

# Color Confinement, Hadron Dynamics, and Hadron Spectroscopy from Light-Front Holography and Superconformal Algebra

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The combined approach of light-front holography and superconformal algebra provides insight into the origin of color confinement and the QCD mass scale. Light-front (LF) holography predicts Lorentz-invariant light-front Schrödinger bound state equations for QCD, analogous to the quantum mechanical Schrödinger equation for atoms in QED. A key feature is the implementation of the de Alfaro, Fubini and Furlan (dAFF) procedure for breaking conformal invariance which allows a mass parameter to appear in the Hamiltonian and the equations of motion while retaining the conformal symmetry of the action. When one applies the dAFF procedure to chiral QCD, a mass scale  $\kappa$  appears which determines hadron masses and universal Regge slopes. It also implies a unique form of the soft-wall dilaton which modifies the action of AdS<sub>5</sub>. The result is a remarkably simple analytic description of quark confinement, as well as nonperturbative hadronic structure and dynamics. The same mass parameter  $\kappa$  controls the Gaussian fall-off of the nonperturbative QCD running coupling:  $\alpha_s(Q^2) \propto \exp(-Q^2/4\kappa^2)$ , in agreement with the effective charge determined from measurements of the Bjorken sum rule. The mass scale  $\kappa$  underlying hadron masses can be connected to the parameter  $\Lambda_{\overline{MS}}$  in the QCD running coupling by matching the magnitude and slope of the nonperturbative coupling to perturbative QCD at large  $Q^2$ . The result is an effective coupling  $\alpha_s(Q^2)$  defined at all momenta and a transition scale  $Q_0$  which separates perturbative and nonperturbative dynamics. QCD is not supersymmetrical in the traditional sense – the QCD Lagrangian is based on quark and gluonic fields, not squarks nor gluinos. However, when one applies superconformal algebra, one obtains a unified spectroscopy of meson, baryon, and tetraquarks as equal-mass members of the same 4-plet representation. The LF resulting Schrödinger equations match the bound state equation obtained from LF holography. The predicted Regge trajectories have a universal slope in both the principal quantum number  $n$  and orbital angular momentum. The meson and baryon Regge trajectories are identical when one compares mesons with baryons with orbital angular momentum  $L_M = L_B + 1$ . The matching of bosonic meson and fermionic baryon masses is a manifestation of a hidden supersymmetry in hadron physics. The pion eigenstate is massless for massless quarks, despite its dynamical structure as a  $q\bar{q}$  bound state. The superconformal relations also can be extended to heavy-light quark mesons and baryons. One also obtains empirically viable predictions for spacelike and timelike hadronic form factors, structure functions, distribution amplitudes, and transverse momentum distributions.

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## 1. Profound Questions for Hadron Physics

Although the foundations of quantum chromodynamics (QCD) were developed 45 years ago [1], there are fundamental and profound questions which are not fully understood. For example, what is the dynamical mechanism for color confinement which prevents quarks and gluons from appearing as asymptotic states? What is the origin of the QCD mass scale which gives the proton its mass, and what is the color confining dynamics which can produce a massless pion  $q\bar{q}$  bound state when the quarks are massless? Why are Regge slopes  $M^2 \propto L, n$  identical for both mesons and baryons, in both angular momentum  $L$  and the principal quantum number  $n$ ? What is the analytic form of the QCD running coupling  $\alpha_s(Q)$  at all scales, *both hard and soft*? What is the full spectroscopy of hadrons in QCD, including tetraquarks, pentaquarks, gluonia, and other exotic states? What are the fundamental frame-independent nonperturbative wavefunctions of the hadronic eigenstates which underly dynamical observables such as structure functions and form factors? How do the off-shell quarks and gluons produced in collisions remain confined and hadronize at the amplitude level to produce the final-state hadrons? What is the nature of the QCD vacuum – can vacuum condensates exist without causing huge contributions to the cosmological constant?

In this report, I will review recent insights to the fundamental features of QCD that can be obtained using a novel theoretical approach based on *light-front holography* [2] and *superconformal algebra* [3, 4]. A key feature is the implementation of the de Alfaro, Fubini and Furlan (dAFF) procedure for breaking conformal invariance which allows a mass parameter to appear in the Hamiltonian and the equations of motion, while retaining the conformal symmetry of the action [5]. A key consequence for chiral QCD is a Lorentz-invariant single-variable *light-front Schrödinger bound-state equation* for  $q\bar{q}$  mesons: [2, 6]

$$\left[ -\frac{d}{d\zeta^2} + \frac{(4L^2 - 1)}{4\zeta^2} + U(\zeta^2) \right] \psi(\zeta) = M^2 \psi(\zeta). \quad (1)$$

The potential has the unique form [6]

$$U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1), \quad (2)$$

where  $\zeta^2 = b_\perp^2 x(1-x)$  is the radial coordinate of light-front (LF) theory,  $x = \frac{k^+}{p^+} = \frac{k^0 + k^z}{p^0 + p^z}$  and  $1-x$  are the quark and antiquark LF momentum fractions,  $S = S^z$  is the  $\hat{z}$  projection of the  $q\bar{q}$  spin and  $L = L^z$  is the relative orbital angular momentum of the  $q$  and  $\bar{q}$ . Quark masses add a term  $\sum_i \frac{m_i^2}{x_i}$  to the LF Hamiltonian. See Fig. 1. The color-confining harmonic oscillator contribution  $\kappa^4 \zeta^2$  to the LF potential for light quarks becomes the usual  $\sigma r$  confining potential for heavy quarkonium in the nonrelativistic limit. [7]

Classical gravity theory based on five-dimensional Anti-de Sitter spacetime AdS<sub>5</sub> provides a geometrical representation of conformal symmetry [8]. It is also holographically dual [2] to field theory in physical 3 + 1 spacetime quantized using LF time  $\tau = t + z/c$ . [9] Eq. (1) with the potential (2) can then be derived using AdS<sub>5</sub> space and the dilaton  $e^{\kappa^2 \tilde{z}^2}$  “soft-wall” modification of its action, where the fifth coordinate of AdS space  $\tilde{z}$  is identified with the light-front coordinate  $\zeta$  using light-front holography. This specific dilaton is mandated by the dAFF procedure.

The  $\kappa^4 \zeta^2$  harmonic oscillator contribution to the potential  $U(\zeta^2)$  confines the colored constituent quarks. The eigenvalues of this equation

$$M^2 = 4\kappa^2(n + L + S/2), \quad (3)$$

provide a surprisingly accurate representation of meson spectroscopy for  $\kappa = 0.53 \text{ GeV}$ . The Regge slopes in  $n$  and  $L$  are predicted to be identical, consistent with observed hadron spectroscopy. The pion with  $n = L = S = 0$  is massless for massless quarks, despite its dynamical structure as a  $q\bar{q}$  bound state. The eigensolutions of Eq. (1)  $\psi_{n,L}(x, k_{\perp}^2)$  provide the LF wave functions underlying hadron structure and dynamics in the nonperturbative domain, as well as hadronization, the conversion of the off-shell constituent quarks to the mesons  $q\bar{q} \rightarrow H$  at the amplitude level. Since the units of mass  $\text{GeV}$  are irrelevant to QCD, only ratios of masses can be predicted by QCD; thus the value of  $\kappa$  is not a fixed parameter – the only parameters entering the light-front holographic predictions are the quark masses predicted by the Higgs mechanism in the Standard Model.

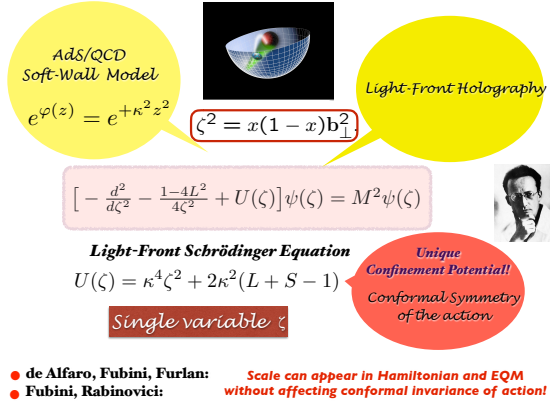
Remarkably, a similar LF Schrodinger equation is predicted for baryons by superconformal algebra where the baryons  $|q[qq]\rangle$  are represented as a  $3_C$  quark and a  $\bar{3}_C$  composite diquark [10, 11] with relative orbital angular momentum  $L_B$ . A crucial difference with the meson equation is that  $L_B - 1$  replaces  $L = L_M$ . The meson and baryon Regge trajectories in  $n$  and  $L$  are then identical when one compares the mesons and baryons spectra with orbital angular momentum  $L_M = L_B + 1$ . This equality of bosonic meson and fermionic baryon masses is a manifestation of a hidden supersymmetry in hadron physics. Superconformal algebra also predicts that each baryon wave function has two components with  $L_B$  and  $L_B + 1$  with equal weight so that there is equal probability for the quark to have spin parallel and antiparallel to the total baryon spin  $J^z$  [3]. Thus the total angular momentum of the quark-scalar diquark proton is carried by the quark's orbital angular momentum. This is a novel realization of the chiral symmetry of the QCD Lagrangian. The  $|q\bar{q}\rangle$  mesons and the  $|q[qq]\rangle$  baryons are also degenerate in mass with  $|[qq][\bar{q}\bar{q}]\rangle$  tetraquarks. Thus another remarkable consequence is the organization of the hadron spectrum into “4-plet” supersymmetric representations. Solving for relativistic hadron spectroscopy and dynamics using this LF formalism is analogous to solving for atomic hadron spectroscopy and dynamics in nonrelativistic Schrödinger theory.

The running coupling of QCD is predicted to have the form  $\alpha_s(Q^2) = \alpha_s(0)e^{-Q^2/4\kappa^2}$  in the nonperturbative domain. This is derived using LF holography by identifying the AdS<sub>5</sub> coupling defined by the conformally breaking dilaton  $e^{\kappa^2 z^2}$  with the QCD coupling [12, 13, 14, 15, 16, 17]. The matching of the magnitude and derivative of this nonperturbative prediction at its inflection point  $Q_0$  with the pQCD coupling then determines  $\alpha_s(Q^2)$  at all scales for any choice of renormalization scheme. We have also used effective nonperturbative LF holographic wave functions, together with BFKL, ERBL and DGLAP evolution equations, to predict structure functions, distribution amplitudes and hard exclusive amplitudes beyond the transition scale  $Q_0$  [18, 19, 20]. Superconformal algebra predicts that the resulting generalized parton distributions of mesons and baryons have a universal structure [20]. One can also predict the distributions of higher Fock states such as  $|uudQ\bar{Q}\rangle$  with intrinsic heavy quarks with their  $Q\bar{Q}$  asymmetries [21].

Quark counting rules for hadron form factors and other hard exclusive amplitudes are also a property of these nonperturbative solutions. One can thus use counting rules to identify the field content of mesons, baryons, tetraquarks, and gluonium. For example, the asymptotic power-law falloff of the exclusive annihilation cross sections for  $\sigma(e^+e^- \rightarrow H_a + H_b) \propto (1/s)^{1+n_H a+n_H b}$ , where  $n_H$  is the number of valence constituent fields (the leading twist) of a meson ( $n=2$ ), baryon ( $n=3$ ), tetraquark ( $n=4$ ), gluonium ( $n=2,3$ ), or pentaquark ( $n=5$ ). [22, 23].

Light-front perturbation theory has many important simplifications for computing scattering amplitudes perturbatively, including explicit unitarity, recursion relations, overall  $J^z$  conservation, and limits on the change in  $L^z$  at each vertex [24].

In Dirac's front form, the vacuum state is the state of lowest invariant mass  $H_{LF}|0\rangle = 0$ . Disconnected vacuum diagrams do not appear, so  $\langle 0|T^{\mu\nu}|0\rangle_{LF} = 0$  [25]. Thus in the front form, QCD effects associated with the vacuum are contained in the hadron wavefunctions [26], and one predicts zero contribution to the cosmological constant [27].



**Figure 1:** The convergence of theoretical methods for generating a model of hadron spectroscopy and dynamics with color confinement and meson-baryon supersymmetric relations.

In the following sections I will discuss these points in detail.

## 2. Conformal Invariance of QCD and the Principle of Maximum Conformality

Conformal symmetry is an underlying symmetry of QCD. If one sets the quark masses to zero in the QCD Lagrangian, the theory has no evident mass scale, and it is manifestly scale invariant. In effect, the classical chiral QCD theory is conformal.

A key tool is the remarkable observation of dAFF [5] which shows how a mass scale can appear in the Hamiltonian and the equations of motion of a theory while retaining the conformal symmetry of the action. When one applies the dAFF procedure to chiral light-front QCD, a mass scale  $\kappa$  appears which determines universal Regge slopes and hadron masses in the absence of the Higgs coupling.

Conformal symmetry also leads to a rigorous way to eliminate the renormalization scale ambiguity [28, 29, 30, 31]. A primary problem for perturbative QCD analyses is how to set the renormalization scale of the QCD running coupling in order to achieve precise fixed-order predictions for physical observables. The Principle of Maximal Conformality (PMC) provides a systematic way to set the renormalization scales order-by-order for any perturbative QCD process, eliminating the ambiguities associated with the conventional renormalization scale-setting procedure. The resulting predictions are independent of the choice of renormalization scheme, a requirement of renormalization group invariance. The scales of the QCD couplings and the effective number of quark flavors are set order-by-order by absorbing the nonconformal  $\beta$  terms in the pQCD series into the running coupling. The resulting pQCD series then matches the  $\beta = 0$  conformal series. The PMC generalizes the BLM procedure to all orders [32] and it reduces to the Gell Mann-Low scale

setting procedure for Abelian QED in the  $N_C \rightarrow 0$  limit. The divergent renormalon series does not appear. The PMC satisfies renormalization group invariance and all of the other self-consistency conditions derived from the renormalization group. It thus systematically eliminates a major theoretical uncertainty for pQCD predictions and increases the sensitivity of experiment to new physics beyond the Standard Model.

The Crewther relation [33], which was originally derived for conformal theory, provides a remarkable connection between two observables when the  $\beta$  function vanishes. Specifically, it connects the non-singlet Adler function to the Bjorken sum rule coefficient for polarized deep-inelastic electron scattering at leading twist. The “Generalized Crewther Relation” [33, 34, 35] relates these observables for physical QCD with nonzero  $\beta$  function. The resulting relation is independent of the choice of the renormalization scheme at any finite order, and the dependence on the choice of the initial scale is negligible. Similar scale-fixed “commensurate scale relations” also connect other physical observables at their physical momentum scales, thus providing new convention-independent precision tests of QCD.

### 3. The Origin of the QCD Mass Scale and de Alfaro, Fubini and Furlan Procedure

A fundamental question for QCD is the origin of the mass of the proton and other hadrons when the quark masses are zero. It is often stated that the mass scale  $\Lambda_{\overline{MS}}$  of the renormalized perturbative theory generates the nonperturbative QCD mass scale; however, this “dimensional transmutation” solution is problematic since the perturbative scale is renormalization-scheme dependent, whereas hadron masses cannot depend on a theoretical convention. It is conventional to measure hadron masses in MeV units; however, QCD has no knowledge of units such as electronvolts. Thus QCD at  $m_q = 0$  can at best only predict ratios of masses such as  $m_p/m_p$  and other dimensionless quantities. It is often argued that the QCD mass scale reflects the presence of quark and gluon condensates in the QCD vacuum state. However, such condensates lead to a cosmological constant a factor of  $10^{42}$  larger than measured. Nontrivial vacuum structure does not appear in QCD if one defines the vacuum state as the eigenstate of lowest invariant mass of the QCD LF Hamiltonian. In fact, in Dirac’s boost invariant “front form” [9], where the time variable is the time  $x^+ = t + z/c$  along the light-front, the light-front vacuum  $|0\rangle_{LF}$  is both causal and frame-independent; one thus has  $\langle 0|T^{\mu\nu}|0\rangle_{LF} = 0$  [25] and zero cosmological constant [27, 26]. In the case of the Higgs theory, the traditional Higgs vacuum expectation value (VEV) is replaced by a “zero mode” in the LF theory, analogous to a classical Stark or Zeeman field [36]. The Higgs LF zero mode [36] has no energy-momentum density, so it also gives zero contribution to the cosmological constant.

The remarkable work of de Alfaro, Fubini, and Furlan [5] provides a novel solution for the origin of the hadron mass scale in QCD. dAFF have shown that one can introduce a nonzero mass scale  $\kappa$  into the Hamiltonian of a conformal theory without affecting the conformal invariance of the action. The essential step is to add to the Hamiltonian  $H$  a term proportional to the dilation operator  $D$  and the special conformal operator  $K$  to fulfill the algebra of the generators of the conformal group. In the case of one-dimensional quantum mechanics, the resulting Hamiltonian acquires a confining harmonic oscillator potential; however, after a redefinition of the time variable, the action remains conformal.

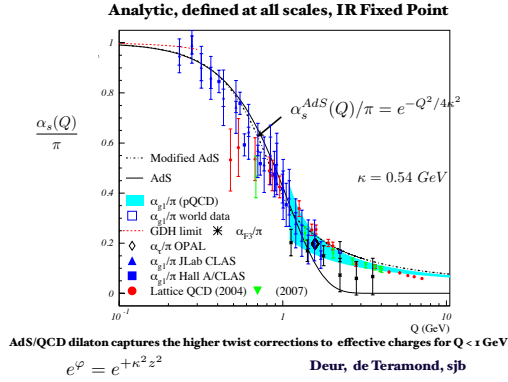
The application of dAFF then leads in fact to a color-confining LF harmonic oscillator potential, where again the action remains conformal. In fact, de Téramond, Dosch, and Brodsky [6] have shown that a mass gap and a fundamental color confinement scale also appear when one extends the dAFF procedure to LF Hamiltonian theory in physical 3+1 spacetime. The LF equation for  $q\bar{q}$  bound states for  $m_q = 0$  can be systematically reduced to Eq. (1), a differential equation in a single LF radial variable  $\zeta$ ; where  $\zeta^2 = b_{\perp}^2 x(1-x)$  is the radial variable of the front form and  $L = \max |L^z|$  is the LF orbital angular momentum [2]. This is in analogy to the non-relativistic radial Schrödinger equation for bound states such as positronium in QCD. Thus in the case of QCD(3+1) – using the causal, frame-independent light-front Hamiltonian formalism – the confining LF potential has the unique form  $\kappa^4 \zeta^2$ . The same procedure can be applied to relativistic quantum field theory using LF quantization [37].

#### 4. Light-Front Holography

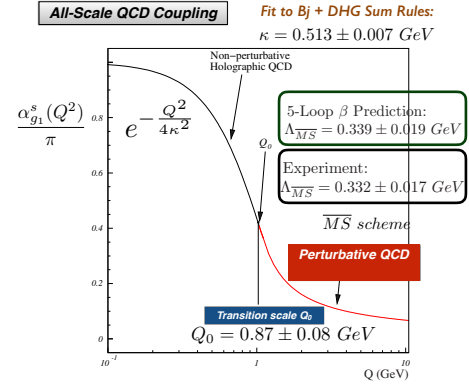
As noted by Maldacena [8], anti-deSitter space in five space-time dimensions (AdS<sub>5</sub>) provides a geometrical representation of the conformal group. Thus AdS<sub>5</sub> can be used as a starting point for a conformal theory such as chiral QCD. In fact AdS<sub>5</sub> is holographically dual to gauge theory quantized using light-front time, Dirac’s front form. Exclusive hadron amplitudes, such as elastic and transition form factors are given in terms of convolutions of light-front wavefunctions [38]. The light-front Drell-Yan-West formulae for electromagnetic and gravitational form factors is identical to the Polchinski-Strassler [39] formula for form factors in AdS<sub>5</sub>. This identification “light-front holography” also provides a nonperturbative derivation of scaling laws [40, 41] for form factors at large momentum transfer. Additional references and reviews of LF Holography may be found in refs. [42, 43, 44, 45, 46].

Remarkably, the identical LF potential and the same LF equation of motion are obtained from AdS<sub>5</sub> when one identifies the fifth coordinate  $\tilde{z}$  with the LF radial coordinate  $\zeta$  and introduces a specific modification of the AdS<sub>5</sub> metric – the “dilaton”  $e^{\phi(\tilde{z})} = e^{+\kappa^2 \tilde{z}^2}$ . This dilaton also leads to a Gaussian functional form of the nonperturbative QCD running coupling:  $\alpha_s(Q^2) = \alpha_s(0) \exp(-Q^2/4\kappa^2)$ , in agreement with the effective charge determined from measurements [47, 48, 49] of the Bjorken sum rule. See Fig. 2. Deur, de Téramond, and Brodsky [12, 13, 14, 15] have also shown how the parameter  $\kappa$ , which determines the mass scale of hadrons and Regge slopes in the zero quark mass limit, can be connected to the mass scale  $\Lambda_s$  controlling the evolution of the perturbative QCD coupling. The high momentum transfer dependence of the coupling  $\alpha_{g_1}(Q^2)$  is predicted by pQCD. The matching of the high and low momentum transfer regimes of  $\alpha_{g_1}(Q^2)$  – both its value and its slope – then determines a scale  $Q_0 = 0.87 \pm 0.08$  GeV which sets the interface between perturbative and nonperturbative hadron dynamics [17]. This connection can be done for any choice of renormalization scheme, such as the  $\overline{MS}$  scheme. The mass scale  $\kappa$  underlying hadron masses can thus be connected to the parameter  $\Lambda_{\overline{MS}}$  in the QCD running coupling by matching its predicted nonperturbative form to the perturbative QCD regime. The result is an effective coupling  $\alpha_s(Q^2)$  defined at all momenta. See Fig. 3.

The identification of AdS<sub>5</sub> with the light-front Hamiltonian theory automatically introduces an extra spin-dependent constant term  $2\kappa^2(L+S-1)$  in the LF Hamiltonian, where  $L = \max L^z, S = \max S^z$  with  $J^z = L^z + S^z$  are the LF spins. The resulting holographic prediction is the single variable



**Figure 2:** Comparison of the nonperturbative running coupling  $\alpha_s/\pi = \exp(-Q^2/4\kappa^2)$  predicted by light front holography with measurements [47, 48, 49] of the effective charge  $\alpha_{g1}(Q^2)$  from the  $Q^2$  dependence of the Bjorken sum rule.



**Figure 3:** Connecting the nonperturbative running coupling  $\alpha_s/\pi = \exp(-Q^2/4\kappa^2)$  with the 5-loop QCD prediction for the effective charge of the Bjorken sum rule  $\alpha_{g1}(Q^2)$ . See Refs. [12, 13, 14, 16]

“LF Schrödinger equation” in  $\zeta$ , where  $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$ . The synthesis of holographic QCD and conformal quantum mechanics is illustrated in Fig. 1. The eigenvalues for the meson spectrum are  $M^2(L, n) = 4\kappa^2(n + L + S/2)$ . The mesonic spectrum of  $q\bar{q}$  bound states is thus described as Regge trajectories in both the principal quantum number  $n$  (the number of nodes in the radial wavefunction) and the orbital angular momentum  $L$  with the same slope  $4\kappa^2$ . Color confinement is a consequence of the light-front potential  $U(\zeta^2)$ . Remarkably, the pion ( $n = 0, J = L = 0$ ) is massless:  $m_\pi = 0$  for  $m_q = 0$ . The LF potential contains a term  $2\kappa^2(J - 1)$  which is analogous to the hyperfine spin-spin splitting interaction in atoms. This term vanishes for the  $\rho$  since  $J = L + S = 1$ . However, in the case of the pion with  $J = 0$ , it gives a negative contribution  $-2\kappa^2$  to the pion mass squared which exactly cancels the positive contributions from the LF kinetic energy and confining potential. Thus light-front holography explains another fundamental question in hadron physics – how a zero mass  $q\bar{q}$  pseudoscalar pion bound state can emerge, despite its composite structure.

The eigensolutions of the LF Schrödinger equation generate both the mass spectrum and the light front wave functions  $\psi_M(x, k_\perp, \lambda)$  for all  $q\bar{q}$  meson bound states. Nonzero quark masses appear in the “LF kinetic energy” (LFKE)  $\sum_i [k_\perp^2 + m_i^2]_i$  contribution to the LF Hamiltonian – the square of the invariant mass of the constituents:  $\mathcal{M}^2 = (\sum_i p_i^\mu)^2$ . One can identify the  $\frac{m^2}{x}$  contribution to the LFKE as arising in the Higgs theory from the coupling  $\frac{m}{x} \times m$  of each quark to the background zero-mode Higgs field [36] which replaces the usual VEV of the standard time “instant form”. The resulting nonperturbative hadronic LF wavefunctions are Gaussian functions of the parton invariant mass squared  $\mathcal{M}^2$ ; they do not factor as functions of  $k_\perp^2$ . The pion distribution amplitude has the form  $\phi_\pi(x) \propto \sqrt{x(1-x)}$  in the nonperturbative domain, which then evolves by ERBL evolution to  $x(1-x)$  at infinite  $Q^2$  [18]. In the heavy quark limit, one recovers the usual  $\sigma r$  confining potential for heavy quarkonium [7].



## 5. Superconformal Algebra and Supersymmetric Hadron Spectroscopy

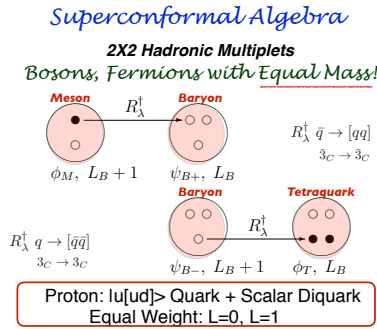
Another advance in LF hadron physics is the application of *superconformal algebra*, a feature of the underlying conformal symmetry of chiral QCD, originally discovered by Haag, Lopuszanski, and Sohnius [50]. The superconformal group in one dimension has an elegant  $2 \times 2$  Pauli matrix representation. The conformal Hamiltonian operator and the special conformal operator can be represented as anticommutators of fermionic generators  $H = 1/2\{Q, Q^\dagger\}$  and  $K = 1/2\{S, S^\dagger\}$ . As shown by Fubini and Rabinovici, [51], a nonconformal Hamiltonian with a mass scale can then be obtained by shifting  $Q \rightarrow Q + \sqrt{\omega}K$ , the supersymmetric analog of the dAFF procedure. When extended to hadron physics in the light front this procedure leads to universal confinement for mesons and baryons and a zero mass pion in the chiral limit [3, 4]. In effect, one has obtained extended representations of the superconformal algebra [51, 52]. This ansatz extends the predictions for the hadron spectrum to a “4-plet” – consisting of a mass-degenerate quark-antiquark meson, a quark-diquark baryon, and a diquark-antidiquark tetraquark, as shown in Fig. 4. The 4-plet contains two entries  $\Psi^\pm$  for each baryon, corresponding to internal orbital angular momentum  $L$  and  $L + 1$ . This property of the baryon LFWFs is the analog of the eigensolution of the Dirac-Coulomb equation which has both an upper component  $\Psi^+$  and a lower component  $\Psi^- = \frac{\vec{\sigma} \cdot \vec{p}}{m+E-V} \Psi^+$ .

Bound-state LF Schrödinger equations for both baryons and mesons can be derived from superconformal algebra [3, 4]. Remarkably, the predicted meson equation is identical to the LF Schrödinger equation with the same confinement potential and spin term that was derived from LF holography from AdS<sub>5</sub> with the soft-wall dilaton  $e^{+\kappa^2 z^2}$ , with the same confinement potential and spin term. See Eqs. (1) and (2). The baryonic eigensolutions correspond to bound states of  $3_C$  quarks to a  $\bar{3}_C$  spin-0 or spin-1  $qq$  diquark cluster; they each have two amplitudes  $L_B, L_B + 1$  with have equal Fock state probability – a feature of “quark chirality invariance”. For example, the proton’s Fock state components  $\psi^+$  (with parallel quark and baryon spins) and  $\psi^-$  (with anti-parallel quark and baryon spins) have equal Fock state probability; thus the proton’s spin is carried by quark orbital angular momentum in the nonperturbative domain. Predictions for the static properties of the nucleons are discussed in Ref. [53]. The predicted spectrum,  $M^2(n, L) = 4\kappa^2(n + L)$  for mesons and  $M^2(n, L) = 4\kappa^2(n + L + 1)$  for nucleons, is remarkably consistent with observed hadronic spectroscopy: The Regge-slopes in  $n$  and  $L$  are identical. The extension of the superconformal structure to include the spin-spin interaction is described [54]. The predicted meson, baryon and tetraquark masses coincide if one identifies a meson with internal orbital angular momentum  $L_M$  with its superpartner baryon or tetraquark with  $L_B = L_M - 1$ . Superconformal algebra thus predicts that mesons with  $L_M = L_B + 1$  have the same mass as the baryons in the supermultiplet. An example of the mass degeneracy of the  $\rho/\omega$  meson Regge trajectory with the  $J = 3/2$   $\Delta$ -baryon trajectory is shown in Fig. 5. The value of  $\kappa$  is not a fixed parameter, since only ratios of masses are predicted by QCD.

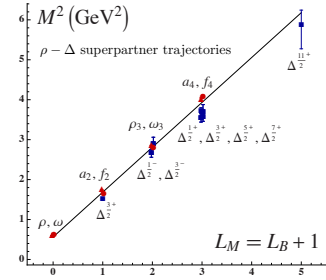
The combination of superconformal algebra and light-front dynamics thus leads to the novel prediction that hadron physics has supersymmetric properties in both spectroscopy and dynamics. The excitation spectra of relativistic light-quark meson, baryon and tetraquark bound states all lie on linear Regge trajectories with identical slopes in the radial and orbital quantum numbers. Detailed predictions for the tetraquark spectroscopy and comparisons with the observed hadron spectrum are presented in Ref. [55].

## 6. Supersymmetric Hadron Spectroscopy for Heavy Quarks

The predictions from light-front holography and superconformal algebra have been extended to mesons, baryons, and tetraquarks with strange, charm and bottom quarks in Refs. [10, 11]. Although conformal symmetry is strongly broken by the heavy quark mass, the basic underlying supersymmetric mechanism, which transforms mesons to baryons (and baryons to tetraquarks) into each other, still holds and gives remarkable connections and mass degeneracy across the entire spectrum of light, heavy-light and double-heavy hadrons. The excitation spectra of the heavy quark meson, baryon and tetraquark bound states continue to lie on universal linear Regge trajectories with identical slopes in the radial and orbital quantum numbers, but with an increased value for the slope. For example, the mass of the lightest double-charm baryon  $|c[ cq ]\rangle$ , where the composite  $[cq]$  is a scalar diquark, is predicted to be identical to the mass of the  $L = 1$  orbital excitation of the  $|c\bar{c}\rangle$  ( the  $1^{++}$   $h'_c(L = 1)$  ) and also the mass of the  $[[cq][\bar{c}\bar{q}]]$  double-charm tetraquark. In fact, the mass of the  $h_c(3525)$  matches the mass of the double-charm baryon  $\Xi_{cc}^+(3520)$  identified by SELEX and a tetraquark candidate the  $\Xi_{cc}(3415)$ . For more details, see Refs. [55, 56]. The effective supersymmetric properties of QCD can be used to identify the structure of the heavy quark mesons, baryons and tetraquark states [55].



**Figure 4:** The 4-plet representation of mass-degenerate hadronic states predicted by superconformal algebra [6]. Mesons are  $q\bar{q}$  bound states, baryons are quark – antiquark bound states and tetraquarks are diquark-antidiquark bound states. The supersymmetric ladder operator  $R_\lambda^\dagger$  connects quarks and antiquark clusters of the same color representation.



**Figure 5:** Comparison of the  $\rho/\omega$  meson Regge trajectory with the  $J = 3/2$   $\Delta$  baryon trajectory. Superconformal algebra predicts the degeneracy of the meson and baryon trajectories if one identifies a meson with internal orbital angular momentum  $L_M$  with its superpartner baryon with  $L_M = L_B + 1$ . See Refs. [3, 4].

Thus one predicts supersymmetric hadron spectroscopy – bosons and fermions with the same mass and twist. The members of the 4-plet not only have identical masses for the bosonic and fermionic hadron eigenvalues, but also supersymmetric relations between their eigenfunctions – their light-front wavefunctions. The baryonic eigensolutions correspond to bound states of  $3_C$  quarks to a  $\bar{3}_C$  spin-0 or spin-1  $qq$  diquark cluster; the tetraquarks in the 4-plet are bound states of diquarks and anti-diquarks. In the case of a nucleon, the overlap of the  $L = 0$  and  $L = 1$  LF wavefunctions in the Drell-Yan-West formula is required to have a non-zero Pauli form factor  $F_2(Q^2)$  and anomalous magnetic moment [38]. The existence of both components is also necessary to generate the pseudo-T-odd Sivers single-spin asymmetry in deep inelastic lepton-nucleon scattering [57].

## 7. Summary

The combination of conformal symmetry, light-front dynamics, the dAFF procedure (conformal quantum mechanics) and its holographic embedding in  $\text{AdS}_5$  space, has provided new insights, not only into the physics underlying color confinement, but also into the form of the nonperturbative QCD coupling and the QCD mass scale. A comprehensive review is given in Ref. [45].

The QCD Lagrangian is not supersymmetrical; it is based on quark and gluonic fields, not squarks nor gluinos. However, when one applies superconformal algebra [50] using the extended dAFF procedure of Fubini and Rabinovici [51], one obtains a unified spectroscopy of meson, baryon, and tetraquarks as equal-mass members of the same 4-plet representation, reflecting the underlying conformal symmetry of semi-classical QCD for massless quarks as well as the color dynamics underlying the supersymmetric connection of mesons, baryons and tetraquarks from the fundamental  $SU(3)_C$  representation  $\bar{\mathbf{3}} \sim \mathbf{3} \times \mathbf{3}$ . The set of “Light-front Schrödinger equations” predicted from superconformal algebra [3, 4], match the bound-state equation obtained from  $\text{AdS}_5$  and LF holography. They incorporate color confinement and other essential spectroscopic and dynamical features of hadron physics, including a massless pion for zero quark mass and linear Regge trajectories with the *same slope* in the radial quantum number  $n$  and internal orbital angular momentum  $L$  for mesons, baryons, and tetraquarks. In fact, the meson and baryon Regge trajectories and spectra are identical when one compares mesons with baryons with orbital angular momentum  $L_M = L_B + 1$ . Their LF wavefunctions, the eigensolutions of the LF Schrödinger equations, have the same underlying universal form. A new method for solving nonperturbative QCD “Basis Light-Front Quantization” (BLFQ) [58], uses the eigensolutions of a color-confining approximation to QCD (such as LF holography) as the basis functions, rather than the plane-wave basis used in DLCQ, thus incorporating the full dynamics of QCD.

The light-front holographic approach, with the constraints imposed by the superconformal algebraic structure, predicts novel supersymmetric relations between mesons, baryons, and tetraquarks of the same parity as members of the same 4-plet representation of superconformal algebra. Empirically viable predictions for spacelike and timelike hadronic form factors, structure functions, distribution amplitudes, and transverse momentum distributions have also been obtained [20, 59]. One can also observe features of superconformal symmetry in the spectroscopy and dynamics of heavy-light as well as double-heavy mesons and baryons [11, 55]. The combination of light-front field theory with superconformal algebra thus leads to the novel prediction that hadron physics has supersymmetric properties in both spectroscopy and dynamics. One can test the similarities of their wavefunctions and form factors in exclusive reactions such as  $e^+e^- \rightarrow \pi T$  where  $T$  is a tetraquark [23]. Quark counting rules for hadron form factors and other hard exclusive amplitudes are also a property of these nonperturbative solutions [22, 23]. One can thus use counting rules to identify the field content of mesons, baryons, tetraquarks, and gluonium. Light-front holography thus gives a remarkably simple analytic description of quark confinement, as well as nonperturbative hadronic structure and dynamics.

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