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Large-N_c QCD and tetraquarks

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Tetraquark properties are examined in the limit of large N_c of color in QCD. The qualitative differences between molecular and compact tetraquarks are outlined. Consequences of the possible existence of compact tetraquarks are analyzed and shown to lead to upper bounds in the N_c-behavior of their decay widths. Open questions on theoretical grounds, related to the dynamics of systems composed of two quarks and two antiquarks, are addressed.

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1. Multiquark states in QCD

Hadrons are color-singlet bound states of quarks and gluons. Mesons are essentially made of a quark and and an antiquark (plus gluons and sea quarks), while baryons are essentially made of three quarks (plus gluons and sea quarks). A schematic representation of mesons and baryons is given in Fig. 1.

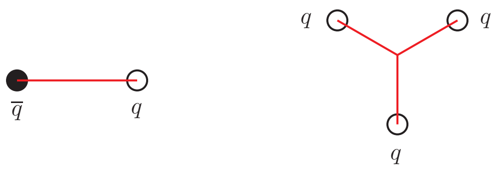

Figure 1: Schematic representation of mesons and baryons. The straight lines represent gauge links.

Are there other types of structure for bound states in QCD, which might be classified as exotics with respect to the conventional hadronic states? In principle, this would be possible by constructing gauge invariant composite operators which might generate exotic states. Examples of such states are tetraquarks, made essentially of two quarks and two antiquarks, and pentaquarks, made essentially of four quarks and one antiquark (Fig. 2).

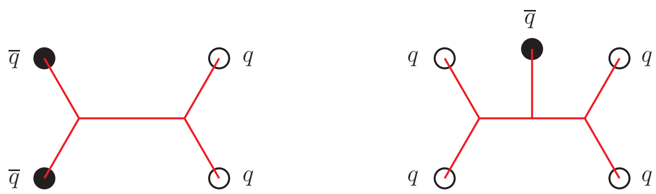

Figure 2: Schematic representation of tetraquarks and pentaquarks. The straight lines represent gauge links.

The possibility of the existence of multiquark states has been considered by many authors and in particular by Jaffe in the framework of the bag model [1]. However, theoretical difficulties arise in QCD. We concentrate in the following on the tetraquark problem. The difficulty is related to the fact that a tetraquark field, local or nonlocal, made of a pair of quark and antiquark fields, which would be color-gauge invariant, can be decomposed, by Fierz transformations, into a combination of products of color-singlet bilinear operators of quark-antiquark pairs. For instance, in local form, a four-quark color-singlet operator $T(x) = (\overline{q}\overline{q}qq)(x)$ can be decomposed as

$$T(x) \equiv (\overline{q}\overline{q}qq)(x) \sim \sum (\overline{q}q)(x)(\overline{q}q)(x), \tag{1.1}$$

where the $(\overline{q}q)(x)$s are themselves color-singlet operators.

However, color-singlet bilinears esssentially describe ordinary meson fields or states. The above decomposition is suggestive of a property that tetraquarks would be factorizable into independent mesons and could at best be bound states or resonances of mesons [2–7], also called *molecular tetraquarks*, and not genuine bound states of two quarks and two antiquarks, which would result from the direct confinement of the four constituents.

What would be, on phenomenological grounds, the difference of the two types of bound state, since both of them would be represented by poles in the hadronic sector?

Meson-meson interaction forces are short-range and weak, as compared to the strong long-distance confining forces. Therefore, molecular type tetraquarks would be loosely bound states, with relatively large space extensions, while tetraquarks formed directly by confining forces would be more tightly bound. The latter are also called *compact tetraquarks*. Compact tetraquarks would also exist in flavor multiplicities, since confinement is independent of flavor.

The above qualitative differences have their influence on phenomenological quantities, like the number of states, decay modes, decay widths and transition amplitudes.

For more than ten years, many tetraquark candidate states have been signalled by several experiments: BaBar, Belle, BESIII, CDF, CLEO, D0, ATLAS, CMS, LHCb. Ordinary meson structures could not fit their properties. An intense theoretical activity has been developed around the extraction of their properties and their interpretation. However, there are difficulties to explain all data by a single model or mechanism. Many review articles give thorough descriptions of the various facets of the problem [8–15].

In lattice calculations, a general consensus does not yet seem to exist, a majority of investigations not providing evidence for tetraquarks [16–20], while evidence is found in a few sectors involving the b quark [21, 22].

We shall be interested here by the qualitative properties of compact tetraquarks, since the very existence of compact tetraquarks is intimately related to fundamental properties of QCD, not yet well understood. The existence of molecular type tetraquarks does not raise any conceptual difficulty.

The *diquark model* has been proposed to explain the formation of compact multiquark states [23–26]. It has been applied to tetraquark phenomenology by Maiani, Piccinini, Polosa and Riquer [27, 28]. Other approaches with the diquark mechanism can be found in [29–32].

We shall study the tetraquark problem in the large-N_c limit of QCD, which might give us complementary informations about it.

2. QCD at large N_c

The framework that is considered is that of $\mathrm{SU}(N_c)$ gauge theories with quark fields belonging to the fundamental representation. N_c is considered as a free parameter and the limit of large values of N_c is taken, while, to ensure a stable limit, the coupling constant is assumed to scale as $g \sim 1/N_c^{1/2}$. This limit has been introduced by 't Hooft, who has studied its general properties [33, 34]. It has been found that in this limit QCD catches the main properties of confinement, while being simplified with respect to secondary complications, like quark pair creation or inelasticities. $1/N_c$ plays here the role of a perturbative parameter and allows the classification of Feynman diagrams according to their topology and relevance (planar, nonplanar, etc.).

The properties of the theory concerning meson and baryon states have been analyzed by Witten [35]. The analysis is done with the aid of two-point, three-point and four-point functions of quark color-singlet bilinear operators (currents), $j(x) = (\overline{q}\Gamma q)(x)$, where Γ represents Dirac matrices, and the study of their large-N_c behavior.

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It is found, from the study of the two-point functions, that, at large-N_c, the hadronic spectrum is saturated by an infinite number of free stable mesons, made of a quark, an antiquark and gluons. (The infinite number is dictated by asymptotic freedom.) Their masses are in general finite:

$$M_n = O(N_c^0), \quad n = 1, 2, \dots \,. \tag{2.1}$$

Many-meson states contribute only at subleading orders in $1/N_c$. Baryons, which at large N_c are essentially made of N_c quarks, have masses that grow like N_c.

From the study of the three-point and four-point functions, one deduces the behavior of the three-meson and four-meson effective couplings, which are vanishing at large N_c (Fig. 3):

$$g(MMM) = O(N_c^{-1/2}); \qquad g(MMMM) = O(N_c^{-1}). \tag{2.2}$$

These behaviors imply that, at large N_c, the mesons decouple from each other and become free

Figure 3: Generic three-meson and four-meson effective couplings.

particles.

From Eqs. (2.2) one deduces the strong decay width behavior of the mesons:

$$\Gamma(M) = O(N_c^{-1}), \tag{2.3}$$

which also expresses their stability property at large N_c.

3. Tetraquarks at large N_c

Can we have similar predictions with tetraquarks? To this end, one may consider the analog of a bilinear current, a four-quark color-singlet current, Eq. (1.1), and its two-point function. It turns out that, at large N_c, the latter is dominated by disconnected pieces of two-point bilinear currents:

$$\langle T(x)T^\dagger(0)\rangle \underset{N_c\to\infty}{=} \langle j(x)j^\dagger(0)\rangle \, \langle j(x)j^\dagger(0)\rangle. \tag{3.1}$$

The right-hand side describes the propagation of two free ordinary mesons [36]. No tetraquark pole can appear at this order.

This fact has been considered as a theoretical proof of the non-existence of tetraquarks as elementary stable particles, which could survive, like the ordinary mesons, in the large-N_c limit.

Recently, Weinberg has observed that if tetraquarks exist as bound states in the large-N_c limit with finite masses, even if they contribute to subleading diagrams, the crucial point is the qualitative property of their decay widths: are they broad or narrow? In the latter case, they might be observable. He has shown that, generally, they should be narrow, with decay widths of the order of $1/N_c$, which is compatible with the stability assumption in the large-N_c limit [37].

Knecht and Peris have shown that in a particular exotic channel, tetraquarks should even be narrower, with decay widths of the order of $1/N_c^2$ [38].

Cohen and Lebed have shown, for more general exotic channels, with an analysis based on the analyticity properties of two-meson scattering amplitudes, that the decay widths should be of the order of $1/N_c^2$ [39].

Lucha, Melikhov and Sazdjian show, on the basis of the singularity analysis of Feynman diagrams with the Landau equations [40, 41], that tetraquark two-meson decay widths of order $1/N_c^2$ actually represent the most general case. For fully exotic tetraquarks (four different quark flavors) two different tetraquarks, each having a preferred decay channel, would be needed [42, 43].

Maiani, Polosa and Riquer impose additional selection rules to make the behaviors compatible with the diquark model and predict two-meson decay widths of order $1/N_c^3$ [44] and $1/N_c^4$ [45].

The particular case of the light meson scattering amplitudes has been studied by Peláez and Rios within the framework of chiral perturbation theory [46, 47] . The existence of scalar mesons with a tetraquark type structure has been found. In the large-N_c limit, their masses and decay widths diverge as $N_c^{1/2}$. Such behaviors do not fit the characteristics of compact tetraquarks, as conjectured by Weinberg. These scalar mesons would rather fit a molecular type structure; this is also supported by a direct resolution of the four-body Bethe-Salpeter equation [48].

The large-N_c behavior of meson scattering amplitudes in connection with lattice calculations has also been studied in [49].

4. Weak point of the large-N_c analysis

Contrary to the case of two-point functions of quark bilinear currents and their saturation by ordinary meson states, possible tetraquark contributions to two-meson scattering amplitudes are competing with the background of two-meson states, which consistently saturate, on qualitative grounds, the corresponding correlation functions. Unless one does detailed quantitative calculations of the numerical coefficients of Feynman diagrams and compares them with the contributions of hadronic intermediate states, the hypothesis of an eventual absence of tetraquark states does not lead to any qualitative inconsistency [43].

Therefore, the predictions made for tetraquark decay amplitudes are based on the assumption of their possible existence and should be considered as upper bounds.

5. Line of approach

The study of possibly existing tetraquark properties is done through the analysis of meson-meson scattering amplitudes [39, 42, 43].

One considers four-point correlation functions of color-singlet quark bilinears,

$$j_{ab} = \overline{q}_a q_b, \tag{5.1}$$

having a coupling with a meson M_{ab}:

$$\langle 0|j_{ab}|M_{ab}\rangle = f_{M_{ab}}; \qquad f_M = O(N_c^{1/2}). \tag{5.2}$$

(a, b refer to flavor indices.) Spin and parity are ignored here, since they are not relevant for the qualitative aspects that are deduced.

One should consider all possible channels where a tetraquark may be present.

To be sure that a QCD diagram may contain a tetraquark contribution through a pole term, one has to check that it receives a four-quark contribution in its s-channel singularities, plus additional gluon singularities that do not modify the N_c-behavior of the diagram. Their existence is checked with the use of the Landau equations [40, 41].

Diagrams that do not have s-channel singularities, or have only two-particle singularities (quark-antiquark), cannot contribute to the formation of tetraquarks at their N_c-leading order. They should not be taken into account for the N_c-behavior analysis of the tetraquark properties.

We consider here the case of fully exotic tetraquarks, containing four distinct quark flavors, which we denote by 1,2,3,4, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{34} = \overline{q}_3 q_4, \quad j_{14} = \overline{q}_1 q_4, \quad j_{32} = \overline{q}_3 q_2. \tag{5.3}$$

The following scattering processes are considered:

$$M_{12} + M_{34} \to M_{12} + M_{34}, \quad \text{Direct channel I;} \tag{5.4}$$

$$M_{14} + M_{32} \to M_{14} + M_{32}, \quad \text{Direct channel II;} \tag{5.5}$$

$$M_{12} + M_{34} \to M_{14} + M_{32}, \quad \text{Recombination channel.} \tag{5.6}$$

The 'direct' four-point functions are

$$\Gamma_I^{(\mathrm{dir})} = \langle j_{12} j_{34} j_{34}^\dagger j_{12}^\dagger \rangle , \qquad \Gamma_{II}^{(\mathrm{dir})} = \langle j_{14} j_{32} j_{32}^\dagger j_{14}^\dagger \rangle . \tag{5.7}$$

Samples of N_c-leading and subleading diagrams for $\Gamma_I^{(dir)}$ are presented in Fig. 4.

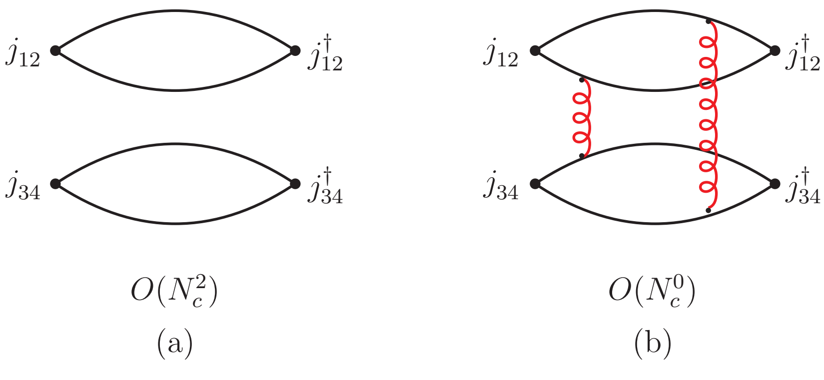

Figure 4: Samples of leading and subleading diagrams for the direct channel I.

Similar diagrams also exist for $\Gamma_{II}^{(dir)}$.

Only diagram (b) may receive contributions from tetraquark states.

The 'recombination' 4-point function is

$$\Gamma^{(\mathrm{recomb})} = \langle j_{12} j_{34} j_{32}^\dagger j_{14}^\dagger \rangle . \tag{5.8}$$

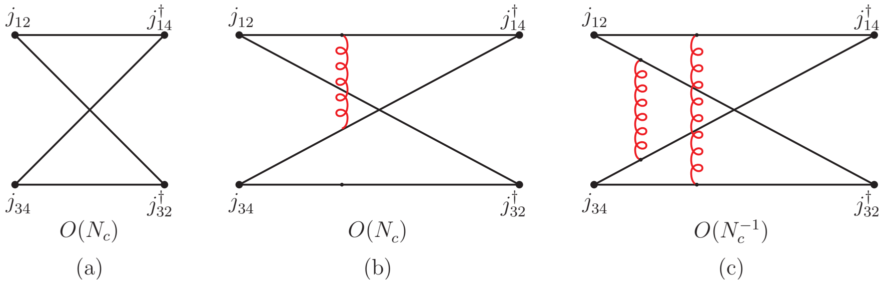

Figure 5: Samples of leading and subleading diagrams for the recombination channel.

Samples of leading and subleading diagrams are presented in Fig. 5. Only diagram (c) may receive contributions from tetraquark states. Contrary to apparencies, diagrams (a) and (b) do not have s-channel singularities. Their singularities lie either in the t- and u-channels, or contribute to the external meson propagators or vertices.

We note that direct and recombination scattering amplitudes have different behaviors in N_c. Therefore, each channel imposes a different constraint. Consideration of only one channel would lead to incomplete solutions.

The solution requires the contribution of two different tetraquarks, T_A and T_B, say, each having different couplings to the meson pairs. One finds for the tetraquark – two-meson transition amplitudes:

$$A(T_A \to M_{12}M_{34}) = O(N_c^{-1}), \qquad A(T_A \to M_{14}M_{32}) = O(N_c^{-2}), \tag{5.9}$$

$$A(T_B \to M_{12}M_{34}) = O(N_c^{-2}), \qquad A(T_B \to M_{14}M_{32}) = O(N_c^{-1}). \tag{5.10}$$

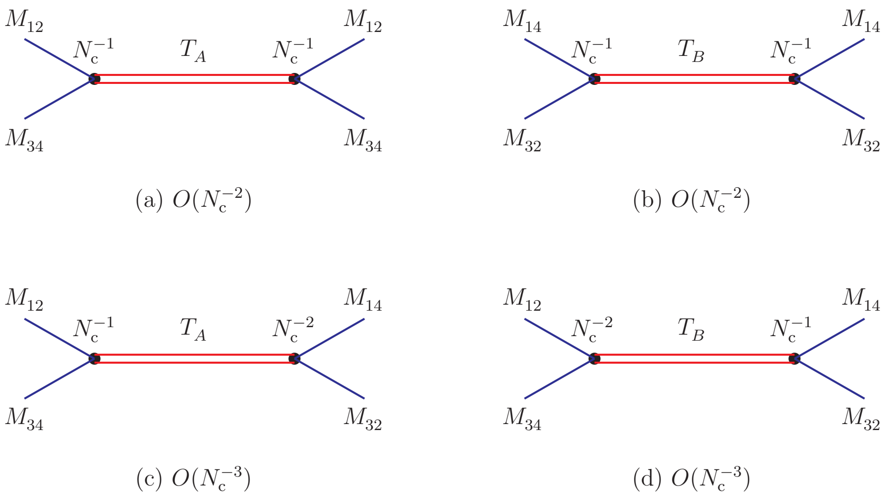

Figure 6: Dominant tetraquark intermediate states in meson-meson scattering. (a): direct channel I; (b): direct channel II; (c) and (d): recombination channel.

The two-meson decay widths of the teraquarks are

$$\Gamma(T_A) \sim \Gamma(T_B) = O(N_c^{-2}), \qquad (5.11)$$

which are much smaller than those of ordinary mesons [Eq. (2.3)]. A diagrammatic representation of the tetraquark intermediate states with their couplings to the external mesons is given in Fig. 6.

There is also a generation of background meson-meson effective interaction by means of four-meson effective couplings represented in Fig. 7. Notice that the recombination (quark exchange) effective coupling dominates; it is actually generated by the leading diagrams of the recombination channel ((a) and (b) of Fig. 5).

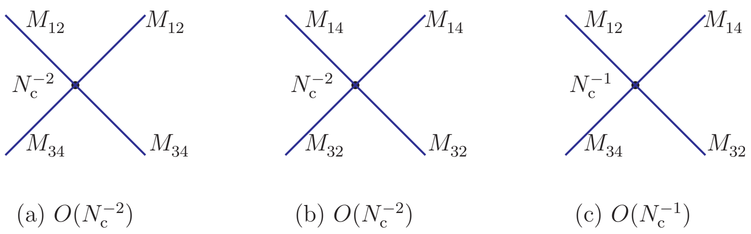

Figure 7: Four-meson effective couplings generated in the direct channel I (diagram (a)), direct channel II (diagram (b)) and the recombination channel (diagram (c)).

These, in turn, generate meson loops (Fig. 8).

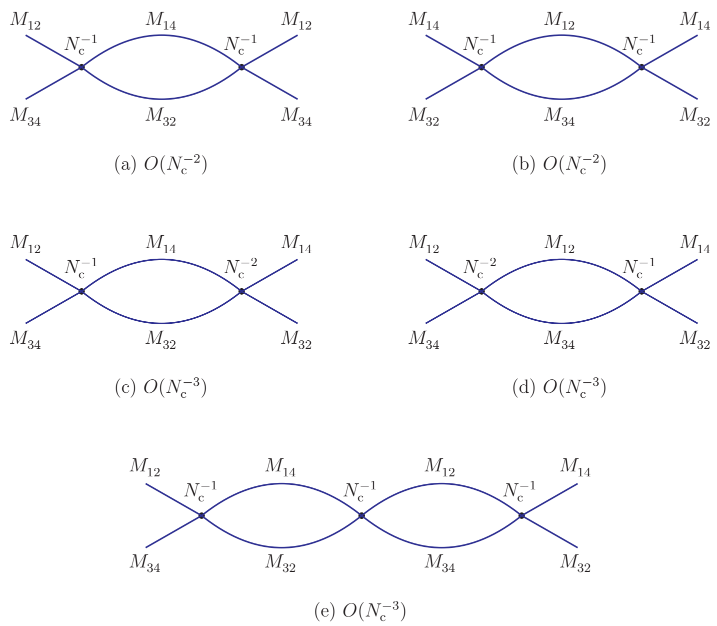

Figure 8: Two-meson intermediate state contributions.

From the four-meson couplings of Fig. 7 and the transition amplitudes of Eqs. (5.9) and (5.10), one can reconstitute an effective interaction Lagrangian expressed in terms of quark color-singlet bilinears, from which one deduces the dominant structure of the two tetraquarks T_A and T_B in terms of the latter quantities [43]. One finds:

$$T_A \sim (\overline{q}_1 q_4)(\overline{q}_3 q_2), \qquad T_B \sim (\overline{q}_1 q_2)(\overline{q}_3 q_4), \tag{5.12}$$

mixings of order $1/N_c$ between the two configurations being possible.

The above result favors a color singlet-singlet structure of the tetraquarks.

6. Dynamical aspects

The fact that we have two different tetraquarks, each having a structure made of two color-singlet clusters, raises a few questions.

First, once the color-singlet clusters are formed inside the four-body system, their mutual interaction can no longer be confining (Fig. 9).

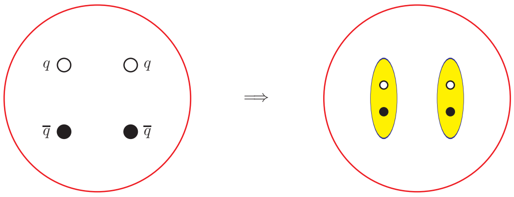

Figure 9: Formation of two color-singlet clusters inside the four-body system. (The external circles are a schematic representation of the system and do not represent confining bags.)

In general, one expects in such a case a short-range interaction between the two clusters, due to meson exchanges or contact terms, which would eventually produce a molecular type tetraquark. At most, one might expect long-range type Van der Waals forces, reminiscent of the confining forces. In that case, the tetraquarks, if they exist, would still be loosely bound, similarly to the molecular type tetraquarks. This possibility has already been foreseen by Jaffe, stressing that there would not be a clear phenomenological distinction between the two situations [50].

Second, the necessity of having two different tetraquarks to accomodate the N_c-counting constraints does not fit the diquark formation scheme, where only one type of diquark is expected to be formed, in its color-antisymmetric representation (Fig. 10).

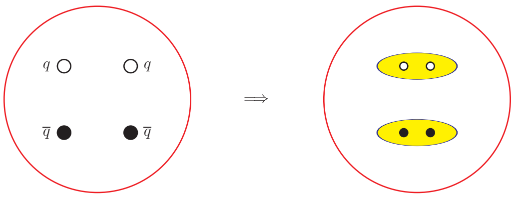

Figure 10: The diquark formation scheme.

The binding of the diquark and antidiquark clusters is expected to be realized by means of confining forces, hence favoring the appearance of compact tetraquarks.

To make the diquark scheme compatible with the N_c-analysis, Maiani, Polosa and Riquer impose additional constraints for the selection of diagrams contributing to the formation of tetraquarks [45]. Only non-planar diagram contributions are retained; this lowers the contribution of direct channel diagrams by two degrees in N_c. To obtain a consistent solution with one tetraquark, it is then assumed that, at leading order in N_c, tetraquarks contribute only to the direct channel diagrams. The decay width into two-mesons is found of the order of $1/N_c^4$.

These constraints, while mathematically correct, require, however, a more detailed analysis of the corresponding dynamical mechanism that is at their origin.

7. Conclusion

The large-N_c limit of QCD allows us to have a complementary insight into the problem of multiquark states. Tetraquark and multiquark states, if they exist, do not generally appear in N_c-leading-order terms and are competing with multimeson background contributions. The conventional large-N_c-based analysis does not lead to a proof of their existence, but simply gives upper bounds for their decay or transition amplitudes.

The generic results, in the case of four quark flavors, have the tendancy to favor the formation of tetraquarks with two color-singlet internal clusters. In such a case, the tetraquarks would probably be loosely bound.

The diquark scheme, which might lead to the emergence of compact tetraquarks, requires fine tuning dynamical mechanisms.

The resolution of four-body bound state equations in conjunction with large-N_c analysis might bring further information for a better understanding of the question.

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