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QCD at non-zero density and phenomenology

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In the last few years, numerical simulations of QCD on the lattice have reached a new level of accuracy. A wide range of thermodynamic observables is now available in the continuum limit and for physical quark masses. This allows a quantitative comparison with measurements from heavy ion collisions. I will review the state-of-the-art results from lattice simulations of QCD thermodynamics and connect them to experimental measurements from RHIC, in view of the second Beam Energy Scan scheduled for 2019-2020.

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1. Introduction

The study of finite-density QCD aims at answering a series of fundamental questions in the next few years. In particular, the existence and location of a critical point separating the crossover at chemical potential $\mu_B = 0$ [1] from a hypothetical first order phase transition, is one of the main open questions in our field. Other topics which will be addressed are the location of the transition line and the phases of QCD at high density. By systematically decreasing the collision energy, heavy ion collisions produce the high density phases of matter in the laboratory. The RHIC facility at BNL is particularly suited for this purpose: the second Beam Energy Scan is scheduled for 2019 and 2020. RHIC will run both in collider and fixed target modes, and will be able to reach high values of the baryonic chemical potentials. The study of dense matter will not stop after RHIC: NICA, CBM and JPARC will pursue the study of critical point, onset of deconfinement and dense hadronic matter at least till 2025.

Such a rich experimental program needs the support of fundamental theory and phenomenology. In this contribution I will focus on observables obtained from lattice QCD simulations, which are so precise to be employed in a quantitative comparison to experiments. These include the equation of state, which is needed as input of hydrodynamic codes which describe the evolution of matter created in heavy-ion collisions (see e.g. [2] and references therein); the information on the phase diagram and constraints on the critical point location; the fluctuations of conserved charges. The latter can be measured in experiments and in principle they allow a direct comparison between theory and heavy ion data, provided that non-thermal effects are understood and corrected for.

2. Low-temperature phase: the Hadron Resonance Gas model

In the low temperature phase, the results of lattice simulations for QCD thermodynamics are generally well described by the Hadron Resonance Gas (HRG) model, which has its roots in the theorem by Dashen, Ma and Bernstein [3]. This theorem allows one to calculate the microcanonical partition function of an interacting system, in the thermodynamic limit $V \to \infty$, assuming that it is a gas of non-interacting free hadrons and resonances [4]. This model needs as an input the list of all known baryons and mesons with their mass and quantum numbers. It was recently pointed out that additional resonances are needed in order to improve the agreement between lattice QCD and HRG model predictions [5, 6].

It is interesting to note that, setting $\mu_S = \mu_Q = 0$, the dependence of the baryonic pressure on the baryonic chemical potential is extremely simple:

$$\frac{p}{T^4} = \sum_{i \in B} \frac{d_i}{\pi^2} \left(\frac{m_i}{T}\right)^2 \sum_{N=1}^{\infty} (-1)^{N+1} N^{-2} K_2(N\frac{m_i}{T}) \cosh[N\frac{\mu_B}{T}] \tag{2.1}$$

any baryon contribution to the pressure is given by a function of the baryon mass m_i and degeneracy d_i, times $\cosh[N\frac{\mu_B}{T}]$. The latter is the same for all baryons. The Boltzmann approximation corresponds to $N = 1$ in the above fugacity expansion. If we now include a finite μ_S and μ_Q, all baryons with the same strangeness and electric charge content will have the same factor, which contains the chemical potentials. This expansion can be used to separate the contribution of particles according to their quantum numbers. This idea, first developed in [5], has led to the identification of the missing resonance states according to their flavor content in Ref. [6]. We will come back to this point later, when discussing kaon fluctuations.

3. QCD equation of state at zero and finite μ_B

The equation of state of QCD with 2+1 flavors at $\mu_B = 0$ is known for a few years. The WB collaboration published continuum extrapolated results at the physical quark mass based on a tree-level Symanzik improved gauge action with 2-step stout-link improved staggered fermions[7, 8, 9]; more recently, also the charm quark has been included in the system [10]. From this analysis, it turns out that the charm quark is a relevant degree of freedom already at $T \sim 250$ MeV, which should be taken into account in hydrodynamic simulations of heavy ion collisions at the LHC energies [11]. The hotQCD collaboration found similar results [12] using the highly improved staggered quark (HISQ) action introduced in [13].

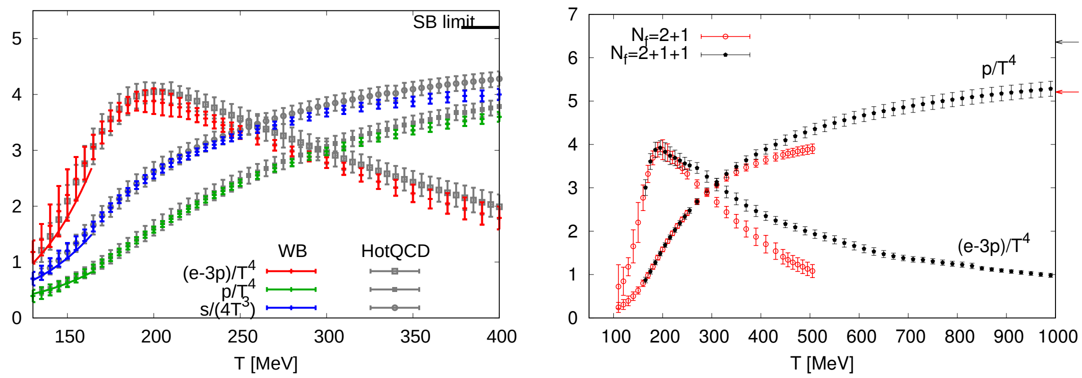

Figure 1: Left: Continuum extrapolated results for trace anomaly, entropy density and pressure. The gray points are from the HotQCD collaboration [12], while the colored ones are from the WB collaboration [9]. The figure also shows the Stefan-Boltzmann limit for the pressure and the scaled entropy; the curves at low temperature correspond to the HRG model predictions. Right: the trace anomaly and pressure in the 2+1 and 2+1+1 flavor theories (from Ref. [10]).

The left panel of Fig. 1 shows the comparison between the hotQCD (gray) and WB (colored) results for the trace anomaly, entropy density and pressure. The right panel is a comparison between the trace anomaly and pressure for a system of 2+1 (red) and 2+1+1 (black) dynamical quark flavors.

An important validation of the lattice QCD Equation of State has been obtained from a Bayesian analysis [14]. In this framework, a large number of observables from RHIC and the LHC has been compared to theoretical models, while varying the model parameters. The posterior distribution over possible equations of states turned out to be consistent with results from lattice QCD. This analysis has also been successfully applied to infer the behavior of other quantities, such as the shear viscosity of the QGP at zero [15] and finite density [16].

Direct simulation of QCD thermodynamics at finite density is not possible, due to the sign problem. Nevertheless, the need for finite-μ_B results to support the experimental program has driven the thermal lattice QCD community to find alternative approaches. Several methods have been proposed to calculate thermodynamics quantities at small chemical potential. These include

Taylor expansion around $\mu_B = 0$ [17, 18, 19, 20, 21], analytic continuation from imaginary μ_B [22, 23, 23, 24, 25, 26, 27, 28, 29], reweighting of the generated configurations [30, 31, 32, 33], use of the canonical ensemble [34, 35, 36] and density of state methods [37, 38]. Here I will focus on the first two. The Taylor expansion for the pressure can be written as

$$\frac{p(T,\mu_B)}{T^4} = \frac{p(T,0)}{T^4} + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left. \frac{d^{2n}(p/T^4)}{d(\frac{\mu_B}{T})^{2n}} \right|_{\mu_B=0} \left(\frac{\mu_B}{T}\right)^{2n} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n} . \tag{3.1}$$

The Taylor coefficients can be calculated in two ways, either by direct simulations, or simulations at imaginary chemical potentials. Another important point is that, at finite μ_B, one has to make choices also for the other two conserved charge chemical potentials, namely μ_S and μ_Q. The two most popular choices are either $\mu_S = \mu_Q = 0$, or μ_S and μ_Q functions of T and μ_B, such that we have $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$, to match the experimental situation in a heavy-ion collision.

More in detail, in the direct method a derivative of the partition function can be written in terms of the action with all fermionic degrees of freedom already integrated out, S_{eff}, as follows:

$$\partial_i \log Z = \frac{1}{Z} \int \mathscr{D}U \, \partial_i e^{-S_{\rm eff}} = \langle A_i \rangle \, . \tag{3.2}$$

Here i indicates the variable of the derivative, the chemical potential μ_i in this case. A_i is the first derivative of $S_{\rm eff}$ without the factor $e^{-S_{\rm eff}}$. Its ensemble average is calculated with the same weight used for generating the configurations. In particular,

$$A_i \; = \; \frac{1}{4} {\rm tr} M_i^{-1}(m_i,\mu_i) M_i'(m_i,\mu_i) \, , \tag{3.3}$$

where $M_i(m_i,\mu_i) = m_i + \not{D}(\mu_i)$ is the fermion operator with the bare mass m_i; $M_i'(m_i,\mu_i)$ stands for its first derivative with respect to μ_i. Higher order derivatives can be evaluated in a similar way. The most expensive part of this method is the calculation of the trace in Eq. (3.3), which contains disconnected contributions and appears in almost all susceptibilities.

After the early results for c_2, c_4 and c_6 [18], the first continuum extrapolated results for c_2 were published in Ref. [39]; in Ref. [40] c_4 was shown, but only at finite lattice spacing. The continuum limit for c_6 was published for the first time in [41] in the case of strangeness neutrality, and later in [42]. In the latter, both strangeness neutrality and $\mu_S = \mu_Q = 0$ results are presented. A continuum estimate is performed, based on $N_t = 6$ and 8 lattices. In [43], a first determination of c_8, at two values of the temperature and $N_t = 8$ was presented. More recently, diagonal and non-diagonal coefficients up to c_8 have been calculated at $N_t = 12$ in Ref. [44] at $\mu_S = \mu_Q = 0$.

Figure 2 shows possible landscapes for simulations at imaginary μ_B and μ_S. The black dot corresponds to direct simulation of all coefficients at $\mu_B = 0$, as performed e.g. by the HotQCD collaboration. The red squares correspond to finite μ_B and $\mu_S = 0$, while the green triangles are trajectories which ensure the strangeness-neutrality condition at $T = 150$ MeV (full) and $T = 200$ MeV (empty). The idea is to simulate lower order fluctuations at imaginary μ_B and use them in a combined fit whose coefficients at $\mu_B = 0$ are the higher order fluctuations. The formulas used for the combined fit of $\chi_1, \ldots \chi_4$ are

$$\chi_1^B(\hat\mu_B) \; = \; 2c_2\hat\mu_B + 4c_4\hat\mu_B^3 + 6c_6\hat\mu_B^5 + \frac{4!}{7!}c_4\varepsilon_1\hat\mu_B^7 + \frac{4!}{9!}c_4\varepsilon_2\hat\mu_B^9$$

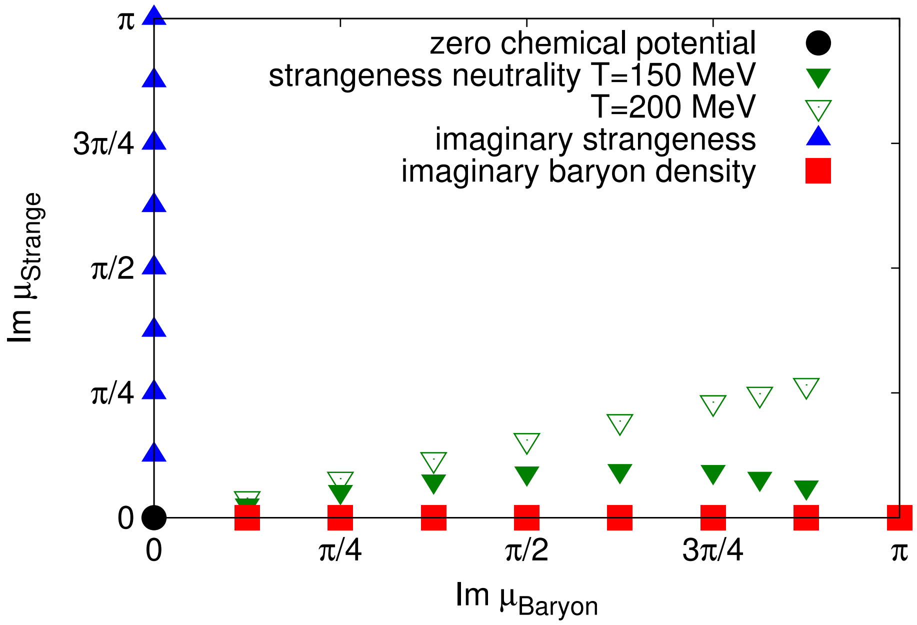

Figure 2: Simulation landscape in the imaginary $\mu_B/T - \mu_S/T$ plane of QCD. The QCD observables are periodic in the imaginary chemical potentials, thus only the range $(0 \ldots \pi) \times (0 \ldots \pi)$ has to be explored.

$$\begin{aligned}
\chi_2^B(\hat{\mu}_B) &= 2c_2 + 12c_4\hat{\mu}_B^2 + 30c_6\hat{\mu}_B^4 + \frac{4!}{6!}c_4\varepsilon_1\hat{\mu}_B^6 + \frac{4!}{8!}c_4\varepsilon_2\hat{\mu}_B^8 \\
\chi_3^B(\hat{\mu}_B) &= 24c_4\hat{\mu}_B + 120c_6\hat{\mu}_B^3 + \frac{4!}{5!}c_4\varepsilon_1\hat{\mu}_B^5 + \frac{4!}{7!}c_4\varepsilon_2\hat{\mu}_B^7 \\
\chi_4^B(\hat{\mu}_B) &= 24c_4 + 360c_6\hat{\mu}_B^2 + c_4\varepsilon_1\hat{\mu}_B^4 + \frac{4!}{6!}c_4\varepsilon_2\hat{\mu}_B^6
\end{aligned} \tag{3.4}$$

where ε_1 and ε_2 are drawn randomly from a normal distribution with mean -1.25 and variance 2.75. The authors of Ref. [44] perform a correlated fit for the four measured observables, thus obtaining the values of c_2, c_4 and c_6 for each temperature, and the corresponding χ_2^B, χ_4^B and χ_6^B (notice that $n!c_n = \chi_n^B$). These results are shown in Fig. 3, together with an estimate of χ_8^B.

With the coefficients we have today we are able to cover the phase diagram for $\mu_B/T < 2.5$.

I would like to mention two approaches, which allow us to bring the EoS closer to the experimental needs. The first one uses the lattice EoS up to order $(\mu_B/T)^4$ and introduces a critical point in the 3D Ising model universality class for QCD [45]. This family of equations of states will be useful to test the effect of the critical point on hydrodynamics simulations, to be compared to the experimental data from the second Beam Energy Scan. The other approach starts with a fugacity expansion for the baryonic density, motivated by the HRG model formula (2.1), and postulates all higher order coefficients to follow this behavior. This approach does not admit a critical point, but allows to predict the behavior of all Taylor expansion coefficients [46].

4. QCD phase diagram

In 2006, lattice QCD simulations showed that the QCD transition at $\mu_B = 0$ is an analytical crossover [1]. The transition temperature is known since a few years [1, 47, 48, 49], with differ-

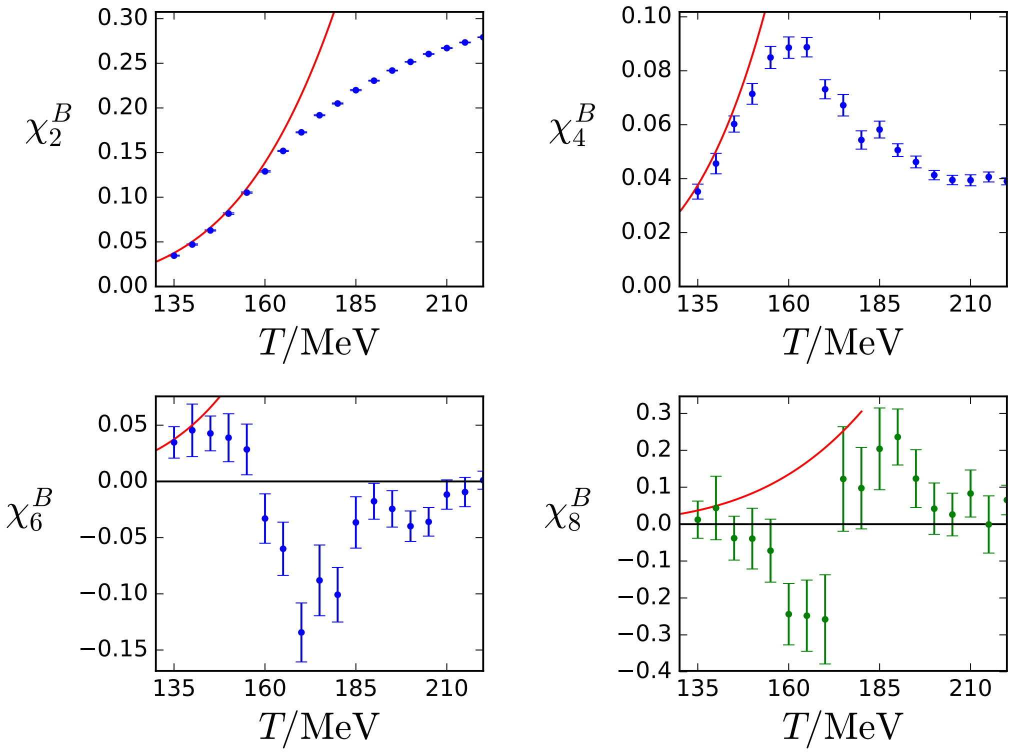

Figure 3: Results for χ_2^B, χ_4^B, χ_6^B and an estimate for χ_8^B as functions of the temperature, obtained from the single-temperature analysis. We plot χ_8^B in green to point out that its determination is guided by a prior. The red curve in each panel corresponds to the Hadron Resonance Gas (HRG) model result.

ent observables leading to slightly different results and defining the width of the crossover. The HotQCD collaboration has recently updated their value for the transition temperature based on chiral observables, considerably shrinking the error-bar [50]. The quoted value is $T_c = 156.5 \pm 1.5$ MeV.

Following the transition temperature location at finite chemical potential, one can define the curvature κ of the QCD phase diagram

$$\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B=0)}\right)^2 - \lambda \left(\frac{\mu_B}{T_c(\mu_B=0)}\right)^4 . \tag{4.1}$$

Several results for κ exist in the literature; they differ by approach and choices of strangeness and electric charge chemical potentials. A compilation of results is shown in Fig. 4.

The Taylor expansion of thermodynamic observables will break down if there is a critical point on the QCD phase diagram. The position of the critical point would then be identified with the radius of convergence of the Taylor series, provided that there is no other singularity in the imaginary μ_B plane, closer to $\mu_B = 0$. The radius of convergence can be obtained from the ratio of subsequent coefficients in the Taylor expansion e.g. of the pressure or its derivatives. For the Taylor expansion of χ_2^B, it is defined as follows

$$r_{2n}^{\chi} = \left| \frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|^{1/2} . \tag{4.2}$$

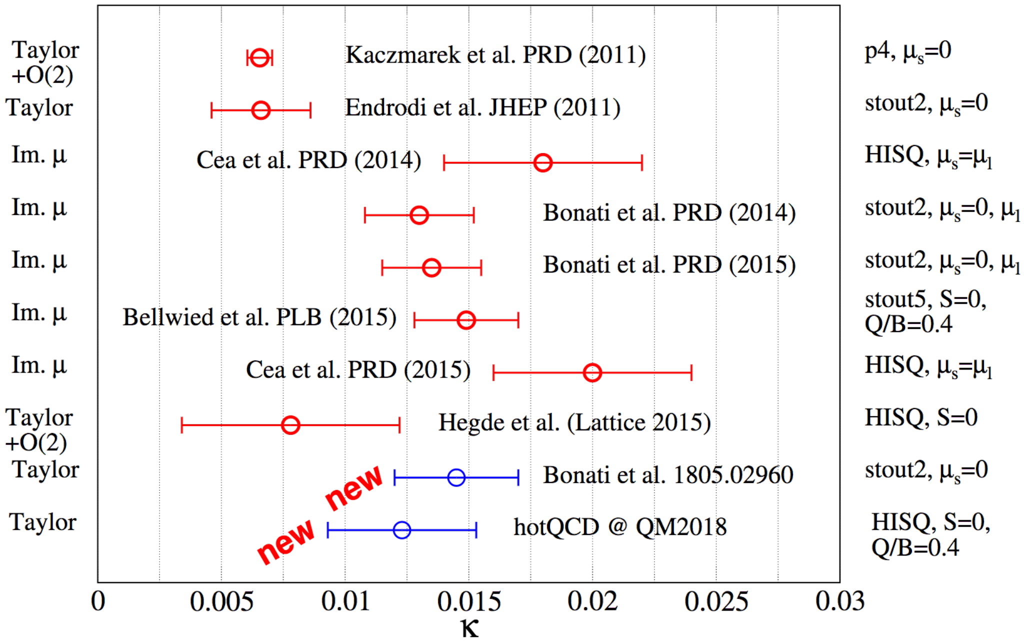

Figure 4: (From Ref. [51]) Compilation of results for the curvature of the phase diagram of QCD, obtained with different methods, different actions and different conditions on μ_S and μ_Q [21, 52, 53, 54, 55, 56, 57, 58, 59].

Strictly speaking, r^{χ}_{2n} converges for $n \to \infty$. The authors of Ref. [60] re-did the analysis on the $N_t = 4$ lattices with unimproved action by Fodor and Katz [31] with statistics increased by a factor 50, and found the same critical point previously quoted. They found that the radius of convergence in that case seems to converge to the right value with just a few coefficients, probably due to the fact that the critical point is relatively close to $\mu_B = 0$. For QCD on finer lattices, the most up-to-date estimate for the radius of convergence is from Refs. [42] and [43]. In the range 135 MeV$\leq T \leq$ 155 MeV a critical point at $\mu_B/T < 2$ is highly disfavored.

5. Fluctuations of conserved charges

Fluctuations of conserved charges allow us to characterize the chemical freeze-out, the moment in the evolution of a heavy ion collision at which all inelastic collisions between hadrons cease: the chemical composition of the system is fixed at this point. The cumulants of the experimental event-by-event distribution of a conserved charge are fluctuation observables which we can calculate on the lattice, and which are defined as

$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n} \tag{5.1}$$

These quantities are fixed at the chemical freeze-out; the lattice curves are functions of the temperature, chemical potential and volume. The volume factor cancels out by taking ratios:

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma = \chi_3/\chi_2 \tag{5.2}$$
$$S\sigma^3/M = \chi_3/\chi_1 \qquad \kappa\sigma^2 = \chi_4/\chi_2, \tag{5.3}$$

where M, σ, S, κ are the mean, variance, skewness and kurtosis of the experimental net-charge distribution, respectively. By comparing the experimental value to the lattice curves we can obtain the temperature and chemical potential at the chemical freeze-out [61, 62, 63].

The STAR collaboration published results for the net-proton [64] and net-charge [65] fluctuations at different collision energies. Possible experimental sources of non-thermal fluctuations are corrected for in the STAR data analysis: the centrality-bin-width correction method minimizes effects due to volume variation because of finite centrality bin width (such effects have been studied e.g. in [66, 67, 68]); the moments are corrected for the finite reconstruction efficiency based on binomial probability distribution [69]. A p_T-dependent efficiency correction method [70] has been recently implemented; multiplicity-dependent and non-binomial efficiency corrections have also been studied [71], as well as the effect of baryon number conservation on the cumulants of net-proton distribution [72].

Final-state interactions in the hadronic phase and non-equilibrium effects might become relevant and affect fluctuations [73, 74, 75, 76, 77, 78, 79]; a fundamental check in favor of the equilibrium scenario is e.g. the consistency between the freeze-out parameters yielded by different quantum numbers, like electric charge and baryon number.

One more caveat is in order, since experimentally only the net-proton multiplicity distribution is measured, as opposed to the lattice net-baryon number fluctuations. It was shown that, once the effects of resonance feed-down and isospin randomization are taken into account [80, 81], the net-proton and net-baryon number fluctuations are numerically very similar, at least in the case of low-order fluctuations [82].

In 2014 it was shown that two independent analyses based on fluctuations of baryon number and electric charge give consistent results for the freeze-out parameters [83]. More recently, a combined fit of the ratio of first-to-second order fluctuations for these two charges has been performed: since their experimental and theoretical uncertainties are smaller, the freeze-out temperature and chemical potential can be determined with a higher precision. The results of this analysis are shown in the left panel of Fig. 5. The colored lines are the trajectories in the (T, μ_B) plane which satisfy the experimental value: the points where they cross yield the desired freeze-out T and μ_B. Also shown in the figure are the isentropic expansion trajectories that the system in a heavy-ion collision would follow in the case of strictly zero viscosity; the red shaded area is the crossover transition region in the QCD phase diagram [56].

Experimentally, it is very difficult to measure strangeness fluctuations. The STAR collaboration has recently presented results for net-kaon fluctuations [88]. On the lattice it is possible to calculate fluctuations of conserved charges: the issue of isolating a single particle contribution is not trivial. For example, as shown in Eq. (2.1), in the hadronic phase it is possible to separate the contribution to the pressure of particles according to their quantum numbers: non- strange mesons, non-strange baryons, mesons with strangeness 1, baryons with strangeness 1, 2, and 3. Following this idea, and taking only the derivatives of the contribution of mesons with strangeness one, one can obtain the fluctuations of mesons with strangeness one. In a heavy ion collision there are two sources of mesons: the primordial ones, which are formed at hadronization, and the ones coming from resonance decays. The HRG model can provide both. In Ref [89] χ_2/χ_1 for kaons has been calculated in the HRG model in two ways, just by means of the Boltzmann approximation using the idea illustrated above, and by calculating the full contribution including primordial kaons and de-

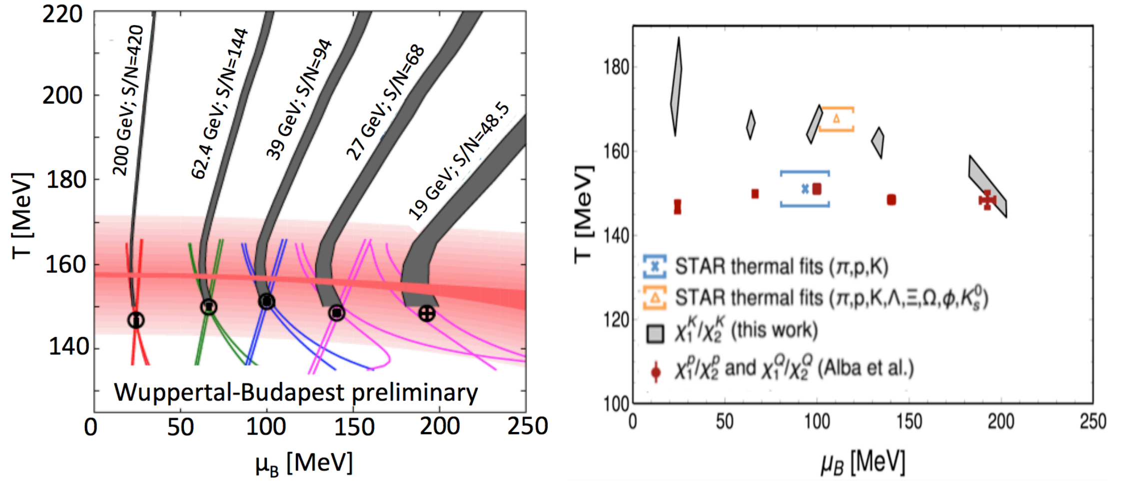

Figure 5: Left: Preliminary results of the WB collaboration [84, 85]. The colored lines are the contours at constant mean/variance ratios of the net electric charge from lattice simulations. The contours that correspond to STAR data intersect in the freeze-out points of Ref. [86]. The red band is the QCD phase diagram from Ref. [56]. Also shown are the isentropic contours that match the chemical freeze-out data. Right: Freeze-out parameters across the highest five energies from the Beam Energy Scan. The red points were obtained from the combined fit of χ_1^p/χ_2^p and χ_1^Q/χ_2^Q [86], while the gray bands are obtained from the fit of χ_1^K/χ_2^K in this work. Also shown are the freeze-out parameters obtained by the STAR collaboration at $\sqrt{s} = 39$ GeV [87] from thermal fits to all measured ground-state yields (orange triangle) and only to protons, pions and kaons (blue x-shaped symbol).

cays. Since the two curves are very close to each other for all values of T and μ_B, it was concluded that it is possible to isolate the net-kaon χ_2/χ_1 on the lattice. It will be interesting to see whether the lattice QCD results for this observables confirm the HRG model analysis of kaons fluctuations, which led to a higher freeze-out temperature for kaons compared to the light particles [90]. The results of this analysis are shown in the right panel of Fig. 5.

Fluctuations can be used to study criticality, as they are expected to diverge with powers of the correlation length near the critical point [19, 91, 92]. Besides, fourth-order fluctuations are expected to be non-monotonic near the critical point [93, 94].

Recent results by the HotQCD collaboration showed the baryon number variance and the disconnected chiral susceptibility, extrapolated to finite μ_B along the crossover line. Both are expected to diverge at the critical point, but none of them shows any signs of criticality up to $\mu_B = 250$ MeV [95].

As for the higher order fluctuations, χ_3/χ_1 and χ_4/χ_2 can be expanded in Taylor series in powers of μ_B/T. The HotQCD collaboration pointed out that the Taylor expansion coefficients for χ_4/χ_2 is three times larger than the one for χ_3/χ_1, a trend which seems to be confirmed by the experimental data. However, recently it was pointed out in Ref. [96] that it is possible to explain the same trend in the data in terms of baryon number conservation and volume fluctuation effects. Results from the WB collaboration for χ_3/χ_1 and χ_4/χ_2 are consistent with these findings [44]. The extrapolation of these quantities to finite μ_B is shown in the two panels of Fig. 6.

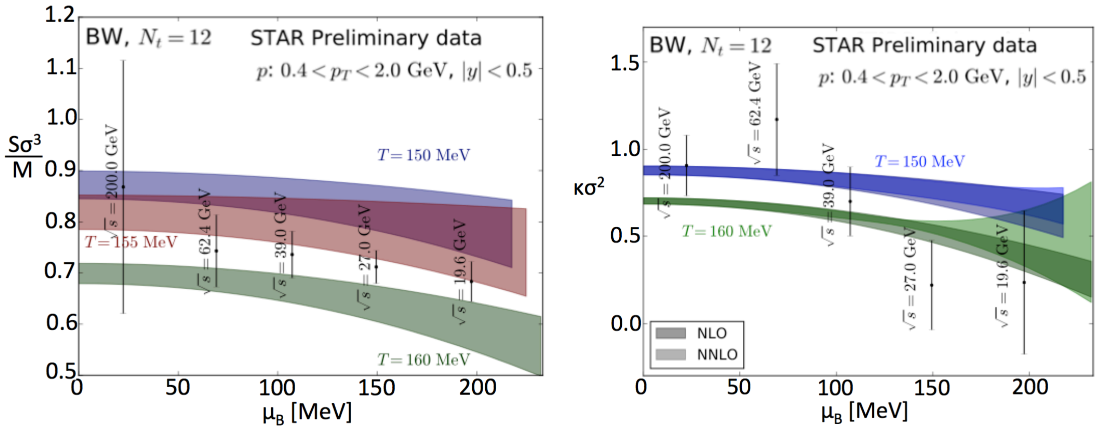

Figure 6: $S_B\sigma_B^3/M_B$ (left panel) and $\kappa_B\sigma_B^2$ (right panel) extrapolated to finite chemical potential. The left panel is extrapolated up to $\mathcal{O}(\hat{\mu}_B^2)$. In the right panel, the darker bands correspond to the extrapolation up to $\mathcal{O}(\hat{\mu}_B^2)$, whereas the lighter bands also include the $\mathcal{O}(\hat{\mu}_B^4)$ term.

6. Conclusions

The forthcoming experimental program, dedicated to the study of finite-density QCD, needs the theoretical support from lattice QCD simulations.This contribution collects an extensive compilation of results on equation of state, phase diagram and fluctuations of conserved charges. So far we have no indication of a critical point from lattice QCD. This is a very exciting time for the study of strongly interacting matter at finite density, and hopefully the joint effort of theory and experiment can lead to a map of the whole phase diagram of QCD relatively soon.

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