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New results on the determination of the strong coupling

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We discuss the status of determinations of the strong coupling using event shape observables based on data collected at the Large Electron Positron collider and theoretical predictions at highest accuracy available at present. We argue that such extractions can be competitive with lattice determinations if the observables are selected carefully such that both higher order perturbative as well as non-perturbative contributions are suppressed. We give a list of such observables and study one particular class–the soft groomed event shapes–in detail. We present predictions for the soft drop thrust and study the scale dependence as a function of the grooming parameters.

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According to the Particle Data Group [1] the current world average of the determinations of the strong coupling $\alpha_s = 0.1181$ has an uncertainty of slightly below 1 %. The average is dominated [2] by the lattice determinations [3] that show the smallest uncertainties by far. Determinations based on experimental data span a much larger range, over 4 %, which suggests that measuring the strong coupling in experiments cannot cope with the precision of lattice determination as the only limitation on the latter is the CPU time used for the computations. Yet it is interesting that the average of α_s extractions from collider data is about one standard deviation smaller than the world average, leaving some uneasy feeling related to the value of this important parameter of nature.

The largest spread of α_s values appears among the determinations based on measuring the geometrical properties of hadronic final states in electron-positron annihilation, which is somewhat counter intuitive as such collisions provide a clean environment with strong interactions affecting only the final state. The main reasons for the large uncertainties lie in the usually large perturbative and non-perturbative (hadronisation) effects. This makes the inclusion of higher-order corrections mandatory. After the closure of LEP significant advances were made in this respect. On the one hand the next-to-next-to-leading order (NNLO) corrections have been computed for three-jet like observables [4, 5, 6], while on the other resummation of large logarithms to all orders have been performed at the next-to-next-to-leading logarithmic (NNLL or N^{2}LL)) and in some cases even at N^{3}LL accuracy [7, 8, 9].

Fig. 1 (left) shows the predictions for the thrust ($\tau = 1 - T$) distribution [10, 11] at LO, NLO and NNLO accuracy, as given by the perturbative expansion for the normalized cross section,[1]

$$\frac{\tau}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \left(\frac{\alpha_s}{2\pi}\right) A(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\tau) + \left(\frac{\alpha_s}{2\pi}\right)^3 C(\tau)\,. \tag{1}$$

Even the most precise prediction falls short significantly over the whole kinematic range, especially for small values of τ where the logarithms $L = -\ln\tau$ become large. This is readily understood from the analytic structure of perturbative predictions:

$$\begin{aligned} A(\tau) &= A_1 L + A_0\,, \\ B(\tau) &= B_3 L^3 + B_2 L^2 + B_1 L + B_0\,, \\ C(\tau) &= C_5 L^5 + C_4 L^4 + C_3 L^3 + C_2 L^2 + C_1 L + C_0 \end{aligned} \tag{2}$$

where the dependence of the coefficients on τ is suppressed. The logarithmic contributions have to be resummed in order to obtain a reliable prediction for small values of τ. As shown in Fig. 1 (left), combining the NNLO and N^{3}LL predictions, using R-matching to account for the double counting of logarithmic terms, improves the agreement between the prediction and data for the thrust distribution significantly. Nevertheless, there remains a large gap between the two in the peak region where most of the data fall. One might expect that the difference between the perturbative prediction and the data is mainly due to hadronisation corrections.

As for estimating the hadronisation corrections, there are two options: (i) use an analytic model (power corrections, PC) for the non-perturbative corrections [7, 15] essentially in the form of a shift of the differential distribution

$$\frac{\tau}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau) \to \frac{\tau}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau - 2a_0)\,, \tag{3}$$

[1] The A, B and C coefficients were computed using the `MCCSM` program [12] that implements the CoLoRFulNNLO subtraction method [13, 14].

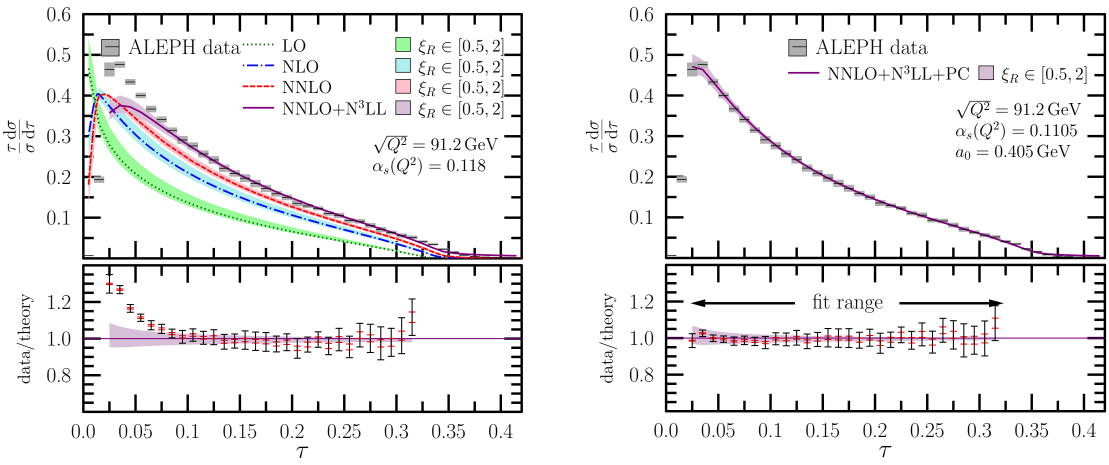

Figure 1: Distributions of thrust in electron-positron annihilation. Left: predictions at first three orders in perturbation theory and also matched NNLO and N^{3}LL predictions obtained with the world average for the strong coupling. Right: distribution of matched NNLO and N^{3}LL predictions supplemented with analytic model for power corrections fitted to ALEPH data. The bands represent the variation of the renormalization scale around the default one in the range $[\mu_0/2, 2\mu_0]$.

(ii) or use modern Monte Carlo (MC) tools [16, 17] to estimate the effect by simulating the final states both at the parton and at the hadron level and use the ratio of the two as a multiplicative correction factor (fitted with a smooth function). Both were applied in the past in α_s measurements. The simultaneous fits for the strong coupling and the non-perturbative parameter based on using NLO+NLL accurate predictions together with analytic model for the power corrections did not show universality [18]. One could hope that with improved perturbative predictions a more universal picture would emerge. However, this expectation did not fulfill [19], in spite of the much better agreement between predictions and data, shown for the thrust distribution in Fig. 1 (right) and for the distribution of heavy jet mass (ρ) in Fig. 2 (left). As Fig. 2 (right) shows, the matched NNLO+N^{3}LL+PC prediction fitted to LEP1 data provide overlapping regions for thrust and heavy-jet mass only if the non-perturbative shift depends logarithmically[2] on ρ in the form $a_0 \ln \frac{1}{\rho}$. Nevertheless, the two parameters are strongly anti-correlated, resulting in large uncertainties of the measured parameters.

A similar analysis was performed for the energy-energy correlation [20] recently in Ref. [21] where the matching of the fixed-order prediction at NNLO [6] and the resummed NNLL one [22] was performed. For this observable the non-perturbative correction depends on two parameters a_1 and a_2. A fit of the NNLO+NNLL+NP prediction to OPAL and SLD data showed again very strong anti-correlations among α_s, a_1 and a_2. Taking into account the energy dependence helps to reduce the anti-correlation for thrust [15], but it remains sizable as data away from the Z boson peak have much reduced statistics. Thus, we may conclude that the analytic models for hadronisation are not sufficient to provide a precise and robust simultaneous estimation of the strong coupling and the non-perturbative parameters, which also questions the utility of some of the α_s determinations

[2]No physical argument is known for such a dependence

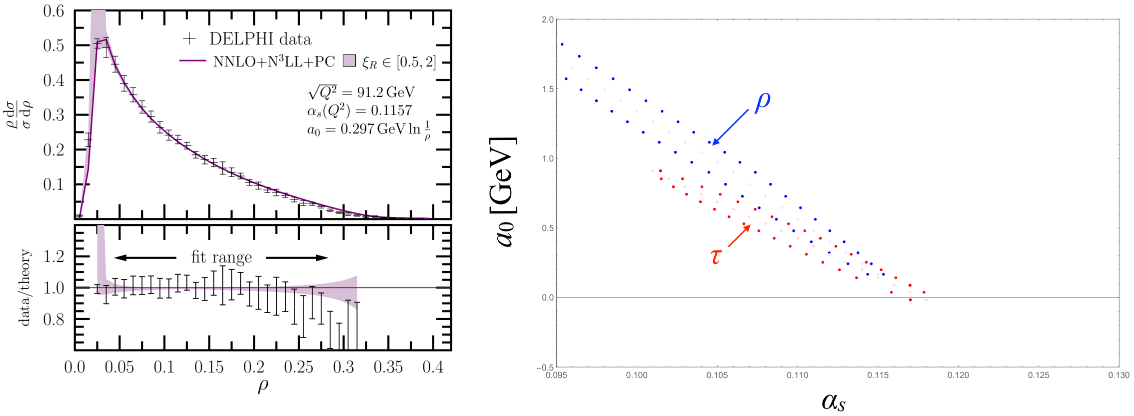

Figure 2: Left: distribution of heavy jet mass in electron-positron annihilation using matched NNLO and N^{3}LL predictions supplemented with power corrections for hadronisation fitted to Delphi data. The bands represent the variation of the renormalization scale around the default one in the range $[\mu_0/2, 2\mu_0]$. Right: fitted values of the strong coupling and the non-perturbative parameter using matched NNLO+N^{3}LL+PC predictions for thrust (τ) and heavy-jet mass (ρ) distributions. The ellipses correspond to 67 % confidence level, taking into account statistical and scale variation uncertainties only.

quoted in the PDG [1].

Besides the analytic models for hadronisation, at LEP it was customary to use MC estimations of its effect on the distributions. As these are not based on first principles, the correct estimation of the hadronisation uncertainty is ambiguous. Accepting that “large uncertainty in small quantity is small uncertainty” we choose the strategy of identifying observables for which the hadronisation corrections are small.[3] An example is the jet cone energy fraction [23], for which the NNLO corrections are very small except near the edges of the phase space [6]. There is a study for the determination of α_s from jet rates [17] that have smaller hadronisation corrections than event shapes. Unfortunately, the perturbative control is worse in this case because the resummation of large logarithmic contributions is known only in the case of two-jet rate at NNLL accuracy [24], while for higher rates only at the next-to-double logarithmic accuracy [25]. One may try to use event shapes from pre-clustered hadrons [26]. For instance, computing energy-energy correlation of jets by pre-clustering the hadrons into exactly five jets results in significantly reduced hadronization corrections [27].

Here we consider a new class of observables, for which hadronisation is reduced by a special kind of grooming called soft drop. The first variant of the soft drop grooming technique was introduced in Ref. [28], further developed in Ref. [29] and defined for jets produced in lepton collisions in Ref. [30]. We take the definition of the soft-drop thrust from Ref. [31] (version $T'_{\rm SD}$ that is free of a transition point in the soft-collinear region). This event shape depends on two grooming parameters β and $z_{\rm cut}$. The effect of these parameters on hadronization corrections was studied in Ref. [31] where it was found that with decreasing β and increasing $z_{\rm cut}$, i.e. stronger grooming ($\beta = \infty$ and $z_{\rm cut} = 0$ means no grooming), the hadronisation corrections to the distribution of $T'_{\rm SD}$ are much

[3] It may not be sufficient, but is necessary: small hadronisation uncertainty does not immediately imply precise extraction of α_s.

reduced over a wide range of the event shape. As such changes in β and $z_{\rm cut}$ also reduce the cross section, the optimal values of the grooming parameters is influenced by the desire of avoiding the loss of too much data.

The precision of α_s determination is also influenced by the convergence of the perturbative series for the observable, characterized by the NLO and NNLO K-factors defined by ratios of distributions of the observable O as

$$K_{\rm NLO}(\mu) = \frac{{\rm d}\sigma_{\rm NLO}(\mu)}{{\rm d}O} \Bigg/ \frac{{\rm d}\sigma_{\rm LO}(Q)}{{\rm d}O}\,, \quad K_{\rm NNLO}(\mu) = \frac{{\rm d}\sigma_{\rm NNLO}(\mu)}{{\rm d}O} \Bigg/ \frac{{\rm d}\sigma_{\rm LO}(Q)}{{\rm d}O}\,. \tag{4}$$

In order to see the convergence of the perturbation expansion more directly, we also define the ratio of the NNLO predictions to NLO ones, denoted by

$$K_{\rm NNLO'}(\mu) = \frac{{\rm d}\sigma_{\rm NNLO}(\mu)}{{\rm d}O} \Bigg/ \frac{{\rm d}\sigma_{\rm NLO}(Q)}{{\rm d}O}\,. \tag{5}$$

In Eqs. (4) and (5) we chose the normalization such that the cross sections in the denominators are always computed at the default renormalization scale $\mu = Q$, independently of μ. The closer the K-factors to unity, the better the convergence of the perturbative series. In order to check how grooming affects the perturbative stability of the predictions, we scanned the region of $(\beta, z_{\rm cut})$ values over the rectangle spanned by the corners $\{(0,0.05),(1,0.05),(0,0.1),(1,0.1)\}$. We found that the K-factors depend on the grooming parameters smoothly. Fig. 3 shows the distribution and the K-factors at the corners of this rectangle. We see that similarly to the non-perturbative corrections, the stronger the grooming the better the convergence of the perturbation series. The values $\beta = 0$ and $z_{\rm cut} = 0.1$ look optimal in the sense that the cross section still remains sizeable. The same conclusions can be drawn if one uses the soft drop hemisphere mass [32].

In this talk we discussed that precise determination of the strong coupling using hadronic final states in electron-positron annihilation requires (i) careful selection of observables, (ii) estimation of the hadronisation corrections with modern MC tools and (iii) needs methods to reduce hadronisation corrections. The latter could be pre-clustering the hadrons, or grooming techniques, such as soft drop. We used the `MCCSM` program for computing differential distributions for groomed (soft drop) event shapes at the NNLO accuracy. We found that our predictions were stable numerically. We observed that soft drop improves the perturbative convergence of the predictions. The smaller perturbative uncertainty, together with the reduced hadronization corrections makes the soft-drop thrust and hemisphere jet mass appealing candidates for a precise determination of the strong coupling at lepton colliders.

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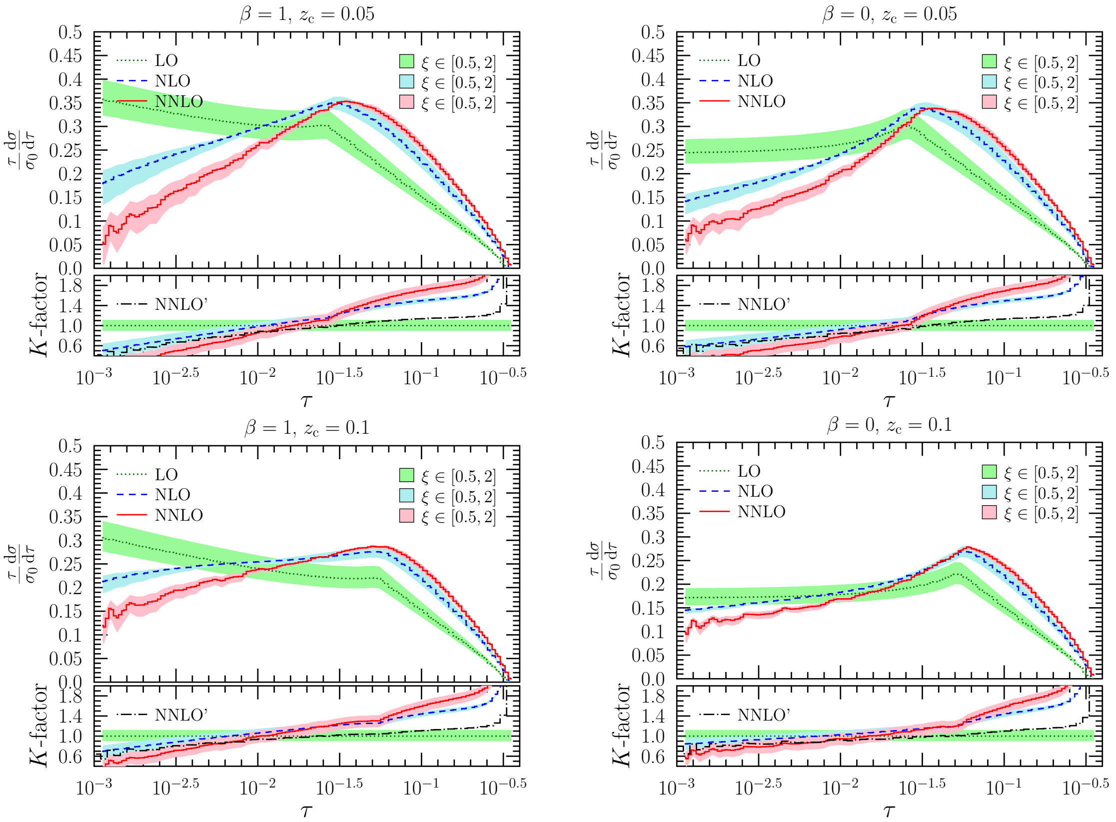

Figure 3: Distributions of the soft drop thrust $\tau = 1 - T'_{\rm SD}$ at LO, NLO and NNLO accuracy for various values of the grooming parameters as indicated above each plot. The lower sections exhibit the K-factors (the LO K-factor, $K_{\rm LO} = 1$ is shown for reference). The bands represent the variation of the renormalization scale around the default one in the range $[\mu_0/2, 2\mu_0]$. In the case of $K_{\rm NNLO'}$ the scale dependence does not provide relevant information, hence not shown.

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