

## $B_c$ spectroscopy using highly improved staggered quarks

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We report on a calculation of  $B_c$  ground state and radial excitation energies, obtained from heavy-charm highly improved staggered quark (HISQ) correlators computed on MILC gauge ensembles, with lattice spacings down to  $a = 0.044$  fm. Using HISQ valence quarks on progressively finer lattices allows us to simulate up to the  $b$ -quark mass. In particular we focus on the  $B_c(2S)$  energy, which we compare with  $\mathcal{O}(\alpha_s)$ -improved non-relativistic QCD results computed on the same ensembles and recent experimental results from ATLAS.

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## 1. Introduction

Recent years have seen a great deal of experimental progress in the study of  $B_c$  mesons at the LHC, including precise lifetime measurements [1, 2, 3], observation of new hadronic [4, 5, 6], and semileptonic [7] decay channels, and excited states [8, 9]. Precision lattice QCD calculations of the  $B_c$  system can provide a number of opportunities when combined with new experimental measurements, including new avenues for determination of  $|V_{cb}|$  [10], shedding light on flavour anomalies [11, 12], and identification of resonance peaks in data.

Based on LHC Run 1 data the ATLAS collaboration observed a resonant structure at the  $5\sigma$  level in decays to the  $B_c$  ground state, which they identified as a radial  $B_c$  excitation with energy  $6842(4)_{\text{syst}}(5)_{\text{stat}}$  MeV [8]. This state was subsequently searched for by the LHCb collaboration, however no corresponding structure was observed in their Run 1 data, despite having a higher yield of  $B_c$  signal candidates [9]. Therefore the location of this state remains an open question.

Here we report our progress on a calculation to determine the  $B_c(2S)$  energy directly from lattice QCD data, using the heavy HISQ methodology described in more detail below, as well as using lattice NRQCD. Section 2 explains the details of the calculation, while Section 3 gives the status of results for the  $B_c$  ground state and  $B_c$  2S-1S splitting.

## 2. Details of calculation

We work on ensembles of  $n_f = 2 + 1 + 1$  gauge configurations generated by the MILC collaboration [13]. All of the ensembles used here have unphysically heavy pion masses in the sea, while the sea strange and charm quark masses are near their physical values. We use the highly improved staggered quark (HISQ) action [14] to compute charm and heavy valence quarks. Throughout the calculation we use charm valence quarks with  $m_c^{\text{val}}$  near to its physical value. We can check the effect of any mistuning by varying  $m_c^{\text{val}}$ .

Because  $am_b$  is large, even on our finest lattice spacing ensemble, quantities calculated at  $m_b$  directly would have potentially large discretisation artifacts proportional to powers of  $(am_b)^2$ , making controlled continuum extrapolations unfeasible. To get around this we work with heavy quark masses (generically referred to as  $m_h$ ), and compute the quantity of interest keeping  $am_h < 0.8$ , and using a range of  $m_h$  values on multiple lattice spacings. In this way we can fit the  $(am_h)^{2n}$  discretisation effects and recover the physical dependence on  $m_h$ , which we model as a power series in  $1/M_{\eta_h}$ . Finally we evaluate this function at  $M_{\eta_h}$  to determine physical results. The ensembles and parameter values used in this calculation are collected in Table 2.

We tie together charm and heavy quark propagators to construct zero momentum two-point functions  $C(t)$ , with the quantum numbers to create and destroy Goldstone pseudoscalar mesons.  $C(t)$  is a sum of exponentials,

$$C(t) = \sum_i a_i e^{-M_i t} + (-1)^t b_i e^{-\tilde{M}_i t} + (t \rightarrow T - t). \quad (2.1)$$

Here we are interested in the lowest and first excited state energies  $M_1$  and  $M_2$  corresponding to the  $H_c$  ground state and first radial excitation. We use multi-exponential Bayesian fits to determine the amplitudes and energies from Eq. (2.1).

ens	$\beta$	$am_l$	$am_s$	$am_c$	$N_s \times N_t$	$am_c^{\text{val}}$	$am_h^{\text{val}}$
c-5	6.00	0.0102	0.0509	0.635	24×64	0.635	-
f-5	6.30	0.0074	0.037	0.440	32×96	0.434	0.6, 0.8
f-10	6.30	0.0036	0.036	0.430	48×96	0.439	0.6, 0.8
sf-5	6.72	0.0048	0.024	0.286	48×144	0.274	0.4, 0.5, 0.6, 0.8
uf-5	7.00	0.00316	0.0158	0.188	64×192	0.188	0.6, 0.8
						0.195	0.4, 0.5, 0.7, 0.9

**Table 1:** Summary of ensembles and run parameters used in this work. The ensemble specifications in the left-hand column appear in the figure legends, and correspond to “coarse” ( $a \approx 0.12$  fm), “fine” ( $a \approx 0.09$  fm), “superfine” ( $a \approx 0.06$  fm) and “ultrafine” ( $a \approx 0.044$  fm) lattice spacings and have  $m_s/m_l = 5$ , or 10 in the case of the ‘f-10’ ensemble.

In addition to the fully relativistic calculation we also calculate the  $B_c$  2S-1S splitting using  $\mathcal{O}(\alpha_s)$ -improved non-relativistic QCD (NRQCD) for the  $b$ -quark on the c-5, f-5, and sf-5 ensembles. The NRQCD propagators are obtained using the time evolution operator constructed from the NRQCD Hamiltonian. The NRQCD  $b$ -quark propagator is combined with a HISQ charm propagator on each gauge background, and averaged to construct the two-point functions. More details of the procedure can be found in [15].

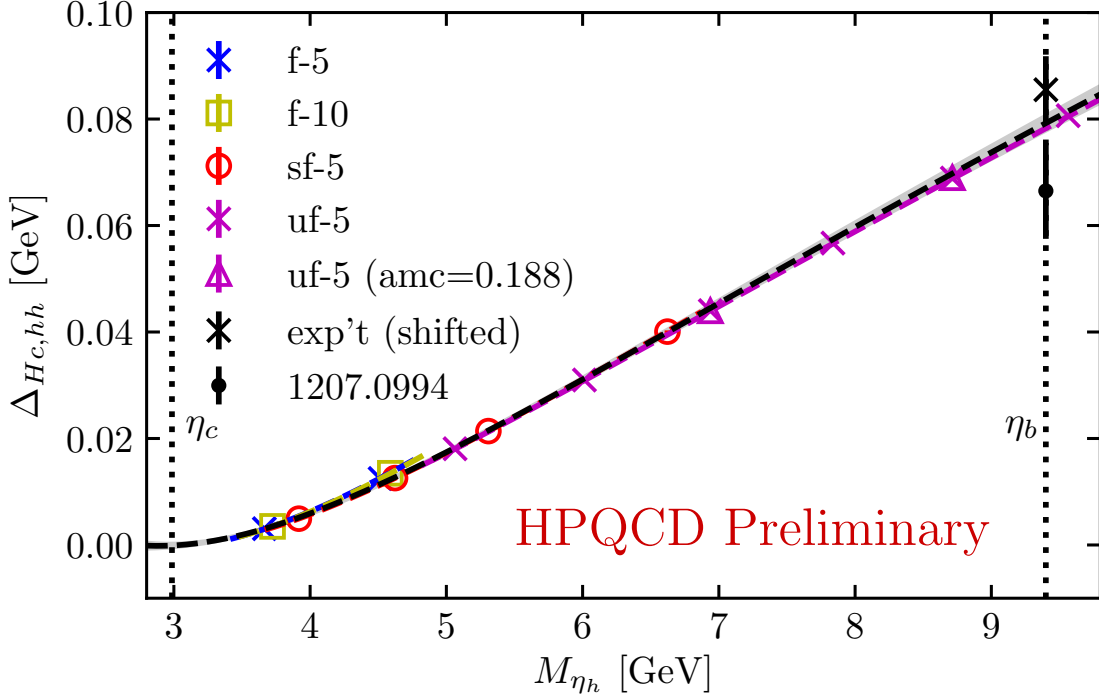
### 3. Results

#### 3.1 $B_c$ mass

The ground state  $H_c$  energies are resolved very precisely from the two-point correlator data. This is shown in Fig. 1, where we plot the quantity  $\Delta_{H_c, hh} = M_{H_c} - (M_{\eta_h} + M_{\eta_c})/2$  as a function of  $M_{\eta_h}$  determined from the corresponding heavy-heavy correlator. Because of this the determination from lattice data is limited by systematic effects due to missing electromagnetism, and the  $\eta_b$  and  $\eta_c$  being unable to annihilate to gluons in our calculation. Using estimates of these effects from [16], we shift the experimental result downward to compare directly with the lattice data. With these taken into account, the lattice data is compatible with the experimental value. For comparison, the earlier lattice determination [16] using the same technique but on  $n_f = 2 + 1$  ensembles is also shown. In that case an error was also estimated for the missing charm quarks in the sea, which are included in the present study.

#### 3.2 $B_c(2S)$ energy

Fig. 2 shows our results for the  $H_c$  2S-1S splitting, as a function of  $M_{\eta_h}$ . To fit the two-point correlators (2.1), we provide Bayesian priors for the amplitudes  $a_i$  of 0(1), and priors for the ground state energy and excited state energy splittings,  $M_i - M_{i-1}$ . We have taken the priors for the mass splittings of  $\approx 600$  MeV, with a width that is 50% of the splitting (using 25% gives consistent results, but with slightly less conservative errors). This covers the expected range of the 2S-1S splitting from charmonium to bottomonium, and is consistent with expectations for higher radial excitations [17]. As shown in Fig. 2, the fit resolves the first excited state energy within a typical



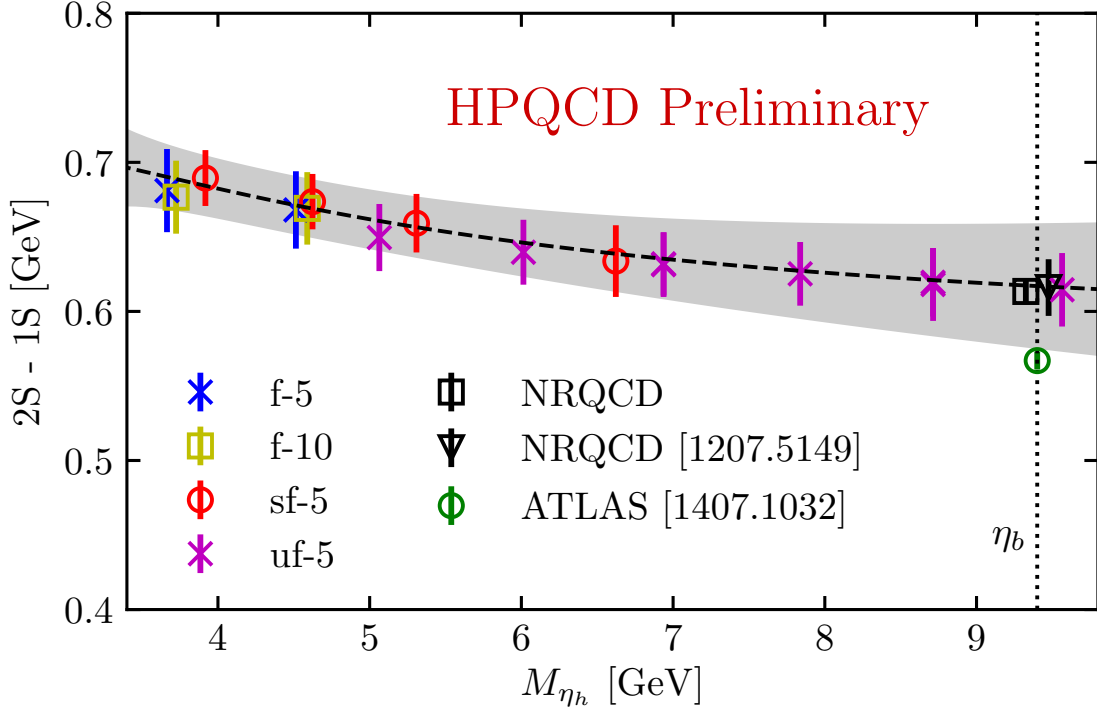
**Figure 1:** The quantity  $M_{H_c} - (M_{\eta_h} + M_{\eta_c})/2$ , plotted as a function of  $M_{\eta_h}$ . The open symbols correspond to values extracted from lattice two-point functions, and the gray band is the fit to this data. We have shifted the value taken from experiments (black burst) to compare directly with the data from the lattice calculation, which is missing electromagnetic and gluon annihilation effects.

uncertainty of around 30 MeV. It should be stressed that only local correlators have been used in this analysis, and that the inclusion of smearing functions should improve the robustness of these determinations.

As in the case of the ground state energy, there is no evidence of significant discretisation artifacts in the data. We have included the data point from the ‘uf-5’ ensemble at  $am_h = 0.9$  on the figure, but it is not used in the fit given by the gray band. For comparison we also show two NRQCD determinations of this quantity, from [18] computed on  $n_f = 2 + 1$  ensembles, and from preliminary results computed here on the present ensembles and shown in Fig. 3. These agree well with the heavy-HISQ determination at  $\eta_b$  and with one another. The observation from ATLAS lies just outside of the one-sigma band from heavy-HISQ.

#### 4. Conclusions

Here we have presented a preliminary result for the  $B_c(2S)$  energy, determined using fully relativistic heavy quarks, as well as NRQCD  $b$ -quarks. We find that we are consistently able to resolve the  $H_c$  radial excitation in our relativistic two-point correlator data. The  $2S$ - $1S$  splitting falls slowly with  $m_h$  over the range from  $m_c$  to  $m_b$ . There is no evidence of large discretisation effects in the energy splitting, and for the moment we use a conservative estimate of these effects.

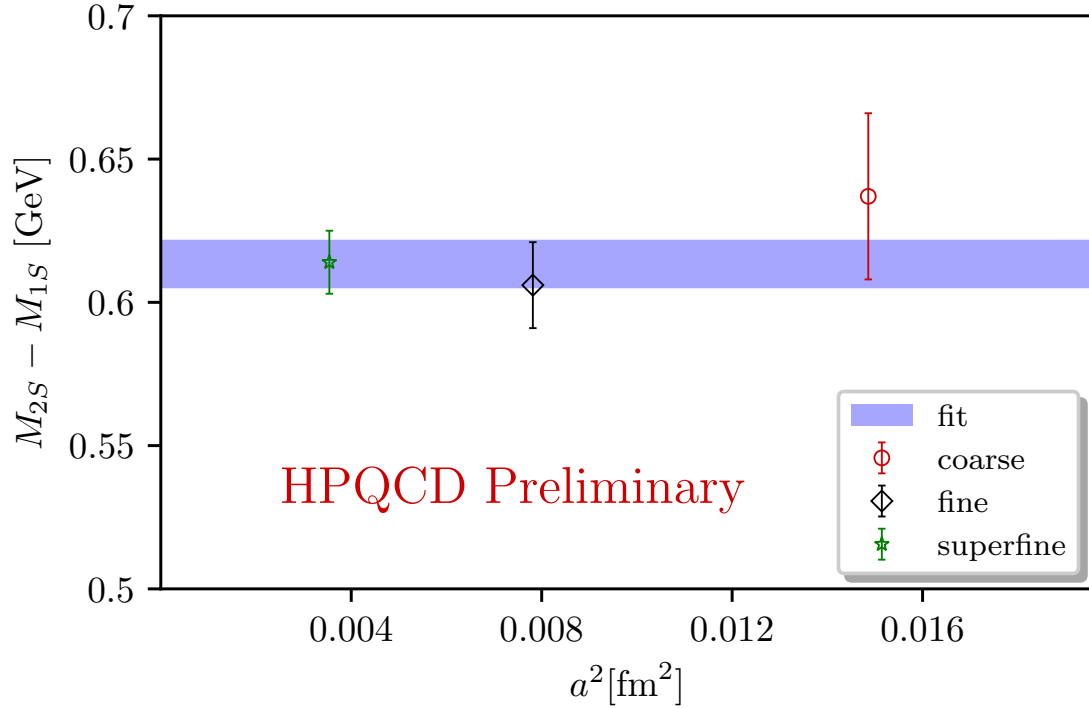


**Figure 2:** Preliminary results for the  $H_c$  2S-1S splitting determined from lattice two-point functions, as a function of  $M_{\eta_h}$ . The fit to the lattice results is given by the gray band, with the physical result for the  $B_c$  on the right at  $\eta_b$ , alongside the observation from ATLAS (green circle) and NRQCD results (black square and triangle).

With this methodology we find a preliminary value of 617(41) MeV for the 2S-1S splitting, or using the PDG average for the  $B_c$  mass 6274.9(8) MeV, 6892(41) MeV for the 2S energy. This is consistent with NRQCD determinations of the same quantity ([18] and Fig. 3), but at present has larger error bars. Both this result and the NRQCD results are above the ATLAS observation [8], at the level of one sigma for heavy-HISQ and two sigma for NRQCD. Because the lattice QCD results have larger uncertainties than the ATLAS result the significance of the discrepancy depends on the lattice errors. One deficiency of the present calculation is that the HISQ two-point correlators are computed without the use of smearing functions. Including one or more smearing functions to expand the basis of correlators will help resolve excited states in the lattice data, and is work in progress.

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**Figure 3:** The  $B_c$  2S-1S splitting obtained from  $\mathcal{O}(\alpha_s)$ -improved NRQCD  $b$  quarks and HISQ charm quarks, calculated on the c-5 (red circle), f-5 (black diamond), and sf-5 (green star) ensembles. The fit value corresponding to the  $a = 0$  determination is shown by the blue band.

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