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Phase structure of multiflavor gauge theories: Critical exponents of Fisher zeros near the endpoint

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A SU(3) gauge theory with 12 flavors is a model of great interest for beyond the standard model physics. Running RHMC simulations for different masses and betas we study the Fisher zeroes in the vicinity of the endpoint of a line of first order phase transitions. The pinching of these zeros with respect to increasing volume provides information about a possible unconventional continuum limit. We also study the mass spectrum of a multiflavor linear sigma model with a splitting of fermion masses.

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PoS(LATTICE2018)206

1. Introduction

As the standard model continues to be tested against experiment there is an ongoing effort to verify the feasibility of multiflavor gauge theories as possible candidates for a more complete description of particle physics. Among other things, the feasibility hinges on a slow running of the coupling, or walking coupling. The renormalization group flows depend on the number of flavors, and through perturbation theory there is evidence for a range of number of flavors in which the theory possesses a non trivial infrared fixed point (IRFP) and exhibits conformality. It is hypothesized that such walking regime can be achieved for theories with N_f just below this conformal window.

Perturbation theory paints a partial picture. Beyond the idea that a conformal window exists, it does not tell us much about its location and properties. This motivates us to study the phase structure of multiflavor lattice gauge theories, and we focus on 12 flavors since it has been the topic of debate about whether it is inside the conformal window or not for some time [1, 2, 3]. The challenge is to understand the phase structure of the lattice model and how its continuum limit behaves if it exhibits an IRFP.

There is evidence from this and other works that there is a first order bulk phase transition (zero temperature) in the space of bare quark mass and inverse coupling $\beta = 6/g^2$. The endpoint of this phase transition is hypothesized to be a second order phase transition and thus a possible unconventional continuum limit [4]. Studies on lattice models suggest the RG (Renormalization Group) flows on complex β of Ising models can be analyzed by the study of Fisher's zeros, zeros of the partition function in this complex analytical continuation, acting as gateways separating confining and symmetric phases of the model [5, 6].

2. Fisher's zeros finite size scaling

It is difficult to calculate the RG flows, but as mentioned earlier, Fisher's zeros block these flows. If the zeros touch the real axis in the infinite volume the two phases are completely separate. Given a RG transformation that acts on lattice spacing as $a \rightarrow ba$, the dimensions of the lattice (in lattice units) transform as

$$L \rightarrow L/b$$

At finite volume, the free energy and consequently the partition function separates into regular and singular part

$$f_{sing} \rightarrow b^4 f_{sing}$$

$$f = -ln(Z)/V \Rightarrow Z_{sing} \rightarrow Z_{sing}$$

The singular part is the part that matters in the infinite volume limit and it is invariant under this transformation. In the case of a complex partition function this implies a scaling of the form $Im\beta(L) \propto L^{-1/\nu}$ near a second order phase transition and $Im\beta(L) \propto L^{-D}$ for a first order phase transition.

3. Methodology and critical exponents

We used the Rational Hybrid Monte Carlo (code by Donald Sinclair) with unimproved staggered fermion action and Wilson gauge action. Running on NERSC computing systems, each simulation (5000 trajectories) gives information at vicinity of simulated β_0 can be used to obtain $Z(\beta_0+\Delta\beta)$. We calculate the average plaquette $\langle U\rangle$ and chiral condensate $\langle\bar{\psi}\psi\rangle$. To connect the simulations we use the FS (Ferrenberg-Swendsen) algorithm, which consists of calculating a density of states of the spectral decomposition of Z

$$Z(\beta)=\int_0^{2N_p} dSn(S)\,e^{-\beta S}$$

$$n(S)=\frac{\sum_\alpha H_\alpha(S)/g_\alpha}{\sum_\alpha\left(e^{F_\alpha-\beta_\alpha S}\right)/g_\alpha}\quad,\quad e^{-F_\alpha}=\sum_S n(S)\,\Delta S e^{-\beta_\alpha S}$$

The iteration between the density of states and the free energy is done with a starting estimate for F. In the equations above $H_\alpha(S)$ is the number of configurations inside the range defined by S for $\beta=\beta_\alpha$ and $g_\alpha=1-2\tau$ where τ is the integrated correlation time.

The convergence of the algorithm is followed by using a χ^2 method, where we compare a quantity directly obtained from the simulation with one calculated with either $n(S)$ or F (figure 1).

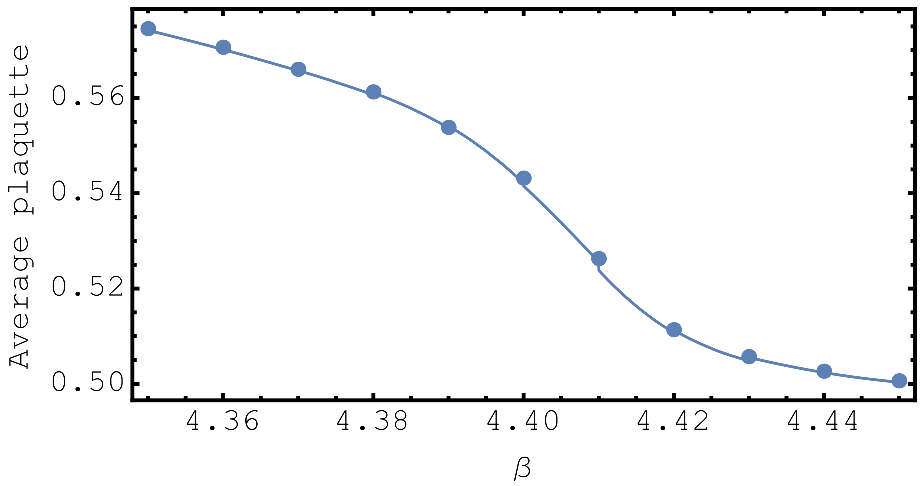

Figure 1: The average plaquette, defined as $1-\frac{1}{3}TrU_p$, measured (points) versus calculated (curve) with $n(s)$ obtained from the FS algorithm, for $V=12^4$ and $m=0.08$

Once in possession of a partition function we can proceed in finding the zeros. The procedure can be done in other ways but we decided to plot the zeros of the ReZ and ImZ and look for intersections between the two functions. We perform a sweep across a range of $Re\beta$ and $Im\beta$ looking for changes of sign in Z, then position the zero between the two analyzed points using a linear interpolation with the value of the function (figure 2).

The intersection is found by a mostly visual inspection. A partial automation was necessary to analyze the error via bootstrapping. Once a zero was found for a particular set of data we resampled the data and used a simple algorithm to find the intersection. The simple algorithm is slow and inefficient but it works when we know the approximate location of the zero, this allows us to resample the data 50 times in a few hours (figure 3).

PoS(LATTICE2018)206

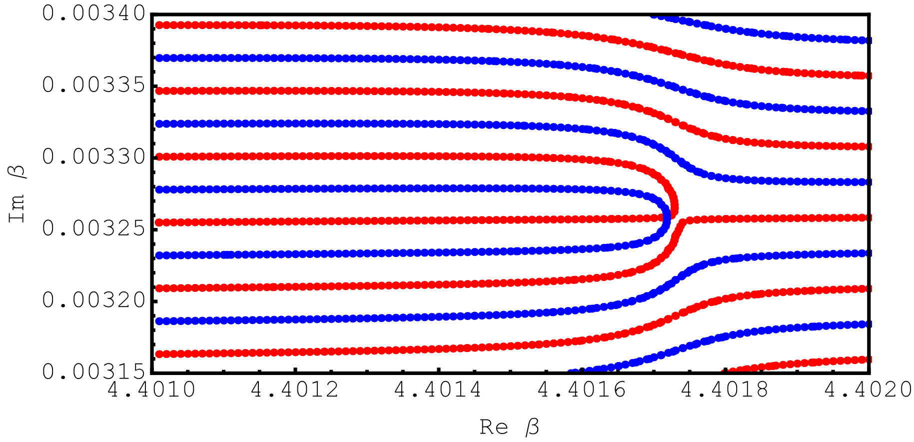

Figure 2: Intersection of $Re[Z] = 0$ and $Im[Z] = 0$ for $L = 12$ and $m = 0.08$.

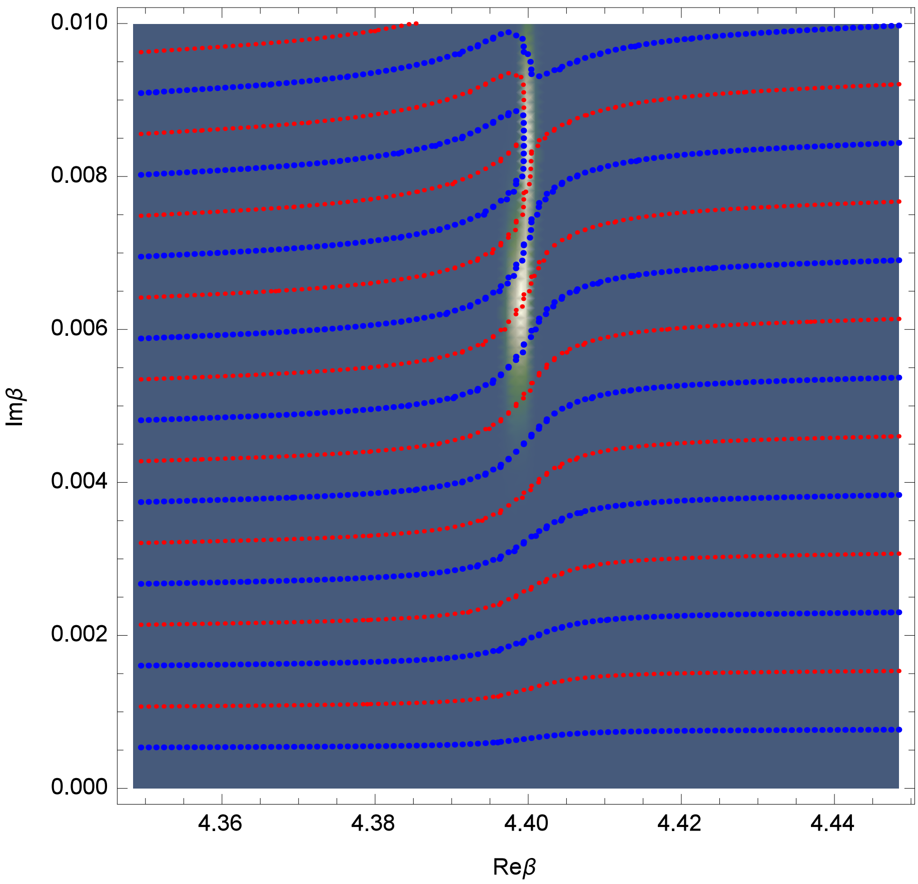

Figure 3: A density map showing how the zeros of each resampled data revolve around the original zero.

This is an ongoing project and the results we have are still under examination. Previous work narrowed down the position of the endpoint to $m \in [0.05, 0.09]$ using 4^4 simulations (figure 4). We repeated this using 12^4 simulations and reduced that range to $m \in [0.05, 0.075]$ (figure 5). There seems to be still a small gap in the $\langle\bar{\psi}\psi\rangle \times \beta$ plane for $m = 0.06$ but it is the closest data we have to the endpoint to analyze the scaling of the zeros so we expect the critical exponent ν to be a close approximation of the target exponent.

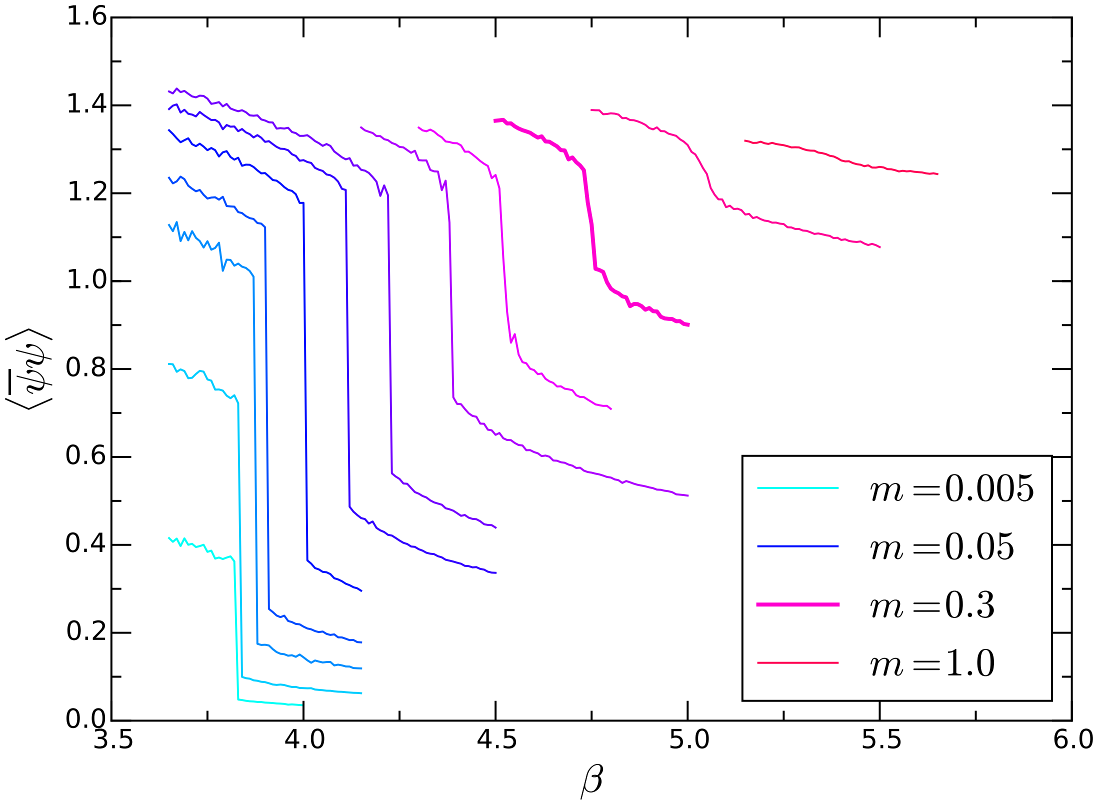

Figure 4: Figure originally from [7]. Chiral condensate for different masses at volume 4^4.

The critical exponents obtained so far (error analysis pending) for $m = 0.02$ is $\nu^{-1} = 3.9$ and for $m = 0.06$ it is $\nu^{-1} = 2.3$. The low mass critical exponent is the expected value for a first order phase transition in a 4D gauge theory. The value near the endpoint could indicate that it is in the same universality class of a mean field theory of free scalar.

4. Future and related work

There is more data to be analyzed and improved, including a mass closer to the endpoint and completing the error analysis. One possible future work is motivated by an effective model of multiflavor gauge theories with two quark masses [8] which could bridge the gap between perturbation theory and lattice. This work is based on the effective model proposed in [9] which successfully described a light sigma particle using lattice results. Our extension of the model with a split mass

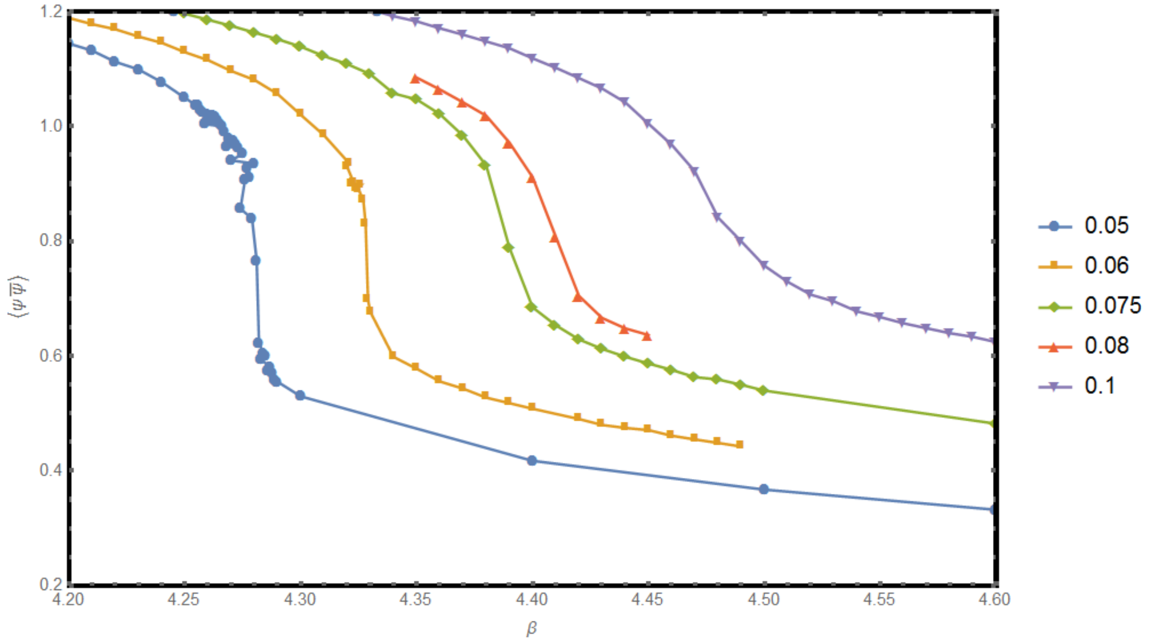

Figure 5: Chiral condensate for different masses at volume 12^4.

(heavy and light but perturbatively similar) makes an interesting prediction, the scalar masses are inverted (M_{a0} composed by two heavy quarks is lighter than the one composed by two light quarks). Whether or not this is an artifact of the model or a feature of the theory, this is a good motivation for future lattice calculations using two quark masses. The results could be either an interesting new feature or it could inform the construction of more precise split mass effective models.

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