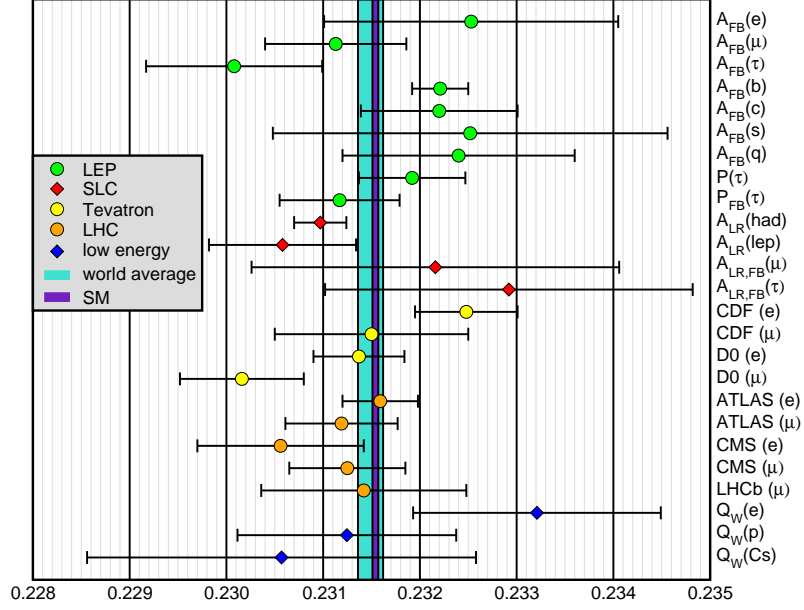
Figure 1: Determinations of the *effective* weak mixing angle, $\sin^2 \theta_W$, entering leptonic Z vector couplings.

necessary for the extraction of $\sin^2 \theta_W$ at $p\bar{p}$ colliders compared to those underlying the symmetric pp initial states at the LHC which is based on rapidity distributions, the corresponding uncertainties can be assumed to be approximately uncorrelated, and we arrive at the world average,

$$\sin^2 \theta_W (\text{world average}) = 0.23149 \pm 0.00013 \ . \tag{2.4}$$

This is in excellent agreement with the result from a global fit to all data,

$$\sin^2 \theta_W (\text{global fit}) = 0.23153 \pm 0.00004 \ . \tag{2.5}$$

3. The W boson mass

Figure 2 shows a similar comparison of M_W determinations. In contrast to the case of $\sin^2 \theta_W$, one observes better mutual agreement among the various measurements at LEP [27], the Tevatron [28], and by ATLAS [29], but their average,

$$M_W (\text{world average}) = 80.379 \pm 0.012 \ \text{GeV} \ , \tag{3.1}$$

is 1.6 σ higher than the indirect determination of all data excluding the direct measurement results of the mass and total width of the W boson,

$$M_W (\text{indirect}) = 80.357 \pm 0.006 \ \text{GeV} \ , \tag{3.2}$$

and 1.5 σ higher than the SM prediction,

$$M_W (\text{global fit}) = 80.361 \pm 0.005 \ \text{GeV} \ . \tag{3.3}$$

The indirect and global fit results for M_W and $\sin^2 \theta_W$ account not only for theoretical uncertainties but also include an estimate and implementation of theoretical correlations [26], which at the achieved level of precision are no longer negligible.
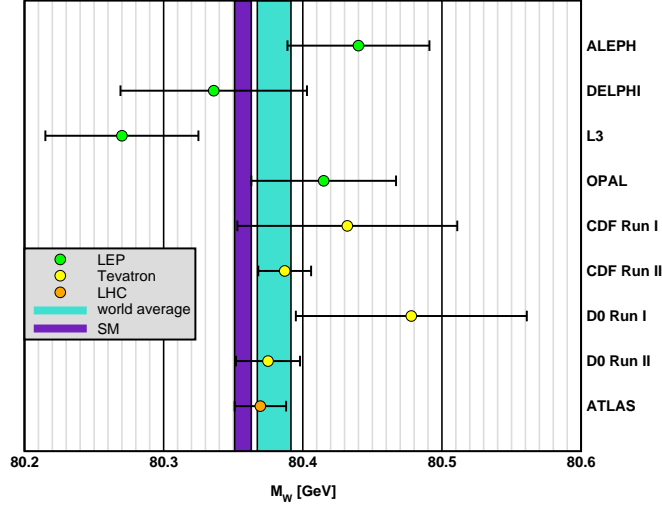
Figure 2: Survey of W boson mass measurements.

4. Theoretical uncertainties and the Higgs boson mass

Indeed, there are various kinds of theory errors entering global fits. For example, there are the hadronic vacuum polarization and light-by-light contributions obstructing a clean and unambiguous determination of the anomalous magnetic moment of the muon, a_μ. There are non-factorizable QCD corrections entering the hadronic Z width [30], as well as non-resonant corrections to the Breit-Wigner shape of the Z resonance, but the most important ones are from unknown higher order contributions to the W and Z boson self-energies. These uncertainties can be estimated by considering the expansion parameters involved. Including the SM fermion content of three full generations as an enhancement factor, these are

$$\frac{3\alpha_s}{\pi} \approx 0.116 \, , \qquad \frac{8\alpha}{\pi} \approx 0.020 \, , \tag{4.1}$$

for QCD and QED, and

$$\frac{3\alpha}{\pi \sin^2 \theta_W} \approx 0.032 \, , \qquad \frac{3 - 6\sin^2 \theta_W + 8\sin^4 \theta_W}{\pi \sin^2 \theta_W \cos^2 \theta_W} \, \alpha \approx 0.029 \, , \tag{4.2}$$

for the charged and neutral current interactions, respectively. For the numerical estimates we evaluated the couplings in the $\overline{\text{MS}}$ scheme at the W boson mass scale, where we have $\alpha^{-1} \approx 128$, $\alpha_s \approx 0.121$, and $\sin^2 \theta_W \approx 0.2311$. Other possible enhancements can arise through the eigenvalue of the quadratic Casimir operator in the adjoint representation in QCD, $C_A = 3$, and through $\hat{m}_t^2 / M_W^2 \approx 4$ (for the $\overline{\text{MS}}$ top quark mass) effects. We translate these loop factors into uncertainties in the so-called oblique parameters $S = S_Z$, T, and $U = S_W - S_Z$ [31], which have been originally introduced to parameterize potential new physics contributions to electroweak radiative corrections. Here, S_W describes the difference of W boson self-energies at the W scale and at very low energies, while T refers to the difference of W and Z boson propagator effects at the electroweak scale, and thus violates weak isospin. Since S_Z is defined in analogy to S_W and receives similar corrections, we rather assume their difference U as a third *uncorrelated* uncertainty parameter. Denoting
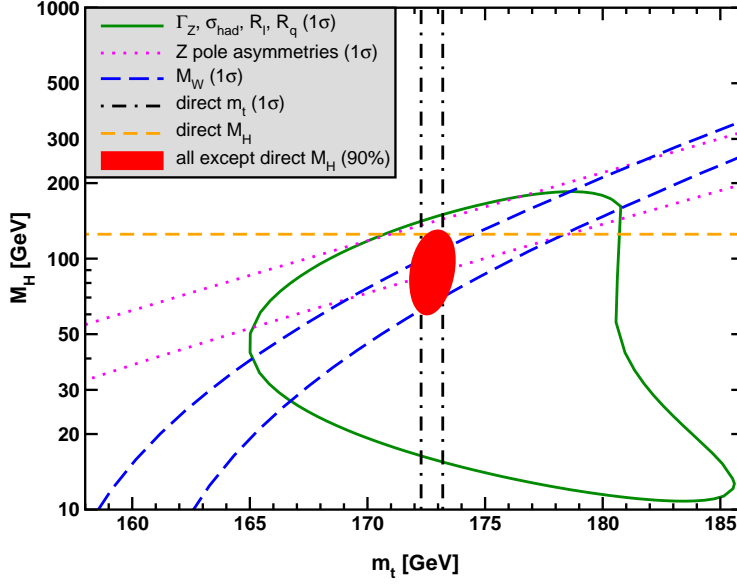
Figure 3: Higgs boson mass *vs.* top quark pole mass for various sets of observables [32].

these uncertainty parameters by ΔS_Z, ΔT and ΔU, and assuming them to be sufficiently different (uncorrelated) *induces* theory correlations between different observables. We find $\Delta S_Z = \pm 0.0034$ (which in practice could be added in quadrature to the hadronic vacuum polarization uncertainty entering the evaluation of the electromagnetic coupling from the Thomson limit to the Z scale), $\Delta T = \pm 0.0073$ from uncertainties associated with the *tb* doublet, and $\Delta U = \pm 0.0051$ [26].

Figure 3 reconfirms that M_W is a little deviant from the SM prediction, while $\sin^2 \theta_W$ is in very good agreement. Indeed, one can determine the top quark mass by means of global fits to all data except for m_t from the Tevatron and the LHC, and including (excluding) theory uncertainties one obtains[1],

$$m_t(\text{indirect}) = 176.5 \pm 1.9 \, (1.8) \text{ GeV}. \tag{4.3}$$

This represents a 1.8 (1.9) σ larger value than the direct measurement [26],

$$m_t(\text{direct}) = 172.90 \pm 0.47 \text{ GeV}. \tag{4.4}$$

Similarly, one can perform global fits to all data except for the direct M_H constraint from the LHC,

$$M_H(\text{excluding theory uncertainties}) = 90^{+17}_{-15} \text{ GeV}, \tag{4.5}$$

$$M_H(\text{including correlated theory uncertainties}) = 91^{+18}_{-16} \text{ GeV}, \tag{4.6}$$

showing an only slightly increased central value and uncertainty and a reduced tension (from 1.9 σ to 1.8 σ) with the directly measured value [26],

$$M_H(\text{LHC}) = 125.10 \pm 0.14 \text{ GeV}, \tag{4.7}$$

once theory uncertainties are included.

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[1]Note, that due to incremental updates some of the numerical results presented here may differ very slightly from those presented at the conference.

5. Vacuum polarization in global fits

I now turn to recent activities in the application of vacuum polarization to global fits. Firstly, one needs the electromagnetic coupling at the Z peak, $\alpha(M_Z)$, to predict M_W and $\sin^2\theta_W$. To this end, three groups [34, 35, 36] have analyzed hadron production data in e^+e^- annihilation, and in some cases also τ decay spectral functions which by approximate isospin symmetry yield additional information on the former. Or one can use perturbation theory for at least part of the calculation, and only rely on data in the hadronic region up to about 2 GeV or so (a quantity we call $\Delta\alpha_{\text{had}}^{(3)}(2 \text{ GeV})$), and then use the renormalization group equation [37, 38] — more precisely the anomalous dimension of the photon field — which depends on the strong coupling α_s, and the charm and bottom quark $\overline{\text{MS}}$ masses, $\hat{m}_c$ and $\hat{m}_b$. The results (for the on-shell definition of α) are,

$$\alpha^{-1}(M_Z) = 128.947 \pm 0.012 \text{ [34]}, \tag{5.1}$$

$$\alpha^{-1}(M_Z) = 128.958 \pm 0.016 \text{ [35]}, \tag{5.2}$$

$$\alpha^{-1}(M_Z) = 128.946 \pm 0.015 \text{ [36]}, \tag{5.3}$$

$$\alpha^{-1}(M_Z) = 128.949 \pm 0.010 \text{ [38]}, \tag{5.4}$$

where the various authors use slightly different input values for $\alpha_s(M_Z)$, but this amounts to differences below the level of 0.004 in $\alpha^{-1}(M_Z)$.

Interestingly, the data used for the hadronic part also enter other observables present in global electroweak fits, which induces another source of uncertainty correlation. For example, these data play a crucial role in the evaluation of the SM prediction of a_μ, where they enter first at the two-loop level and generate a correlation with $\alpha(M_Z)$, and both are in turn anti-correlated with the three-loop vacuum polarization contribution to a_μ. Finally, there is a correlation with the quantity $\sin^2\theta_W(0) - \sin^2\theta_W(M_Z)$.

Because the muon mass scale is rather low, most of the evaluation of the hadronic vacuum polarization contribution to a_μ needs to be based on data. However, there is a fraction that can be computed perturbatively. In particular, the heavy quark contributions are fully accessible in perturbation theory [39], which for the charm and bottom contributions yields,

$$a_\mu^c(\text{vacuum polarization; PQCD}) = (14.6 \pm 0.5_{\text{PQCD}} \pm 0.2_{\hat{m}_c} \pm 0.1_{\alpha_s})10^{-10}, \tag{5.5}$$

$$a_\mu^b(\text{vacuum polarization; PQCD}) = 0.3 \times 10^{-10}, \tag{5.6}$$

where the errors in Equation (5.5) are from the truncation of the perturbative series at order α_s^2, and the parametric uncertainties in $\hat{m}_c(\hat{m}_c)$ (taken from Ref. [40]) and from α_s. Equation (5.5) is in excellent agreement with the very recent result obtained from a lattice gauge theory simulation [41],

$$a_\mu^c(\text{vacuum polarization; lattice}) = (14.66 \pm 0.45_{\text{stat.}} \pm 0.06_{\text{syst.}})10^{-10}, \tag{5.7}$$

and is of similar precision. Since the bottom quark introduces a rather different scale, its effect is much harder to study on the lattice, while the perturbative method is even more reliable here, yielding a negligible error. It should also be remarked that Ref. [41] finds a rather large total hadronic vacuum polarization effect, so that if confirmed, there would cease to be a conflict between the measurement of a_μ [42] and the SM, which currently amounts to more than 3 σ. But then there would be a new discrepancy between the dispersive and lattice approaches to vacuum polarization.
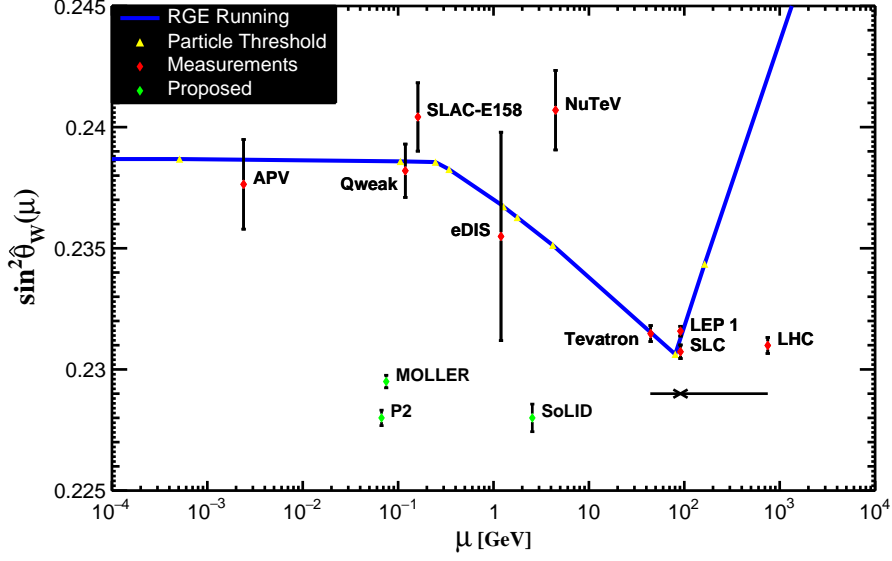
Figure 4: Renormalization group evolution (running) of the weak mixing angle (updated from Ref. [38]).

An important quantity where vacuum polarization enters is $\sin^2\theta_W(0)$. It is needed for many low-energy electroweak observables, and it can be seen from Figure 4 that future experiments in low momentum transfer PVES will be at the precision level of the LEP and SLC measurements. As mentioned earlier, the interpretation of NuTeV and other νDIS experiments suffers from poorly understood and potentially large nuclear effects, and should probably be best removed from this plot which ought to illustrate the experimental verification of the running of $\sin^2\theta_W$.

There are basically three parts in the computation of $\sin^2\theta_W(0)$. The largest piece can be obtained by solving the coupled system of differential equations for the $\overline{\text{MS}}$ quantities $\hat\alpha$ and $\sin^2\hat\theta_W$,

$$\mu^2\frac{d\hat\alpha}{d\mu^2} = \frac{\hat\alpha^2}{\pi}\left[\frac{1}{24}\sum_i K_i\gamma_i Q_i^2 + \sigma\left(\sum_q Q_q\right)^2\right], \tag{5.8}$$

$$\mu^2\frac{d\hat{v}_f}{d\mu^2} = \frac{\hat\alpha Q_f}{24\pi}\left[\sum_i K_i\gamma_i\hat{v}_i Q_i + 12\sigma\left(\sum_q Q_q\right)\left(\sum_q \hat{v}_q\right)\right], \tag{5.9}$$

where $\hat{v}_f = T_f - 2Q_f\sin^2\hat\theta_W$ is the Z boson vector coupling to fermion f, and T_f is the third component of weak isospin of fermion f. The sums are over all active particles in the relevant energy range. The Q_i are the electric charges, while the γ_i are constants depending on the field type [43]. For quarks, K_i is proportional to the color factor $N_C = 3$ and contains QCD corrections up to and including $\mathcal{O}(\alpha_s^4)$ [38], while for both leptons and bosons $K_i = N_C = 1$. The terms involving σ arise due to QCD annihilation (singlet) diagrams with purely gluonic sub-diagrams. We can relate the renormalization group equation for $\hat\alpha$ to that for $\sin^2\hat\theta_W$ since both, the γZ mixing tensor $\Pi_{\gamma Z}$ and the photon vacuum polarization function $\Pi_{\gamma\gamma}$, are pure vector-current correlators. As already mentioned, the remaining non-perturbative part needs the same kind of data that entered the calculations of $\alpha(M_Z)$ to obtain Equations (5.1)–(5.4). This part needs to be subdivided into two pieces
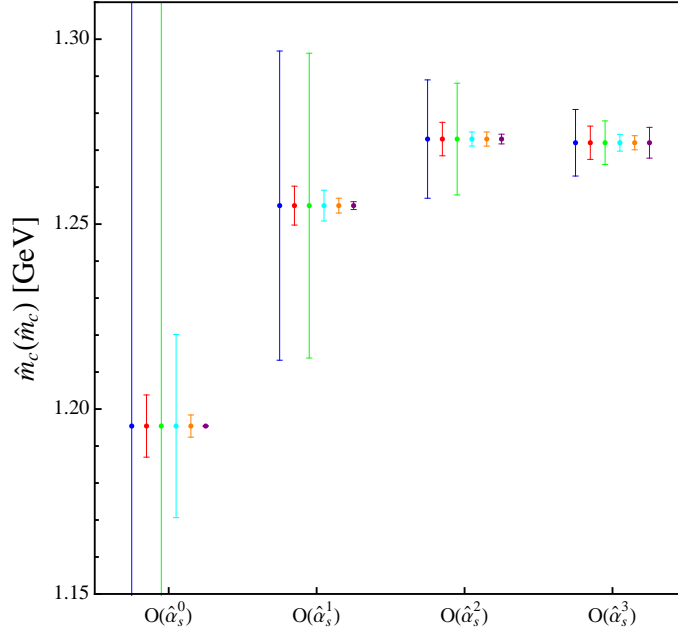
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Figure 5: Error budget for the sum rule analysis for $\hat{m}_c(\hat{m}_c)$ as a function of loop order [40].

because the vector couplings of the Z boson differ from the electric charges, implying that there is a piece that is not directly related to $\alpha(M_Z)$ and necessitating a study of the effect and uncertainty associated with the corresponding flavor separation into up- and down-type quarks. One also needs to estimate the uncertainties from the singlet piece and from isospin breaking effects. The overall uncertainty is negligible compared to any upcoming low-energy determination of $\sin^2\theta_W$ in the foreseeable future. It is interesting that if one were to compute $\sin^2\theta_W(0)$ directly rather than relative to $\sin^2\theta_W(M_Z)$, the hadronic vacuum polarization data would enter twice, and their errors would *add*. This is because

$$\sin^2\theta_W(0) = \hat{\kappa}(0)\sin^2\theta_W(M_Z), \tag{5.10}$$

and the correction factor $\hat{\kappa}(0)$ increases with larger e^+e^- hadronic cross sections, and $\sin^2\theta_W(M_Z)$ itself is proportional to $\alpha(M_Z)$ which can be seen by recalling that

$$M_Z^2 \propto g_Z^2(M_Z)v^2 \propto \frac{\alpha(M_Z)}{\sin^2\theta_W(M_Z)\cos^2\theta_W(M_Z)}G_F^{-1}, \tag{5.11}$$

and solving for $\sin^2\theta_W(M_Z)$ (v is the Higgs boson vacuum expectation value and G_F is the Fermi constant).

The final application of vacuum polarization are charm and bottom quark mass determinations. If one employs as input quantities only the electronic decay widths of the narrow resonances, *e.g.*, the J/ψ and the $\psi(2S)$ in the case of charm, and compares two different moments of the relevant vacuum polarization function, one obtains simultaneous information on the quark mass and the less well-known continuum contribution. The constraint on the latter can then be compared with the corresponding moments of the experimental determinations of electro-production of open charm or bottom. This results in over-constrained systems, where any residual difference can be

M_Z	91.1884 ± 0.0020 GeV	1.00	−0.06	0.00	0.00	0.02	0.03
$\hat{m}_t(\hat{m}_t)$	163.28 ± 0.44 GeV	−0.06	1.00	0.00	−0.13	−0.28	0.03
$\hat{m}_b(\hat{m}_b)$	4.180 ± 0.021 GeV	0.00	0.00	1.00	0.00	0.00	0.00
$\hat{m}_c(\hat{m}_c)$	1.275 ± 0.009 GeV	0.00	−0.13	0.00	1.00	0.45	0.00
$\alpha_s(M_Z)$	0.1187 ± 0.0016	0.02	−0.28	0.00	0.45	1.00	−0.02
$\Delta\alpha_{\text{had}}^{(3)}(2$ GeV	0.00590 ± 0.00005	0.03	0.03	0.00	0.00	−0.02	1.00

Table 1: Standard global fit [32], where the correlation with M_H is Equation (4.7) is negligible.

taken as an error estimate [40, 44] of non-perturbative effects, which are supposedly small but possibly not entirely negligible. This strategy has been applied to $\hat{m}_c$ resulting in the precision determination [40],

$$\hat{m}_c(\hat{m}_c) = 1272 \pm 8 + 2616[\alpha_s(M_Z) - 0.1182] \text{ MeV}, \tag{5.12}$$

where the central value is in very good agreement with recent lattice results [45] and of similar precision. The breakdown of the error in this approach is detailed in Figure 5.

6. Results and conclusions

Table 1 together with Equation (4.7) summarizes the SM global fit [32]. Two simple examples are to suffice to illustrate the application of global fits to constraints on physics beyond the SM.

One is the ρ_0 fit, in which one assumes that the new physics is mainly affecting the ρ parameter, which is a measure of the neutral-to-charged current interaction strengths. Various quantities constrain ρ_0 and there are different ways to interpret it. For example, any electroweak doublet with a non-trivial mass splitting, $\Delta m_i^2 \geq (m_1 - m_2)^2$, contributes to ρ_0 positive definitely,

$$\Delta\rho_0 = \sum_i \frac{G_F N_C}{8\sqrt{2}\pi^2} \Delta m_i^2 \ . \tag{6.1}$$

This equation might appear to suggest that there is no decoupling, so that even a doublet with grand unification or Planck scale masses but electroweak size splitting may give observable effects in experiments at much lower energies. However, this is not the case, as there is a see-saw type suppression of Δm_i^2 in any given model. Another way of saying this is to recall that the leading contribution to ρ_0 in the SM effective field theory is a combination of dimension six operators, so that these effects are suppressed on dimensional grounds by at least two powers of the scale of new physics. The global fit yields [32],

$$\rho_0 = 1.00039 \pm 0.00019, \tag{6.2}$$

which is 2 σ higher than the SM value, $\rho_0 \equiv 1$. This is another manifestation of the tension in the W boson mass discussed earlier. It is amusing to point out that at face value, one even finds a non-trivial 95% CL *lower* bound on the sum of all such mass splittings,

$$(16 \text{ GeV})^2 \leq \sum_i \frac{N_C}{3} \Delta m_i^2 \leq (48 \text{ GeV})^2 \qquad (90\% \text{ CL}). \tag{6.3}$$
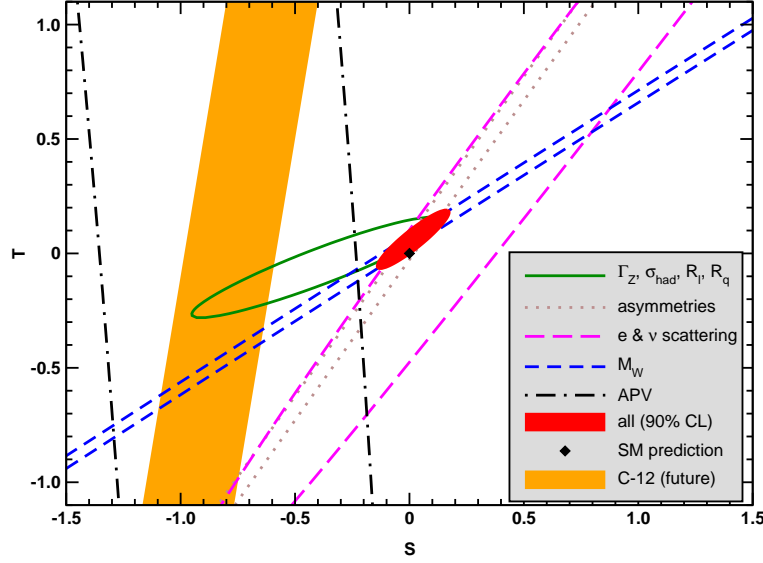
Figure 6: *T versus S* for various data sets (modified from Ref. [32]). For illustration, I also show the impact that the ^{12}C PVES measurement would have if it could be performed with a relative error of 0.3%, dominated by the polarization uncertainty (within the SM another measurement of $\sin^2 \theta_W$ to which the polarization asymmetry is directly proportional). As can be seen, this yields a different slope in the *ST*-plane.

This strongly disfavors, for example, zero hypercharge, $Y = 0$, Higgs triplets for which $\rho_0 < 1$. On the other hand, a Higgs triplet with $|Y| = 1$ is consistent with the data provided its vacuum expectation value is around 1% of that of the SM doublet.

The other example is the fit result [32] for the *S* and *T* parameters [31],

$$S = 0.02 \pm 0.07, \tag{6.4}$$

$$T = 0.06 \pm 0.06, \tag{6.5}$$

with a correlation of 81%. This fit is illustrated in Figure 6 and fixes $U = 0$, as U is expected to be suppressed by two additional factors of the new physics scale compared to *S* and *T* [46]. It is remarkable that the χ^2 value at the minimum of the fit drops by 4.2 units if these two extra degrees of freedom are allowed. This kind of improvement is non-trivial and quite rare, and is again related to M_W. One can interpret the *S* and *T* parameters in a variety of new physics models, if one assumes that non-oblique effects are absent or small. For example, the mass of the lightest Kaluza-Klein state [47] in warped extra dimensions [48] should satisfy the bound $M_{KK} \gtrsim 3.2$ TeV [32], while the lightest vector state in minimal composite Higgs models [49] is bound by $M_V \gtrsim 4$ TeV [32].

To conclude, both, the LHC and low-energy measurements are approaching LEP and SLC precision in $\sin^2 \theta_W$. There are new players represented by COHERENT [4], Qweak [12], and APV isotope ratios [18], which are not quite as precise yet, but they are first measurements with great prospects of improvement. As for today, with the lower precision with which the results of these first measurements came in, it is more interesting to assume the validity of the SM, and use them to constrain neutron skins (the difference of the neutron and proton radii in nuclei), or more generally form factor effects.

For ultra-high precision tests, not only theoretical uncertainties from unknown higher orders need to be estimated and included, but also their correlations across various observables, even though this is a difficult task. As examples, I discussed uncertainties in the *W* and *Z* boson self-energies due to unknown higher orders, as well as hadronic vacuum polarization effects and their uncertainties which enter correlated in an increasing number of quantities.

Acknowledgements

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