

Results for the mass difference between the long- and short- lived K mesons for physical quark masses

Bigeng Wang^{*†}

Department of Physics, Columbia University, New York, NY 10027, USA

E-mail: bw2482@columbia.edu

The two neutral kaon states in nature, the K_L (long-lived) and K_S (short-lived) mesons, are the two time-evolution eigenstates of the $K^0 - \bar{K}^0$ mixing system. The prediction of their mass difference Δm_K based on the Standard Model is an important goal of lattice QCD. In this article, I will present preliminary results from a calculation of Δm_K performed on an ensemble of $64^3 \times 128$ gauge configurations with inverse lattice spacing of 2.36 GeV and physical quark masses. These new results come from 2.5 times the Monte Carlo statistics used for the result presented in last year's conference. Further discussion of the methods employed and the resulting systematic errors will be given.

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1. Introduction

The mass difference between K_L and K_S is generated by K meson mixing through $\Delta S = 2$ weak interaction. With its experimental value of $3.483(6) \times 10^{-12}$ MeV measured with sub percentage error, a discrepancy between the prediction based on the Standard Model and this value implies the existence of physics beyond the Standard Model. This quantity is highly non-perturbative and can be calculated using Lattice QCD from first principles. Since 2013 an exploratory calculation on a $16^3 \times 32$ calculation, with unphysical masses ($m_\pi = 421$ MeV) including only connected diagram[2], the RBC-UKQCD collaborations have been improving the calculation by including disconnected diagrams and extending measurements to finer lattice spacing [3]. Our most recent calculation on a $64^3 \times 128$ lattice with physical masses on 59 configurations gives a preliminary result of $\Delta m_k = (5.5 \pm 1.7) \times 10^{-12}$ MeV [1]. In this article, an update of the methods and results extending our calculation from 59 to 152 configurations is presented.

2. Integrated Correlator and Δm_K

The $K_L - K_S$ mass difference is expressed as:

$$\Delta M_K = 2\text{Re}M_{00} = 2\mathcal{P} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n}. \quad (2.1)$$

To evaluate Δm_K on an Euclidean space lattice, we evaluate the integrated correlators:

$$\mathcal{A}(T) = \frac{1}{2} \sum_{t_2=t_a}^{t_b} \sum_{t_1=t_a}^{t_b} \langle 0 | T \{ \bar{K}^0(t_f) H_W(t_2) H_W(t_1) K^0(t_i) \} | 0 \rangle, \quad (2.2)$$

where H_W is the $\Delta S = 1$ effective Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V_{q's}^* (C_1 Q_1^{qq'} + C_2 Q_2^{qq'}). \quad (2.3)$$

Here the $Q_i^{qq'}$ $_{i=1,2}$ are current-current operators, defined as:

$$Q_1^{qq'} = (\bar{s}_i \gamma^\mu (1 - \gamma_5) d_i) (\bar{q}_j \gamma^\mu (1 - \gamma_5) q'_j), \quad Q_2^{qq'} = (\bar{s}_i \gamma^\mu (1 - \gamma_5) d_j) (\bar{q}_j \gamma^\mu (1 - \gamma_5) q'_i), \quad (2.4)$$

and $V_{q_a q_b}$ are the usual CKM matrix elements and C_i are Wilson coefficients.

If we insert a complete set of intermediate states, we identify the coefficient of the term linear in the size of integration box $T = t_b - t_a + 1$ as proportional to the expression for Δm_K given in Equation 2.1:

$$\mathcal{A}(T) = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \left\{ -T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n} \right\}. \quad (2.5)$$

Before doing a linear fitting with respect to T , the second term in the curly bracket has to be removed. For an intermediate state $|n\rangle$ with energy E_n larger than m_K , for large enough T , the contribution from the second term is negligible. For a state $|n\rangle$ with energy E_n smaller than or close to m_K , we need to subtract its contribution.

In our case of physical quark masses, $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$ and $|\pi\rangle$ states need to be subtracted. With the freedom of adding the operators $\bar{s}d$ and $\bar{s}\gamma_5 d$ to the weak Hamiltonian with properly chosen coefficients c_s and c_p , we are able to remove two of the contributions. Here we choose c_s and c_p to satisfy Equation 2.6 so that contributions from $|0\rangle$ and $|\eta\rangle$ will vanish:

$$\langle 0|H_W - c_p \bar{s}\gamma_5 d|K^0\rangle = 0, \quad \langle \eta|H_W - c_s \bar{s}d|K^0\rangle = 0. \quad (2.6)$$

As a result, the original $\Delta S = 1$ effective weak Hamiltonian in Equation 2.3 and the current-current operators should be modified to be :

$$Q'_i = Q_i - c_{pi} \bar{s}\gamma_5 d - c_{si} \bar{s}d \quad (2.7)$$

with c_{pi} and c_{si} are calculated on lattice using Equation 2.8.

$$c_{si} = \frac{\langle \eta|Q_i|K^0\rangle}{\langle \eta|\bar{s}d|K^0\rangle}, \quad c_{pi} = \frac{\langle 0|Q_i|K^0\rangle}{\langle 0|\bar{s}\gamma_5 d|K^0\rangle}. \quad (2.8)$$

For contractions among Q_i , there are four types of diagrams to be evaluated, as shown in Figure 1. In addition, there are "mixed" diagrams from the contractions between the $\bar{s}d$, $\bar{s}\gamma_5 d$ and Q_i operators, having similar topologies to type 3 and type 4 contractions.

The GIM mechanism removes both quadratic and logarithmic divergences that might otherwise be expected as the two H_W operators approach each other. We therefore include the charm quark in our calculation and as a result always have the difference between up and charm quark propagators for every charge $+2/3$ quark line.

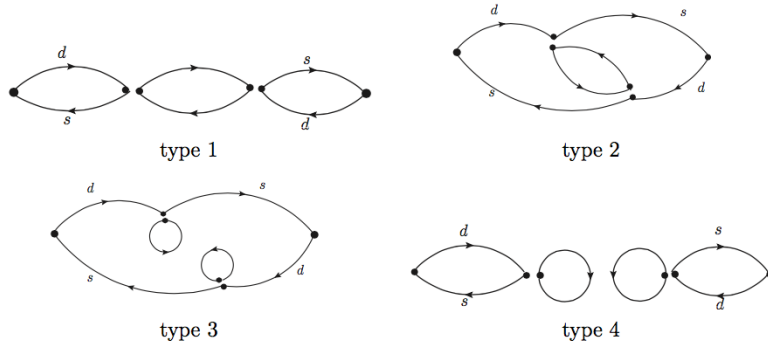


Figure 1: Four types of contractions in the 4-point correlators with Q_1 and Q_2 .

3. From lattice results to physical Δm_K

The fitting of the integrated correlator in Equation 2.2 further breaks into fitting of the integrated correlator with Q_1 and Q_2 :

$$\mathcal{A}_{ij}(T) = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0|Q_i|n\rangle \langle n|Q_j|K^0\rangle}{m_K - E_n} \left\{ -T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n} \right\}. \quad (3.1)$$

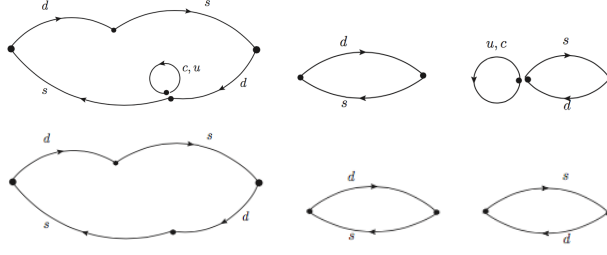


Figure 2: The "mixed" contractions in the 4-point correlators. The top two are products of the $c_{si}\bar{s}d$ and $c_{pi}\bar{s}\gamma_5 d$ with the Q_i and the bottom two are products of the $c_{si}\bar{s}d$ and $c_{pi}\bar{s}\gamma_5 d$. The diagrams on the left are similar to type-3 diagrams and the diagrams on the right are similar to type-4 diagrams.

Considering the GIM mechanism, the relationship between $\mathcal{A}_{ij}(T)$ in Equation 3.1 and $\mathcal{A}(T)$ in Equation 2.2 is:

$$\mathcal{A}(T) = \lambda_u^2 \sum_{i,j=1,2} C_i C_j \mathcal{A}_{ij}(T), \quad (3.2)$$

where the C_i are Wilson coefficients and $\lambda_u = V_{ud}V_{us}^*$.

We fit each $\mathcal{A}_{ij}(T)$ separately and obtain the k_{ij} , coefficient of the linear term of T . The value of Δm_K from the lattice should be:

$$\Delta m_K^{lat} = \frac{G_F^2}{2} \lambda_u^2 \sum_{i,j=1,2} (-2) \times C_i^{lat} C_j^{lat} k_{ij}. \quad (3.3)$$

We obtain the Wilson coefficients of the operators C_i^{lat} in three steps [4] [5]:

- Non-perturbative Renormalization: from lattice to RI-SMOM
- Perturbation theory: from RI-SMOM to \overline{MS}
- Perturbation theory: Wilson coefficients in the \overline{MS} scheme

So C_1^{lat} and C_2^{lat} can be expressed as:

$$C_i^{lat} = C_a^{\overline{MS}} (1 + \Delta r)_{ab}^{RI \rightarrow \overline{MS}} Z_{bi}^{lat \rightarrow RI}. \quad (3.4)$$

4. Sample AMA and super-jackknife method

We use sample All Mode Averaging(AMA) method to reduce the computational cost[6]. The usual AMA correction is applied on each configuration, among different time slices. In contrast the sample AMA correction is applied among configurations: on most configurations, quantities are calculated with a CG stopping residual of 10^{-4} ("sloppy"). On the other configurations the same quantities are calculated with a CG stopping residual of both 10^{-4} ("sloppy") and 10^{-8} ("exact"). The differences between "sloppy" and "exact" measurements are used as corrections to the "sloppy" only configurations.

In our case, we have data for type-3 and type-4 diagrams, three-point and two-point functions from both "sloppy" measurements and corrections. We firstly jackknife the "sloppy" and

β	am_l	am_h	$\alpha = b + c$	L_s
2.25	0.0006203	0.02539	2.0	12

Table 1: Input parameters of the lattice calculation.

Data Set	K^0	π	η	$\pi\pi_{I=0}$
new 152	0.2104(1)	0.0574(1)	0.258(2)	0.1138(5)
old 59	0.2105(2)	0.0576(1)	0.290(29)	0.1137(8)

Table 2: Fitting results for meson masses and $\pi - \pi$ energy in lattice units ($a^{-1} = 2.36$ GeV)

correction data separately and then use the super-jackknife method to estimate the error of the super-jackknife samples. For a certain quantity Y , a pion correlator as an example, from the N_s "sloppy" measurements $\{y_i\}_{i=1,\dots,N_s}$, we obtain the jackknife "sloppy" ensemble $\{Y_i\}_{i=1,\dots,N_s}$ with $Y_i = \frac{1}{N_s-1} \sum_{j \neq i} y_j$. Similarly, from the N_c corrections $\{\Delta y_i\}_{i=1,\dots,N_c}$, we obtain the jackknife correction ensemble $\{\Delta Y_i\}_{i=1,\dots,N_c}$ with $\Delta Y_i = \frac{1}{N_c-1} \sum_{j \neq i} \Delta y_j$. We then combine the two jackknife ensembles to form a super-jackknife ensemble $\{Y'_k\}_{k=1,\dots,N_s+N_c}$ with $N_s + N_c$ elements, where:

$$Y'_k = Y_k + \overline{\Delta Y}, \quad k = 1, \dots, N_s \quad (4.1)$$

$$Y'_k = \overline{Y} + \Delta Y_{k-N_s}, \quad k = N_s + 1, \dots, N_s + N_c \quad (4.2)$$

where $\overline{\Delta Y} = \frac{1}{N_c} \sum_{j=1}^{N_c} \Delta Y_j$ is the mean value of the corrections and $\overline{Y} = \frac{1}{N_s} \sum_{i=1}^{N_s} Y_i$ is the mean value of the "sloppy" measurements.

5. Lattice calculation and results

The calculation was performed on a $64^3 \times 128 \times 12$ lattice with 2+1 flavors of Möbius DWF and the Iwasaki gauge action with physical pion mass (136 MeV) and inverse lattice spacing $a^{-1} = 2.36$ GeV. The inputs parameters are listed in Table 1. We will compare results presented in Lattice 2017 [1] with our updated results. We have in total 152 configurations, among which 116 configurations are "sloppy" and 36 configurations are used for corrections. In the tables below, we refer to the updated data set as "new 152" and data set used in 2017 as "old 59".

The masses from fitting two-point correlators are included in Table 2. Amplitudes and coefficients for subtractions are listed in Table 3, Table 4, and Table 5. These results are consistent within errors. As the statistics increase, the errors scale approximately as $\frac{1}{\sqrt{N}}$, where N is the number of total measurements.

Data Set	$\langle \pi Q_1 K^0 \rangle$	$\langle \pi Q_2 K^0 \rangle$	$\langle 0 Q_1 K^0 \rangle$	$\langle 0 Q_2 K^0 \rangle$
new 152	$-5.02(3) \times 10^{-4}$	$1.407(4) \times 10^{-3}$	$-1.284(3) \times 10^{-2}$	$2.449(4) \times 10^{-2}$
old 59	$-5.08(5) \times 10^{-4}$	$1.407(8) \times 10^{-3}$	$-1.289(4) \times 10^{-2}$	$2.454(7) \times 10^{-2}$

Table 3: The K^0 to π matrix element and the K^0 to vacuum matrix element, without subtracting the $\bar{s}d$ operator.

Data Set	c_{s1}	c_{s2}	c_{p1}	c_{p2}
new 152	$2.13(33) \times 10^{-4}$	$-3.16(25) \times 10^{-4}$	$-1.472(2) \times 10^{-4}$	$2.807(2) \times 10^{-4}$
old 59	$1.53(64) \times 10^{-4}$	$-2.77(42) \times 10^{-4}$	$-1.476(3) \times 10^{-4}$	$2.811(3) \times 10^{-4}$

Table 4: The subtraction coefficients for the scalar and pseudo-scalar operator.

Data Set	$\langle \pi \pi_{I=2} Q_1 K^0 \rangle$	$\langle \pi \pi_{I=2} Q_2 K^0 \rangle$	$\langle \pi \pi_{I=0} Q_1 K^0 \rangle$	$\langle \pi \pi_{I=0} Q_2 K^0 \rangle$
new 152	$1.473(6) \times 10^{-5}$	$1.473(6) \times 10^{-5}$	$-8.7(1.5) \times 10^{-5}$	$9.5(1.5) \times 10^{-5}$
old 59	$1.471(10) \times 10^{-5}$	$1.471(10) \times 10^{-5}$	$-6.6(2.5) \times 10^{-5}$	$7.9(2.3) \times 10^{-5}$

Table 5: The K to $\pi\pi$ matrix element for Isospin 0 and 2. The I=2 matrix element for Q_1 and Q_2 are the same because they come from the same three point diagrams.

5.1 Four-point integrated correlators and Δm_K

The integrated correlators $\mathcal{A}_{ij}(T)$ are plotted in Figure 3. The fitted slopes k_{ij} all should be improved with 2.5 times our earlier statistics. The Δm_K value and separated contributions from different types of diagrams after normalization are shown in Table 6. Based on the formula proposed in [7], our estimated finite-volume correction for Δm_K is:

$$\Delta m_K(FV) = -0.22(7) \times 10^{-12} \text{MeV}$$

and our preliminary result for Δm_K is:

$$\Delta m_K = 7.9(1.3) \times 10^{-12} \text{MeV}.$$

In order to realize the GIM mechanism in our calculation, the charm quark is included in our calculation. The lattice spacing in our calculation is $a^{-1} = 2.36 \text{ GeV}$, which is only twice the charm quark mass. Discretization effects are estimated to be the largest source of systematic error: $\sim (m_c a)^2$ is $\sim 25\%$.

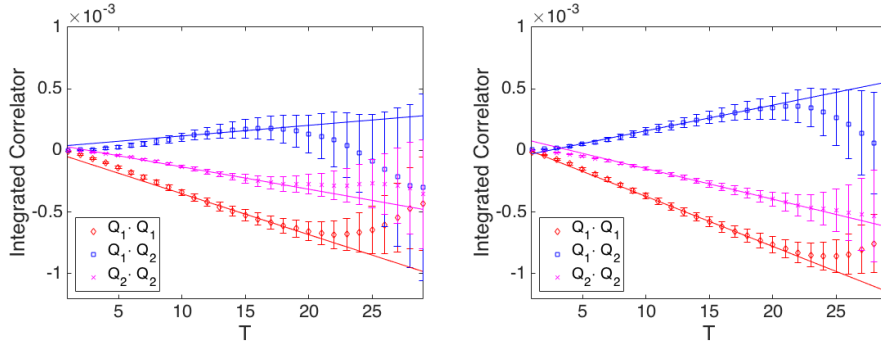


Figure 3: Integrated correlators $\mathcal{A}_{ij}(T)$. The left plot shows the result from "old 59". The right plot shows the result from "new 152".

5.2 Sample AMA correction

Our use of the sample AMA method reduced the computational cost of the calculation by a factor of 2.3, while the statistical error on the correction will add to the total statistical error. Table

Data Set	Δm_K	$\Delta m_K(\text{tp12})$	$\Delta m_K(\text{tp34})$	$\Delta m_K(\text{tp3})$	$\Delta m_K(\text{tp4})$
new 152	8.2(1.3)	8.3(0.6)	0.1(1.1)	1.58(31)	-1.28(94)
old 59	5.8(1.8)	7.0(1.3)	-1.1(1.2)	1.17(43)	-2.16(1.20)

Table 6: Results for Δm_K from uncorrelated fits in units of 10^{-12} MeV with fitting range 10:20.

Data Set	type 3&4 error from "sloppy"	type 3&4 error from correction	type 3&4 error in total
new 152	0.9	0.6	1.1
old 59	1.1	0.6	1.2

Table 7: Error contributions to Δm_K from type-3 and type-4 diagrams (in units of 10^{-12} MeV). From left to right, type-3 and type-4 errors from "sloppy", from correction and in total are shown. In our calculation, type-3 and type-4 diagrams are AMA corrected while type-1 and type-2 diagrams are only calculated as part of the exact measurements that are also used to determine the corrections for the type-3 and type-4 diagrams.

7 shows the size of the error coming from the correction which is added in quadrature to give our final error. We can conclude that the AMA method does not contribute much to the error in our final answer.

6. Conclusion and Outlook

Our preliminary result for Δm_K based on 152 configurations with physical quark masses is:

$$\Delta m_K = 7.9(1.3)(2.1) \times 10^{-12} \text{ MeV}.$$

Here the first error is statistical and the second is an estimate of largest systematic error, the discretization error which results from including a charm quark with $m_c = 0.31$ in our calculation. Our Δm_K value is to be compared with the experimental value $3.483(6) \times 10^{-12}$ MeV. However, we view such a comparison as premature given the possibly large and poorly estimated finite lattice spacing error. In the future, planned Δm_K calculations on SUMMIT with finer lattice spacing will provide a better estimate of the systematic errors coming from discretization effects.

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