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Wigner function approach to polarization-vorticity coupling and hydrodynamics with spin

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Newly introduced equilibrium Wigner functions for particles with spin one-half are used in the semi-classical kinetic equations to study a possible relation between thermal vorticity and spin polarization. It is shown that in global equilibrium both the thermal-vorticity and spin polarization tensors are constant but not necessarily equal. In the case of local equilibrium, we define a procedure leading to hydrodynamic equations with spin. We introduce such equations for the de Groot, van Leeuwen, and van Weert (GLW) formalism as well as for the canonical scheme (these two frameworks differ by the definitions of the energy-momentum and spin tensors). It is found that the GLW and canonical versions are connected by a pseudo-gauge transformation.

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1. Introduction

Recent measurements of the Λ–hyperon spin polarization in heavy-ion collisions by the STAR experiment [1, 2] have inspired a broad interest in the theoretical studies related to spin polarization and vorticity formation. The results of various investigations that refer to: the spin-orbit coupling [3, 4, 5, 6, 7], statistical properties of matter in global equilibrium [8, 9, 10, 11, 12, 13], kinetic models of spin dynamics [14, 15, 16, 17], hydrodynamics with triangle anomalies [18, 19] and the Lagrangian formulation of hydrodynamics [20, 21, 22] have been reported in this context.

A natural framework that can deal simultaneously with polarization and vorticity (dubbed below the hydrodynamics with spin) was proposed in Refs. [23, 24], see also Refs. [25, 26]. This framework is based on a generalized form of the equilibrium distribution functions for particles with spin-1/2 (scalar phase-space distribution functions are replaced by 2×2 relativistic spin density matrices).

In this contribution we discuss the results of our recent work [27]. We introduce the equilibrium Wigner functions for particles with spin-1/2, which satisfy the semi-classical kinetic equation, and study a possible relation between spin polarization and thermal vorticity. We also discuss a procedure leading to the hydrodynamics with spin, for the case of the de Groot, van Leeuwen, and van Weert formalism (GLW) and the canonical formalism.

2. Basic concepts —global and local thermodynamic equilibrium

In the case of spinless particles, the phase space distribution function $f(x,p)$ satisfies the Boltzmann equation

$$p^\mu \partial_\mu f(x,p) = C[f(x,p)], \tag{2.1}$$

where $p^\mu = (E_p, p)$ and $\partial_\mu = (\partial_t, \nabla)$ are the particle four momentum and space-time derivative, while $C[f]$ is the collision integral. The latter vanishes in the case of free streaming particles as well as in global or local thermodynamic equilibrium. In the free-streaming case, the distribution function $f_{\rm fs}(x,p)$ exactly satisfies the drift equation $p^\mu \partial_\mu f_{\rm fs}(x,p) = 0$. In the global thermodynamic equilibrium, the drift equation is also satisfied. In this case it leads to the constraints on the hydrodynamic parameters which specify the form of the equilibrium distribution $f_{\rm eq}(x,p)$. In particular, the parameters $\xi(x)$ (defined as the ratio of the local chemical potential $\mu(x)$ to the local temperature $T(x)$) and $\beta_\mu(x)$ (defined as the ratio of the local four fluid velocity $u_\mu(x)$ to the local temperature $T(x)$) satisfy the conditions: $\partial_\mu \xi = 0$ and $\partial_\mu \beta_\nu(x) + \partial_\nu \beta_\mu(x) = 0$. The first equation implies that $\xi(x) = \mu(x)/T(x) = \xi^0 = \text{const}$. The second one is known as the Killing equation. It has a solution of the form

$$\beta_\mu(x) = \beta^0_\mu + \varpi^0_{\mu\nu} x^\nu, \tag{2.2}$$

where, β^0_μ and the antisymmetric tensor $\varpi^0_{\mu\nu}$ are constants. For a given form of the $\beta_\mu(x)$ field, thermal vorticity is defined as

$$\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right). \tag{2.3}$$

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Using Eq. (2.2) in Eqs. (2.3), one can show that $\varpi_{\mu\nu} = \varpi^0_{\mu\nu}$, *i.e.*, the thermal vorticity in the global equilibrium is constant.

In the local equilibrium, the drift equation does not vanish. This is so, because in this case a correction δf has to be added to the equilibrium function $f_{\rm eq}$ to describe dissipative phenomena. However, if the gradients of the local hydrodynamic variables are small, the dissipative effects can be neglected. Hydrodynamic parameters may be constrained in this case by some specific moments of Eq. (2.1) in momentum space. They yield the conservation laws for charge, energy, and momentum.

For particle with spin, one makes use of the Wigner functions $\mathscr{W}^{\pm}_{\rm eq}(x,k)$ which, in addition to the standard hydrodynamic parameters, depend on an antisymmetric spin polarization tensor $\omega_{\mu\nu}(x)$. This allows us to distinguish between four rather than two different types of equilibrium: 1) *global equilibrium*— in this case the β_μ field is a Killing vector, $\varpi_{\mu\nu} = -\frac{1}{2}\left(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu\right) = \omega_{\mu\nu} = \text{const}$, $\xi = \text{const}$, 2) *extended global equilibrium* — β_μ is a Killing vector, $\varpi_{\mu\nu} = \text{const}$, $\omega_{\mu\nu} = \text{const}$ but $\omega_{\mu\nu} \neq \varpi_{\mu\nu}$, $\xi = \text{const}$, 3) *local equilibrium* — β_μ field is not a Killing vector but one can still have $\omega_{\mu\nu}(x) = \varpi_{\mu\nu}(x)$, $\xi = \xi(x)$, 4) *extended local equilibrium* — β_μ field is not a Killing vector, $\omega_{\mu\nu}(x) \neq \varpi_{\mu\nu}(x)$, $\xi = \xi(x)$.

We emphasize that similarly to the case of spinless particles, the global and extended global equilibrium states correspond to the case where $\mathscr{W}_{\rm eq}(x,k)$ exactly satisfies the kinetic equation with a vanishing collision term, while for the local and extended local equilibrium states only certain moments of the kinetic equation for $\mathscr{W}_{\rm eq}(x,k)$ can be set equal to zero (again with a vanishing collision term), which results in the conservations laws for energy, linear and angular momentum, and charge.

3. Equilibrium Wigner functions

In our approach we make use of the semi-classical connection between Wigner functions and the phase-space dependent spin density matrices $f^{\pm}_{rs}(x,p)$, introduced by de Groot, van Leeuwen, and van Weert in Ref. [28] as follows

$$
\begin{aligned}
\mathscr{W}^{+}_{\rm eq}(x,k) &= \frac{1}{2}\sum_{r,s=1}^{2}\int dP\,\delta^{(4)}(k-p)u^r(p)\bar{u}^s(p)f^{+}_{rs}(x,p),\\
\mathscr{W}^{-}_{\rm eq}(x,k) &= -\frac{1}{2}\sum_{r,s=1}^{2}\int dP\,\delta^{(4)}(k+p)v^s(p)\bar{v}^r(p)f^{-}_{rs}(x,p).
\end{aligned}
$$

Here $dP = \frac{d^3p}{(2\pi)^3 E_p}$ is the Lorentz invariant measure in momentum space with $E_p = \sqrt{m^2+p^2}$ being the on-mass-shell particle energy. Note that four momentum $k = k^\mu = (k^0,\mathbf{k})$ appearing as an argument of the Wigner function is not necessarily on the mass shell.

The equilibrium Wigner functions can be constructed by taking, as an input, the following expressions for $f^{+}_{rs}(x,p)$ and $f^{-}_{rs}(x,p)$ [11]

$$
f^{+}_{rs}(x,p) = \frac{1}{2m}\bar{u}_r(p)X^{+}u_s(p), \qquad f^{-}_{rs}(x,p) = -\frac{1}{2m}\bar{v}_s(p)X^{-}v_r(p).
$$

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Here m is the (anti-)particle mass, $u_r(p)$, and $v_r(p)$ are Dirac bispinors with spin indices r and s running from 1 to 2. The matrices $X^{\pm}$ are defined by the formula $X^{\pm} = \exp\left[\pm\xi(x) - \beta_\mu(x)p^\mu\right]M^{\pm}$ where $M^{\pm} = \exp\left[\pm\frac{1}{2}\omega_{\mu\nu}(x)\Sigma^{\mu\nu}\right]$ and $\Sigma^{\mu\nu} = (i/4)[\gamma^\mu,\gamma^\nu]$ is known as the Dirac spin operator.

As it was shown in Ref. [24], if we assume that the spin polarization tensor $\omega_{\mu\nu}$ satisfies the conditions, $\omega_{\mu\nu}\omega^{\mu\nu} \geq 0$ and $\omega_{\mu\nu}\tilde{\omega}^{\mu\nu} = 0$, where $\tilde{\omega}^{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\omega^{\alpha\beta}$ is the dual spin polarization tensor, we can derive the following expression for the matrix $M^{\pm}$,

$$M^{\pm} = \cosh(\zeta) \pm \frac{\sinh(\zeta)}{2\zeta}\,\omega_{\mu\nu}\Sigma^{\mu\nu}, \quad \text{where} \quad \zeta = \frac{1}{2}\sqrt{\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu}}.$$

The parameter ζ can be interpreted as the ratio of spin chemical potential Ω to the temperature T [23].

Wigner functions are 4×4 matrices which satisfy the relation $\mathscr{W}^{\pm}_{\rm eq}(x,k) = \gamma_0\mathscr{W}^{\pm}_{\rm eq}(x,k)^\dagger\gamma_0$. Therefore, they can always be decomposed in terms of the 16 independent generators of the Clifford algebra

$$\mathscr{W}^{\pm}_{\rm eq}(x,k) = \frac{1}{4}\left[\mathscr{F}^{\pm}_{\rm eq}(x,k) + i\gamma_5\mathscr{P}^{\pm}_{\rm eq}(x,k) + \gamma^\mu\mathscr{V}^{\pm}_{{\rm eq},\mu}(x,k) + \gamma_5\gamma^\mu\mathscr{A}^{\pm}_{{\rm eq},\mu}(x,k) + \Sigma^{\mu\nu}\mathscr{S}^{\pm}_{{\rm eq},\mu\nu}(x,k)\right]. \tag{3.1}$$

Various coefficient functions appearing in the expansion of equilibrium Wigner function can be obtained by contracting $\mathscr{W}^{\pm}_{\rm eq}(x,k)$ with appropriate gamma matrices and then taking the trace, for details see Ref. [27]. The total Wigner function is $\mathscr{W}_{\rm eq}(x,k) = \mathscr{W}^{+}_{\rm eq}(x,k) + \mathscr{W}^{-}_{\rm eq}(x,k)$.

4. Semi-classical expansion and Boltzmann-like equations for particles with spin

For an arbitrary Wigner function, a similar decomposition can be done in terms of the expansion coefficients $\mathscr{F}(x,k)$, $\mathscr{P}(x,k)$, $\mathscr{V}_\mu(x,k)$, $\mathscr{A}_\mu(x,k)$,and $\mathscr{S}_{\mu\nu}(x,k)$. The function $\mathscr{W}(x,k)$ satisfies the equation of the form [29, 30]

$$\left(\gamma_\mu K^\mu - m\right)\mathscr{W}(x,k) = 0, \qquad K^\mu = k^\mu + \frac{i\hbar}{2}\partial^\mu. \tag{4.1}$$

The above equation exactly holds in global equilibrium and should give the constraint on hydrodynamic variables μ, T, u^μ and $\omega_{\mu\nu}$ in local equilibrium. Solution of above equation can be written in the form of a series in $\hbar$,

$$\mathscr{X} = \mathscr{X}^{(0)} + \hbar\mathscr{X}^{(1)} + \hbar^2\mathscr{X}^{(2)} + \cdots. \qquad\qquad \mathscr{X} \in \{\mathscr{F},\mathscr{P},\mathscr{V}_\mu,\mathscr{A}_\mu,\mathscr{S}_{\nu\mu}\}$$

Keeping the terms up to the first order in $\hbar$, we can obtain the following equations for the coefficients functions $\mathscr{F}_{(0)}(x,k)$ and $\mathscr{A}^\nu_{(0)}(x,k)$,

$$k^\mu\partial_\mu\mathscr{F}_{(0)}(x,k) = 0, \quad k^\mu\partial_\mu\mathscr{A}^\nu_{(0)}(x,k) = 0, \quad k_\nu\mathscr{A}^\nu_{(0)}(x,k) = 0.$$

It can be easily shown that the functions $\mathscr{F}^{(0)}$ and $\mathscr{A}^{(0)}_\mu$ are basic independent ones and other coefficient functions can be expressed in terms of these two functions. Also, the algebraic structure of the equilibrium coefficient functions is consistent with the zeroth-order equations obtained from

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the semi-classical expansion of the Wigner function. Therefore, one can take $\mathscr{X}^{(0)} = \mathscr{X}_{\rm eq}$. In this way one can get,

$$k^{\mu}\partial_{\mu}\mathscr{F}_{\rm eq}(x,k)=0, \quad k^{\mu}\partial_{\mu}\mathscr{A}^{\nu}_{\rm eq}(x,k)=0, \quad k_{\nu}\mathscr{A}^{\nu}_{\rm eq}(x,k)=0. \tag{4.2}$$

Using the expressions (obtained by contracting $\mathscr{W}^{\pm}_{\rm eq}(x,k)$ with appropriate gamma matrices and then taking the trace), for the coefficients functions $\mathscr{F}_{\rm eq}(x,k)$ and $\mathscr{A}^{\nu}_{\rm eq}(x,k)$ and then substitutes them into Eqs. (4.2), one can see that the resulting equations will be exactly fulfilled if β^{μ} field satisfies the Killing equation. This suggests that thermal vorticity $\varpi_{\mu\nu}$ is constant, while the parameters ξ and spin polarization tensor $\omega_{\mu\nu}$ are also constant. Note that no conclusion can be drawn if $\omega_{\mu\nu}$ is equal to $\varpi_{\mu\nu}$. This situation corresponds to case of extended global equilibrium.

5. Procedure to formulate hydrodynamics with spin

5.1 Charge current

The charge current $\mathscr{N}^{\alpha}(x)$ can be expressed in terms of the Wigner function $\mathscr{W}(x,k)$ as follows [28]

$$\mathscr{N}^{\alpha}(x) = {\rm tr}\int d^4k\,\gamma^{\alpha}\,\mathscr{W}(x,k) = \int d^4k\,\mathscr{V}^{\alpha}(x,k). \tag{5.1}$$

Using the expression for $\mathscr{V}^{\alpha}(x,k)$ up to the first order in $\hbar$, one can find the equilibrium charge current to be of the form $\mathscr{N}^{\alpha}_{\rm eq}(x) = N^{\alpha}_{\rm eq}(x) + \delta N^{\alpha}_{\rm eq}(x)$, where $\delta N^{\alpha}_{\rm eq}(x)$ is the first order in $\hbar$ correction to $\mathscr{N}^{\alpha}_{\rm eq}(x)$. It can be easily shown that $\partial_{\alpha}\,\delta N^{\alpha}_{\rm eq}(x) = 0$. Thus, the conservation law of charge can be expressed by the equation $\partial_{\alpha}N^{\alpha}_{\rm eq}(x) = 0$, where $N^{\alpha}_{\rm eq}(x) = \int d^4k\,\mathscr{V}^{\alpha}_{(0)}(x,k)$. It turns out, that this expression matches with that derived in Ref. [23].

5.2 Energy-mometum and spin tensors

In the GLW formulation, the energy-momentum and spin tensors are expressed in terms of the Wigner function as [28]

$$T^{\mu\nu}_{\rm GLW}(x) = \frac{1}{m}{\rm tr}\int d^4k\,k^{\mu}\,k^{\nu}\,\mathscr{W}(x,k) = \frac{1}{m}\int d^4k\,k^{\mu}\,k^{\nu}\,\mathscr{F}(x,k), \tag{5.2}$$

$$S^{\lambda,\mu\nu}_{\rm GLW}(x) = \frac{\hbar}{4}\int d^4k\,{\rm tr}\left[\left(\left\{\sigma^{\mu\nu},\gamma^{\lambda}\right\} + \frac{2i}{m}\left(\gamma^{[\mu}k^{\nu]}\gamma^{\lambda} - \gamma^{\lambda}\gamma^{[\mu}k^{\nu]}\right)\right)\mathscr{W}(x,k)\right]. \tag{5.3}$$

Keeping terms only up to the first order in $\hbar$, replacing $\mathscr{F}_{(0)}(x,k)$ by $\mathscr{F}_{\rm eq}(x,k) = 0$, and carrying out the momentum integrations, we can reproduce the perfect-fluid formula for the GLW energy-momentum tensor reported in Ref. [23]. It should obey the conservation law $\partial_{\mu}T^{\mu\nu}_{\rm GLW}(x) = 0$. The expression for the GLW spin tenor can be obtained by replacing $\mathscr{W}(x,k) = \mathscr{W}_{\rm eq}(x,k)$ and carrying out the momentum integration. Note, that if the energy-momentum tensor $T^{\mu\nu}_{\rm GLW}(x)$ is symmetric, the conservations of the orbital and spin parts of the total angular momentum should hold separately, therefore, we must have $\partial_{\lambda}S^{\lambda,\mu\nu}_{\rm GLW} = 0$.

The canonical versions of the energy-momentum $T^{\mu\nu}_{\rm can}(x)$ and spin $S^{\lambda,\mu\nu}_{\rm can}(x)$ tensors can be obtained from the Dirac Lagrangian by applying the Noether theorem [31] and are given by the following expressions

$$T^{\mu\nu}_{\rm can}(x) = \int d^4k\,k^{\nu}\,\mathscr{V}^{\mu}(x,k), \tag{5.4}$$

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$$S^{\lambda,\mu\nu}_{\rm can}(x) = \frac{\hbar}{4}\int d^4k\,{\rm tr}\left[\left\{\sigma^{\mu\nu},\gamma^{\lambda}\right\}\mathscr{W}(x,k)\right] = \frac{\hbar}{2}\varepsilon^{\kappa\lambda\mu\nu}\int d^4k\,\mathscr{A}_{\kappa}(x,k) \equiv \frac{\hbar}{2}\varepsilon^{\kappa\lambda\mu\nu}\,\mathscr{A}_{\kappa}(x). \quad (5.5)$$

In this case, using Eq. (5.4), we obtain $T^{\mu\nu}_{\rm eq,can}(x) = T^{\mu\nu}_{\rm eq,GLW}(x) + \delta T^{\mu\nu}_{\rm can}(x)$, where $\delta T^{\mu\nu}_{\rm eq,can}(x) = -\frac{\hbar}{2m}\int d^4k k^{\nu}\partial_{\lambda}\mathscr{S}^{\lambda\mu}_{\rm eq}(x,k) = -\partial_{\lambda}S^{\nu,\lambda\mu}_{\rm GLW}(x)$. Note that canonical energy-momentum tensor should be conserved as well, *i.e.*, we must have $\partial_{\alpha}T^{\alpha\beta}_{\rm eq,can}(x) = 0$. Since $S^{\nu,\lambda\mu}_{\rm GLW}(x)$ is antisymmetric in the indices λ and μ, therefore, $\partial_{\mu}\,\delta T^{\mu\nu}_{\rm can}(x) = 0$. Thus, the conservation law for the canonical energy-momentum tensor is analogous to the GLW case.

The canonical version of equilibrium spin tensor can be obtained by considering the axial-vector component in Eq. (5.5) in the zeroth order (with the assumption that $\mathscr{A}^{(0)}_{\kappa}(x,k) = \mathscr{A}_{{\rm eq},\kappa}(x,k)$) and then carrying out the integration over the four-momentum k. It can be shown that

$$S^{\lambda,\mu\nu}_{\rm can} = S^{\lambda,\mu\nu}_{\rm GLW} + S^{\mu,\nu\lambda}_{\rm GLW} + S^{\nu,\lambda\mu}_{\rm GLW} \quad (5.6)$$

and

$$\partial_{\lambda}S^{\lambda,\mu\nu}_{\rm can}(x) = -\partial_{\lambda}S^{\mu,\lambda\nu}_{\rm GLW}(x) + \partial_{\lambda}S^{\nu,\lambda\mu}_{\rm GLW}(x) = T^{\nu\mu}_{\rm can} - T^{\mu\nu}_{\rm can}. \quad (5.7)$$

This is interesting, as one can see that the divergence of the canonical spin tensor is equal to the difference of the energy-momentum components (provided the GLW spin tensor is conserved). This result is expected because the energy-momentum tensor is not symmetric in the canonical case.

It is important to note that the two approaches (GLW and canonical) are connected via a pseudo-gauge transformation. In fact, if we define a super-potential $\Phi^{\lambda,\mu\nu} \equiv S^{\mu,\lambda\nu}_{\rm GLW} - S^{\nu,\lambda\mu}_{\rm GLW}$, we can show that

$$S^{\lambda,\mu\nu}_{\rm can} = S^{\lambda,\mu\nu}_{\rm GLW} - \Phi^{\lambda,\mu\nu} \quad \text{and} \quad T^{\mu\nu}_{\rm can} = T^{\mu\nu}_{\rm GLW} + \frac{1}{2}\partial_{\lambda}\left(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda}\right). \quad (5.8)$$

5.2.1 Conservation laws from the kinetic equations

Conservation laws for charge and energy-momentum can be obtained respectively by taking the zeroth and first moments of the kinetic equation $k^{\mu}\partial_{\mu}\mathscr{F}_{\rm eq}(x,k) = 0$. However, the equations for charge and energy-momentum are not closed due additional degrees of freedom arising from the spin polarization. To close them, we need to determine the dynamics of spin. If we multiply the kinetic equation $k^{\alpha}\partial_{\alpha}\mathscr{A}^{\mu}_{\rm eq}(x,k) = 0$, by a factor $\varepsilon^{\mu\beta\gamma\delta}k_{\beta}$ and then integrate over k we can obtain the dynamics of spin. It agrees, in fact, with the conservation of the GLW spin tensor.

6. Summary and conclusions

Using the equilibrium distribution functions of spin-1/2 particles that have been put forward in Ref.[11] we have constructed the equilibrium Wigner functions that satisfy the semi-classical kinetic equation. For a collision-less case, using the semi-classical expansion of Wigner function we obtain the Boltzmann like kinetic equations with spin. Using these kinetic equations we have shown that there is no direct relation between the thermal vorticity and spin polarization, except for the fact that the two should be constant in global equilibrium. Finally, we outline the procedure to construct the conservation laws for hydrodynamics with spin within the framework of de Groot,

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van Leeuwen, and van Weert (GLW) and in the canonical framework. In the GLW case, the energy-momentum tensor is symmetric and spin is conserved, while for the canonical case the energy-momentum tensor is asymmetric and spin is not conserved. Interestingly, the two cases are found to connected by the pseudo-gauge transformation.

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