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The rare $H \to q_i q_j$ decays revisited

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We revisit the rare decays of the Higgs boson into two different quarks at the one-loop level in the Standard Model. We use the GIM mechanism along with Taylor expansions of the decay amplitudes in order to get rid of spurious terms contained in the form factors. We found that $\mathrm{Br}(H \to uc)$=1.63×10^{-18}, $\mathrm{Br}(H \to ds)$=9.07×10^{-15}, $\mathrm{Br}(H \to db)$=1.03×10^{-8} and $\mathrm{Br}(H \to sb)$=2.44×10^{-7}. In particular, our predictions for the $H \to uc, ds$ decays disagree with previous results in the literature.

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1. Introduction

The scalar boson observed in the LHC is compatible with that predicted by the Standard Model (SM) [1, 2], where by means of the Higgs Mechanism is responsible for providing mass to the rest of the known elementary particles. Some interesting properties of the Higgs boson could be related to flavor violation since the SM does not predict at the tree level the existence of flavor changing neutral currents with quarks ($\bar{q}_i q_j H$). Nevertheless, the SM allows this type of couplings through quantum fluctuations at the one-loop level, where such couplings can be studied through the $H \to q_i q_j$ decays, explicitly, $H \to uc, ds, db, sb$.

So far, in the SM these decays have not been deeply studied [3]. Therefore, we revisit and recalculate them in a different approach, where we carefully perform Taylor expansions to the form factors of the decay amplitudes in order to apply the GIM mechanism [4], as a consequence, we find new predictions for two decay modes.

2. The $H \to u_i u_j$ decay

The Higgs decay into two distinct up quarks $H \to u_i u_j$ is described by the Feynman diagrams depicted in Fig. 1. It should be noted that $u_i u_j = uc$ is the only possible channel, where inside the loops circulate the three down quarks $d_k = d_1, d_2, d_3 = d, s, b$. In this sense, the resulting amplitude for the $H \to u_i u_j$ decay can be written as

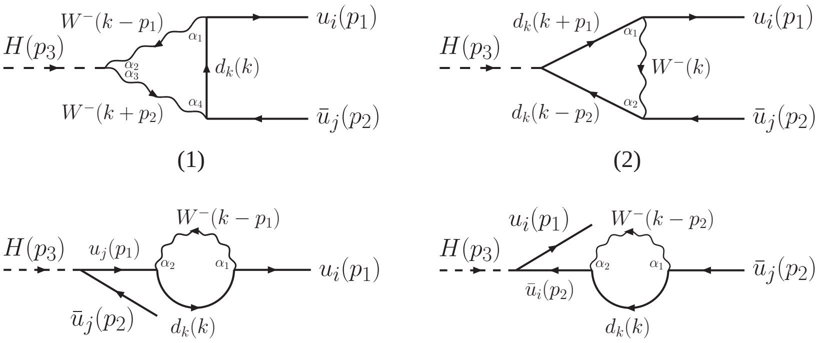

Figure 1: Decay $H \to u_i u_j$, with $u_i u_j = uc$ and $d_k = d, s, b$.

$$\mathscr{M} = \bar{u}(p_1)\left(F_1 + F_2\gamma^5\right) v(p_2)\ , \tag{2.1}$$

where the form factors $F_{1,2}$ have the following generic structure

$$\begin{aligned} F \;=\; \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k}\, [\, & f_{A_1} A_0(1) + f_{A_2} A_0(2) + f_{B_1} B_0(1) + f_{B_2} B_0(2) \\ & + f_{B_3} B_0(3) + f_{B_4} B_0(4) + f_{C_1} C_0(1) + f_{C_2} C_0(2)]\,. \end{aligned} \tag{2.2}$$

In specific, they depend on the Passarino-Veltman scalar functions A_0, B_0, C_0 and on the functions $f_{A1}, \ldots, f_{C2}$ that are given in terms of the masses of the particles. At this stage, the amplitude is ultraviolet divergent (UV) because there still remains the UV pole $\varepsilon_{\rm UV}^{-1}$ coming from A_0 and B_0:

$$\mathscr{M} \sim -\sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} \frac{1}{\varepsilon_{\rm UV}} \frac{i g^3 m_H^2}{256\pi^2 m_W^3} \bar{u}(p_1) \left[(m_{u_i} + m_{u_j}) - (m_{u_i} - m_{u_j})\gamma^5 \right] v(p_2) \,, \tag{2.3}$$

but this can be removed by virtue of the GIM mechanism, which for $H \to u_i u_j$ satisfies

$$\sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} = V_{u_i d} V^*_{u_j d} + V_{u_i s} V^*_{u_j s} + V_{u_i b} V^*_{u_j b} = 0 \,, \tag{2.4}$$

this eliminates any term independent of the m_{d_k} mass. Besides, in order to strictly apply such mechanism we must be able to split the f functions, in Eq. (2.2), into its dependent part of the m_{d_k} mass and the independent one, namely,

$$\begin{aligned} F \; &= \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} \left[f(m_{d_k}) + g(\cancel{m_{d_k}}) \right] = \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} f(m_{d_k}) \\ &= V_{u_i d} V^*_{u_j d} f(m_d) + V_{u_i s} V^*_{u_j s} f(m_s) + V_{u_i b} V^*_{u_j b} f(m_b) \,. \end{aligned} \tag{2.5}$$

To achieve this, we must properly expand the f functions by using Taylor expansions. In this way, the $H \to uc$ receives contributions from all the down quarks $d_k = d,s,b$. Therefore, by taking into account that $m_H > m_W \gg m_u$, m_c, m_d, m_s, m_b, we can expand with respect to m_{u_i}, m_{u_j} and m_{d_k}, which leads to

$$F_{1,2} = \sum_{k=1}^{3} V_{u_i d_k} V^*_{u_j d_k} f_{1,2}(m_{d_k}) \,, \tag{2.6}$$

$$f_{1,2}(m_{d_k}) = \frac{\pm i g^3}{256\pi^2} \frac{m_{u_i} \pm m_{u_j}}{m_W} \frac{\mathscr{F}(r_1)}{1 - r_1} \frac{m_{d_k}^2}{m_W^2} \,, \tag{2.7}$$

$$\begin{aligned} \mathscr{F}(r_1) \;\equiv\; & (r_1 - 1)[2(\beta_1 \log 2 + \beta_1 r_1 \log 4 - 6 r_1 + 4) + \pi^2 \left(2 r_1^2 + r_1 - 1/3\right) - 2 i\pi (4 r_1 - 1)] \\ & + \{-2(\beta_1 + 1) + 2 r_1 [\beta_1 (2 r_1 - 1) + 3] - 2 i\pi (r_1 - 1) r_1 (4 r_1 + 1)\} \mathrm{l}_1 \\ & - (r_1 - 1)[r_1 (4 r_1 + 1) \mathrm{l}_1^2 + 2\beta_1 (2 r_1 + 1) \mathrm{l}_2] - 2(2 r_1 - 1)[-\beta_1 + (\beta_1 - 3) r_1 + 1] \mathrm{l}_3 \\ & + (2 r_1 - 1)\{2(r_1 - 1)[(\beta_1 - 1)\mathrm{l}_4 - (\beta_1 + 1)\mathrm{l}_5] + 2[-\beta_1 + (\beta_1 + 3) r_1 - 1]\mathrm{l}_6\} \\ & - 2(r_1 - 1)[r_1 (4 r_1 + 1)\mathrm{L}_1 - \left(2 r_1^2 - r_1 + 1\right)(\mathrm{L}_2 - \mathrm{L}_3 + \mathrm{L}_4 - \mathrm{L}_5 + \mathrm{L}_6)]. \end{aligned} \tag{2.8}$$

Here $r_1 \equiv m_W^2/m_H^2$, $\beta_1 \equiv \sqrt{1 - 4 r_1}$, being $\mathrm{l}_{1,\ldots,6}$ and $\mathrm{L}_{1,\ldots,6}$ the logarithms and dilogarithms, respectively, which are listed in the Appendix.

3. The $H \to d_i d_j$ decays

The Higgs decays into two different down quarks receive virtual contributions coming from the three up quarks $u_k = u,c,t$, which can be seen in Fig. 2. The amplitude of the $H \to d_i d_j$ decay (with $d_i d_j = ds, db, sb$) is analogous to that of the $H \to u_i u_j$ decay; the new amplitude can be found by exchanging $u_i \to d_i$, $u_j \to d_j$, $W^- \to W^+$ and $V_{u_i d_k} V^*_{u_j d_k} \to V^*_{u_k d_i} V_{u_k d_j}$.

For the case of the $H \to d_i d_j$ process, the GIM mechanism imposes that

$$\sum_{k=1}^{3} V^*_{u_k d_i} V_{u_k d_j} = V^*_{ud_i} V_{ud_j} + V^*_{cd_i} V_{cd_j} + V^*_{td_i} V_{td_j} = 0 \,, \tag{3.1}$$

which eliminates the UV part of the amplitude. But unlike the $H \to u_i u_j$ case, for the $H \to d_i d_j$ decays exist two different mass hierarchy scenarios for the form factors, in consequence, this requires two different Taylor expansion schemes, which will be explained below:

- For the virtual contribution of the u and c quarks, where $m_H > m_W \gg m_{d_i}$, m_{d_j}, m_u, m_c, the expansion is analogous to that implemented in the $H \to u_i u_j$ decay. In this case, its form factors can be expanded with respect to m_{d_i}, m_{d_j} and $m_{u_k} = m_{u_1}, m_{u_2} = m_u, m_c$, hence the result is also analogous to Eq. (2.6):

$$F_{1,2} = \sum_{k=1}^{2} V^*_{u_k d_i} V_{u_k d_j} f_{1,2}(m_{u_k}) = V^*_{ud_i} V_{ud_j} f_{1,2}(m_u) + V^*_{cd_i} V_{cd_j} f_{1,2}(m_c) \,, \tag{3.2}$$

$$f_{1,2}(m_{u_k}) = \frac{\pm i g^3}{256\pi^2} \frac{m_{d_i} \pm m_{d_j}}{m_W} \frac{\mathscr{F}(r_1)}{1 - r_1} \frac{m^2_{u_k}}{m^2_W} \,, \tag{3.3}$$

  where $\mathscr{F}(r_1)$ is defined just as in Eq. (2.8).

- For the virtual contribution of the t quark, where $m_t > m_H > m_W \gg m_{d_i}$, m_{d_j}, the expansion can only be performed with respect to m_{d_i} and m_{d_j}, but not for $m_{u_k} = m_{u_3} = m_t$, this yields to

$$F_{1,2} = \sum_{k=3} V^*_{u_k d_i} V_{u_k d_j} f_{1,2}(m_{u_k}) = V^*_{td_i} V_{td_j} f_{1,2}(m_t) \,, \tag{3.4}$$

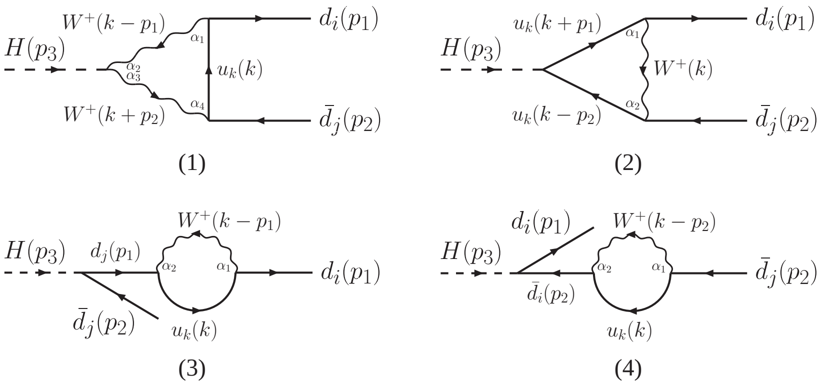

Figure 2: Decays $H \to d_i d_j$, with $d_i d_j = ds, db, sb$ and $u_k = u, c, t$.

$$f_{1,2}(m_{u_k}) = \frac{\pm i g^3}{128\pi^2} \frac{m_{d_i} \pm m_{d_j}}{m_W} \mathscr{F}(r_1, r_2) \frac{m^2_{u_k}}{m^2_W} \,, \tag{3.5}$$

$$\begin{aligned}
\mathscr{F}(r_1,r_2) \equiv\ & 2(r_1+r_2-1)+\mathrm{l}_1-\mathrm{l}_7+\beta_1(2r_1+1)\mathrm{l}_8-\beta_2(4r_1+2r_2-1)\mathrm{l}_9 \\
& +\left(4r_1^2-2r_1r_2+r_1-2r_2^2\right)\left(-\mathrm{L}_7+\mathrm{L}_8+\mathrm{L}_9-\mathrm{L}_{10}+\mathrm{L}_{11}-\mathrm{L}_{12}\right) \\
& -\frac{1}{r_2}\left[-4r_1^3+2r_1^2(r_2+1)+r_1(2r_2-1)r_2+r_2^2+r_2\right] \\
& \times\left(\mathrm{l}_{10}^2/2+\mathrm{L}_{13}-\mathrm{L}_{14}+\mathrm{L}_{15}-\mathrm{L}_{16}+\mathrm{L}_{17}+\mathrm{L}_{18}\right). \qquad (3.6)
\end{aligned}$$

Here, $r_1 \equiv m_W^2/m_H^2$, $\beta_1 \equiv \sqrt{1-4r_1}$, $r_2 \equiv m_{u_k}^2/m_H^2$, $\beta_2 \equiv \sqrt{1-4r_2}$. The logarithms $\mathrm{l}_{1,\ldots,10}$ and the dilogarithms $\mathrm{L}_{1,\ldots,18}$ are explicitly given in the Appendix.

4. Predictions

Finally, because $m_H \gg m_{u_i}, m_{u_j}$, we can express the branching ratio for our decays of interest as follows

$$\mathrm{Br}(H \to q_i q_j) = \frac{\Gamma(H \to q_i q_j)}{\Gamma_H} \simeq \frac{N_C m_H}{4\pi\Gamma_H}\left(|F_1|^2+|F_2|^2\right), \qquad (4.1)$$

where $\Gamma(H \to q_i q_j) = \Gamma(H \to q_i \bar{q}_j) + \Gamma(H \to \bar{q}_i q_j) = 2\Gamma(H \to q_i \bar{q}_j)$ and the total decay width of the Higgs boson is $\Gamma_H = 4.1 \times 10^{-3}$ GeV. To perform the evaluation of the branching ratios we have taken the input values from the PDG Live [5]. Finally, our predictions are listed in the Table 1.

$H \to q_i q_j$	Br
$H \to uc$	1.63×10^{-18}
$H \to ds$	9.07×10^{-15}
$H \to db$	1.03×10^{-8}
$H \to sb$	2.44×10^{-7}

Table 1: Branching ratios for the $H \to q_i q_j$ decays.

5. Conclusions

We have presented analytical results for the rare $H \to q_i q_j$ decays in the context of the SM, which arise at the one-loop level. Specifically, we have performed Taylor expansions to the form factors in order to retain the virtual m_{q_k} mass and eliminate any term independent of it by virtue of the GIM mechanism. Our predictions agree with two of the four numerical values reported in Ref. [3], we agree on the $H \to db, sb$ channels, in contrast, they have reported $\mathrm{Br}(H \to uc) \sim 10^{-15}$ and $\mathrm{Br}(H \to ds) \sim 10^{-8}$, while our approach allow us to predict 10^{-18} and 10^{-15}, respectively.

Appendix

The logarithms and dilogarithms required in the Eqs. (2.8), (3.3) and (3.6) are given below

$$
\begin{aligned}
&\mathrm{l}_1 \equiv \log r_1, && \mathrm{l}_2 \equiv \log(-\beta_1 + 2r_1 - 1), && \mathrm{l}_3 \equiv \log\left(\frac{\beta_1 - 1}{\beta_1 - 2r_1 + 1}\right),\\
&\mathrm{l}_4 \equiv \log\left(\frac{\beta_1 + 1}{\beta_1 - 2r_1 + 1}\right), && \mathrm{l}_5 \equiv \log\left(\frac{\beta_1 - 1}{\beta_1 + 2r_1 - 1}\right), && \mathrm{l}_6 \equiv \log\left(\frac{\beta_1 + 1}{\beta_1 + 2r_1 - 1}\right),\\
&\mathrm{l}_7 \equiv \log r_2, && \mathrm{l}_8 \equiv \log\left(\frac{2r_1}{\beta_1 + 2r_1 - 1}\right), && \mathrm{l}_9 \equiv \log\left(\frac{2r_2}{\beta_2 + 2r_2 - 1}\right),\\
&\mathrm{l}_{10} \equiv \log\left(\frac{\beta_1 + 1}{\beta_1 + 2r_{12} - 1}\right), && \mathrm{L}_1 \equiv \mathrm{Li}_2(r_1 + 1), && \mathrm{L}_2 \equiv \mathrm{Li}_2\left(\frac{r_1 - 1}{r_1}\right),\\
&\mathrm{L}_3 \equiv \mathrm{Li}_2\left(\frac{2 - 2r_1}{-2r_1 + \beta_1 + 1}\right), && \mathrm{L}_4 \equiv \mathrm{Li}_2\left(\frac{-2r_1}{-2r_1 + \beta_1 + 1}\right), && \mathrm{L}_5 \equiv \mathrm{Li}_2\left(\frac{2r_1 - 2}{2r_1 + \beta_1 - 1}\right),\\
&\mathrm{L}_6 \equiv \mathrm{Li}_2\left(\frac{2r_1}{2r_1 + \beta_1 - 1}\right), && \mathrm{L}_7 \equiv \mathrm{Li}_2\left(\frac{r_{12}^2}{r_{12}^2 + r_1}\right), && \mathrm{L}_8 \equiv \mathrm{Li}_2\left(\frac{r_{12}^2 + r_{12}}{r_{12}^2 + r_1}\right),\\
&\mathrm{L}_9 \equiv \mathrm{Li}_2\left(\frac{-2r_{12}}{-2r_{12} + \beta_2 - 1}\right), && \mathrm{L}_{10} \equiv \mathrm{Li}_2\left(\frac{2r_{12} + 2}{2r_{12} - \beta_2 + 1}\right), && \mathrm{L}_{11} \equiv \mathrm{Li}_2\left(\frac{2r_{12}}{2r_{12} + \beta_2 + 1}\right),\\
&\mathrm{L}_{12} \equiv \mathrm{Li}_2\left(\frac{2r_{12} + 2}{2r_{12} + \beta_2 + 1}\right), && \mathrm{L}_{13} \equiv \mathrm{Li}_2\left(\frac{r_{12}^2 - r_{12}}{r_{12}^2 + r_2}\right), && \mathrm{L}_{14} \equiv \mathrm{Li}_2\left(\frac{r_{12}^2}{r_{12}^2 + r_2}\right),\\
&\mathrm{L}_{15} \equiv \mathrm{Li}_2\left(\frac{2 - 2r_{12}}{\beta_1 + 1}\right), && \mathrm{L}_{16} \equiv \mathrm{Li}_2\left(\frac{2 - 2r_{12}}{-2r_{12} + \beta_1 + 1}\right), && \mathrm{L}_{17} \equiv \mathrm{Li}_2\left(\frac{-2r_{12}}{-2r_{12} + \beta_1 + 1}\right),\\
&\mathrm{L}_{18} \equiv \mathrm{Li}_2\left(\frac{2r_{12}}{2r_{12} + \beta_1 - 1}\right).
\end{aligned}
$$

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References

[1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012). doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012). doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].

[3] L. G. Benitez-Guzmán, I. García-Jiménez, M. A. López-Osorio, E. Martínez-Pascual and J. J. Toscano, J. Phys. G **42**, 085002 (2015). doi:10.1088/0954-3899/42/8/085002 [arXiv:1506.02718 [hep-ph]].

[4] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D **2**, 1285 (1970). doi:10.1103/PhysRevD.2.1285

[5] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, 030001 (2018). doi:10.1103/PhysRevD.98.030001