Recent developments on direct CP violation in the kaon system and connection to $K \to \pi \nu \bar{\nu}$ measurements

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The first lattice result from the RBC and UKQCD Collaborations and improved perturbative calculations of $\varepsilon'_K/\varepsilon_K$ have implied that the Standard-Model (SM) expectation deviates from measured values at the $2.8\,\sigma$ level. Since $\varepsilon'_K/\varepsilon_K$ comes from CP-violating FCNC and is significantly suppressed in the SM, the discrepancy can be explained easily in several new physics (NP) models. In this contribution, it is shown that correlations with the other rare decays, especially $K \to \pi \nu \overline{\nu}$ and $K_S \to \mu^+ \mu^-$, are crucial for discrimination of the NP models. These channels can be probed precisely in the future by the NA62 and KOTO experiments for $K \to \pi \nu \overline{\nu}$ and LHCb experiment for $K_S \to \mu^+ \mu^-$.

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1. ε'_K in the Standard Model

Charge-parity (*CP*) violating flavour-changing neutral current (FCNC) decays of kaon are significantly suppressed by a small CKM component of $\mathrm{Im}[V^*_{ts}V_{td}]/|V^*_{us}V_{ud}| \sim 0.6 \times 10^{-3}$ and a loop suppression factor in the Standard Model (SM), and hence are extremely sensitive to new physics (NP). Prime examples of such observables are direct *CP* violation in $K_L \to \pi^+\pi^-$, $\pi^0\pi^0$ decays, the branching fraction of $K_L \to \pi^0 \nu \bar{\nu}$, and the flavour-tagged asymmetry in $K_S \to \mu^+\mu^-$ decay.

In $K_L \to \pi\pi$ decays, one can distinguish between two types of *CP* violation: direct (ε'_K) and indirect *CP* violation (ε_K). Both kinds of *CP* violation have been precisely measured by many kaon experiments. Note that ε'_K is smaller than ε_K by three orders of magnitude. This strong suppression comes from the smallness of the $\Delta I = 3/2$ decay (to $I = 2$ state) compared to the $\Delta I = 1/2$ decay (to $I = 0$ state), namely the $\Delta I = 1/2$ rule, and an accidental cancellation of leading penguin contributions in the SM. Their suppressions lead to high sensitivity to NP. Until recently, large theoretical uncertainties precluded reliable predictions for ε'_K. Although SM predictions of ε'_K using chiral perturbation theory (ChPT) are consistent with the experimental value, their theoretical uncertainties are large. In contrast, a calculation by the dual QCD approach [1,2] finds the SM value much below the experimental one. A major breakthrough has been obtained from the recent lattice-QCD calculations of the hadronic matrix elements by the RBC-UKQCD collaboration [3,4], which supports the latter result.

A compilation of representative SM predictions and the experimental values for $\mathrm{Re}(\varepsilon'_K/\varepsilon_K)$ is given in Fig. 1. The SM predictions (magenta and blue bars) are taken from Refs. [3–12]. The experimental values (black bars) are taken from Refs. [13–16]. The thick black one is the world average of data [17]

$$\mathrm{Re}\left(\varepsilon'_K/\varepsilon_K\right)_{\mathrm{exp}} = (16.6 \pm 2.3) \times 10^{-4}. \tag{1.1}$$

In order to predict ε'_K in the SM, one has to calculate the hadronic matrix elements of four-quark operators using nonperturbative methods. The magenta bars in Fig. 1 have utilized analytic approaches to calculate them: chiral quark model (BEFL '97), ChPT (PPS '01 and GP '18) with the minimal hadronic approximation (HPR '03), and the dual QCD approach (BG '15). On the other hand, a determination of all hadronic matrix elements from lattice QCD has been obtained by the RBC-UKQCD collaboration [3,4], and the blue bars are based on the lattice result:

$$\varepsilon'_K/\varepsilon_K = \begin{cases} (1.9 \pm 4.5) \times 10^{-4} & \text{(BGJJ '15)}, \\ (1.06 \pm 5.07) \times 10^{-4} & \text{(KNT '16)}. \end{cases} \tag{1.2}$$

These results are obtained by next-to-leading order (NLO) calculations exploiting *CP*-conserving data to reduce hadronic uncertainties and include isospin-violating contributions [18] which are not included in the lattice result. Furthermore, the latter result contains an additional $\mathcal{O}(\alpha^2_{EM}/\alpha^2_s)$ correction, which appears only in this order, and also utilizes a new analytic solution of the renormalization group (RG) equation which avoids the problem of singularities in the NLO terms. The two numbers in Eq. (1.2) disagree with the experimental value in Eq. (1.1) by 2.9σ [11] and 2.8σ [12], respectively. The uncertainties are dominated by the lattice statistical and systematic uncertainties for the $I = 0$ amplitude.
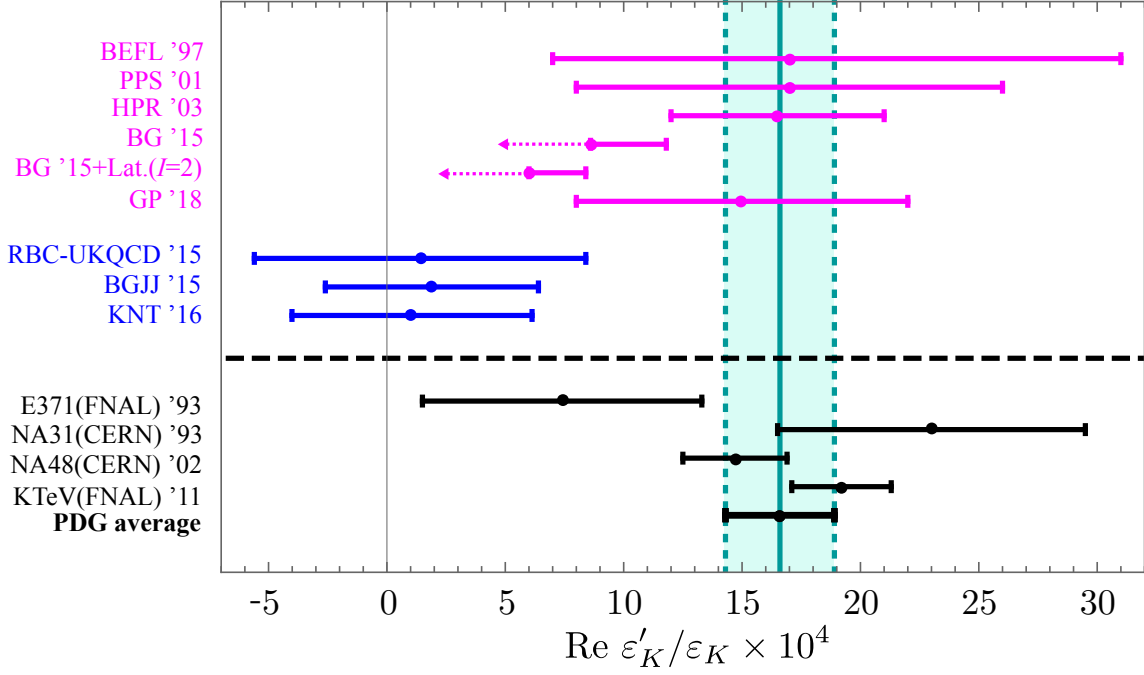
Figure 1: Compilation of representative SM predictions and the experimental values for $\mathrm{Re}(\varepsilon'_K/\varepsilon_K)$. All error bars represent $1\,\sigma$ range. The SM predictions are taken from Bertolini *et al.* (BEFL '97) [5], Pallante *et al.* (PPS '01) [6], Hambye *et al.* (HPR '03) [7], Buras and Gérard (BG '15) [8, 9], Gisbert and Pich (GP '18) [10], RBC-UKQCD lattice result [3, 4], Buras *et al.* (BGJJ '15) [11], and Kitahara *et al.* (KNT '16) [12], where magenta bars are based on analytic approaches to hadronic matrix elements while blue bars are based on lattice results. The thick black one is the world average of the experimental values [17].

The main difference between the analytic approach and the lattice result comes from a hadronic matrix element $\langle (\pi\pi)_{I=0} | Q_6 | K^0 \rangle \propto B_6^{(1/2)}$ which controls the largest positive contribution to $\varepsilon'_K/\varepsilon_K$ $[Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}]$. In ChPT, a large value has been obtained: $B_6^{(1/2)} \sim 1.6$ (BEFL '97), ~ 1.6 (PPS '01), and ~ 3 (HPR '03, see Ref. [8]). On the other hand, the dual QCD approach predicts a small number, $B_6^{(1/2)} \le B_8^{(3/2)} \sim 0.8$ (BG '15). The current lattice result is consistent with the latter result: $B_6^{(1/2)} = 0.56 \pm 0.20$ [4, 12].

Although the lattice simulation [4] includes final-state interactions partially along the line of Ref. [19], the lattice result of the strong phase shift δ_0 is smaller than the phenomenological expectation at $2.8\,\sigma$ level [20]. Meanwhile, the phase shift δ_2 of the lattice result is consistent with the phenomenological expectation. Also, the lattice result explains the $\Delta I = 1/2$ rule for the first time at $1\,\sigma$ level [3, 4, 9],

$$(\mathrm{Re}A_0/\mathrm{Re}A_2)_{\mathrm{exp}} = 22.45 \pm 0.05, \quad (\mathrm{Re}A_0/\mathrm{Re}A_2)_{\mathrm{Lat.}} = 31.0 \pm 11.1. \tag{1.3}$$

In the near future, the increasing precision of lattice calculations using more sophisticated methods will further sharpen the SM predictions in Eq. (1.2) and answer the question about NP in $\varepsilon'_K/\varepsilon_K$. In the lattice preliminary result, the discrepancy of the strong phase shift δ_0 is resolved [21].

Several NP models including supersymmetry (SUSY) can explain the discrepancy of $\varepsilon'_K/\varepsilon_K$. It is known that such NP models are likely to predict deviations of the other rare decay branching

ratios from the SM predictions, especially $K \to \pi \nu \bar{\nu}$ which includes *CP*-violating FCNC decay and can be probed precisely in the near future by the NA62 and KOTO experiments. In this contribution, based on the lattice result of $\varepsilon'_K / \varepsilon_K$ and Eq. (1.2), we present correlations between $\varepsilon'_K / \varepsilon_K$ and $\mathscr{B}(K \to \pi \nu \bar{\nu})$ in two types of NP scenarios: a box dominated scenario and a Z-penguin dominated one, and discuss how to distinguish between them.

2. Box dominated scenario

We first focus on the box dominated scenario, where all NP contributions to $|\Delta S| = 1$ and $|\Delta S| = 2$ processes are dominated by four-fermion box diagrams. Such a situation is realized in the minimal supersymmetric standard model (MSSM) [22]. The desired effect in ε'_K is generated via gluino-squark box diagrams when a mass difference between the right-handed up and down squarks exists [23, 24].

While sizable effects in ε'_K are obtained by the gluino box contributions, simultaneous efficient suppression of the SUSY QCD contributions to ε_K can also be achieved. The Majorana nature of the gluino leads to a suppression of $|\Delta S| = 2$ gluino box contributions to ε_K, where there are two such diagrams (crossed and uncrossed boxes) with opposite signs. If the gluino mass $m_{\tilde{g}}$ equals roughly 1.5 times the average down squark mass M_S, both contributions to ε_K cancel [25]. Note that this suppression appears only when a hierarchy $\Delta_{Q,12} \gg \Delta_{\bar{D},12}$ or $\Delta_{Q,12} \ll \Delta_{\bar{D},12}$ is satisfied, where the following notation is used for the squark mass matrices: $M_{X,ij}^2 = m_X^2 \left(\delta_{ij} + \Delta_{X,ij} \right)$, with $X = Q$, $\bar{U}$, or $\bar{D}$.

2.1 Contributions to ε'_K

The master equation for $\varepsilon'_K / \varepsilon_K$ (see e.g., Ref. [11]) reads:

$$\frac{\varepsilon'_K}{\varepsilon_K} = \frac{\omega_+}{\sqrt{2}|\varepsilon_K^{\exp}|\mathrm{Re}A_0^{\exp}} \left[\frac{\mathrm{Im}A_2}{\omega_+} - \left(1 - \hat{\Omega}_{\mathrm{eff}} \right) \mathrm{Im}A_0 \right], \tag{2.1}$$

with $\hat{\Omega}_{\mathrm{eff}} = (14.8 \pm 8.0) \times 10^{-2}$, the measured $|\varepsilon_K^{\exp}|$, $\omega_+ = (4.53 \pm 0.02) \times 10^{-2}$, and the amplitudes $A_I = \langle (\pi\pi)_I | \mathscr{H}^{|\Delta S|=1} | K^0 \rangle$ involving the effective $|\Delta S| = 1$ Hamiltonian $\mathscr{H}^{|\Delta S|}$. $I = 0, 2$ represents the strong isospin of the final two-pion state. The gluino box diagrams contribute to $\mathrm{Im}A_2$ when $m_{\bar{U}} \neq m_{\bar{D}}$. Because these contributions are governed by the strong interaction and there is an enhancement factor $1/\omega_+ = 22.1$ for the $\mathrm{Im}A_2$ term in (2.1), they easily become the largest contribution to $\varepsilon'_K / \varepsilon_K$. To obtain the desired large effect in ε'_K, the flavour mixing has to be in the left-handed squark mass matrix. The opposite situation with right-handed flavour mixing and $\tilde{u}_L$-$\tilde{d}_L$ mass splitting is not possible because of the $SU(2)_L$ invariance.

In the left panel of Fig. 2, the portion of the squark mass plane which simultaneously explains $\varepsilon'_K / \varepsilon_K$ discrepancy and ε_K constraint is shown. As input, we take the grand-unified theory (GUT) relation for gaugino masses, $m_{\tilde{g}}/M_S = 1.5$ for the suppressed ε_K, and $m_Q = m_{\bar{D}} = \mu_{\mathrm{SUSY}} = M_S$ with varying $m_{\bar{U}}$. The universal slepton mass is set to be $m_L = 300$ GeV. Furthermore, the trilinear SUSY-breaking matrices A_q are set to zero, $\tan\beta = 10$, and the only nonzero off-diagonal element of the squark mass matrices is $\Delta_{Q,12} = 0.1 \exp(-i\pi/4)$ for the left-handed squark sectors for $m_{\bar{U}} > m_{\bar{D}} = M_S$ (upper branch) and $\Delta_{Q,12} = 0.1 \exp(i3\pi/4)$ for $m_{\bar{U}} < m_{\bar{D}} = M_S$ (lower branch).
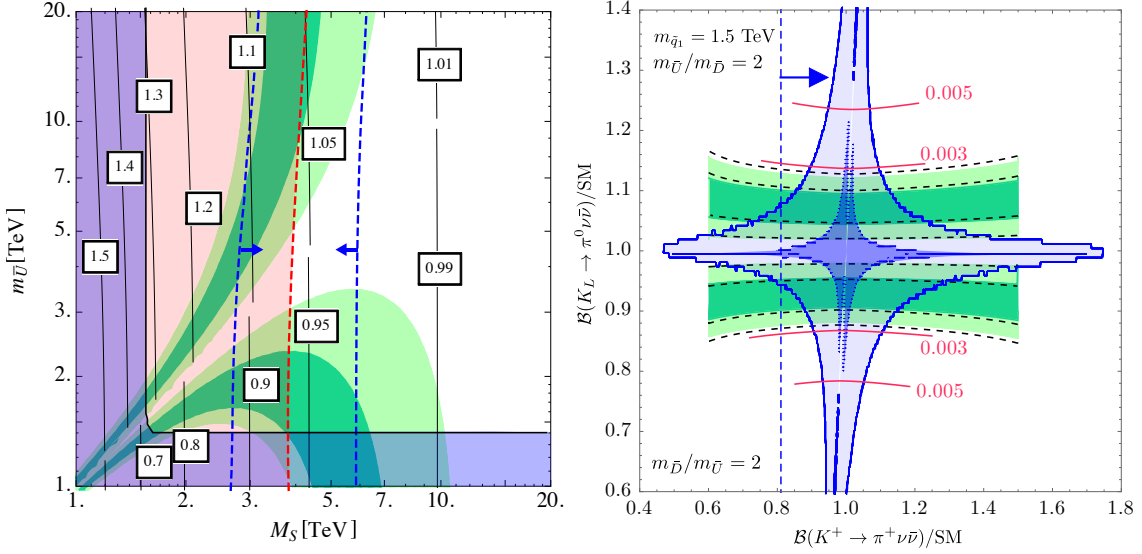
Figure 2: In the left panel, parameter constraints from ε_K and the LHC results are shown by the red and blue regions. The correlation with $\mathscr{B}(K \to \pi\nu\bar{\nu})$ is also shown in the right panel. The $\varepsilon'_K/\varepsilon_K$ discrepancy is resolved at the 1σ (2σ) level within the dark (light) green region in both panels. The light (dark) blue region requires a milder parameter tuning than 1% (10%) of the gluino mass and the *CP* violating phase in order to suppress contributions to ε_K. The red contour represents the SUSY contributions to $\varepsilon'_K/\varepsilon_K$.

2.2 $\mathscr{B}(K_L \to \pi^0\nu\bar{\nu})$ and $\mathscr{B}(K^+ \to \pi^+\nu\bar{\nu})$

The SUSY contributions to ε_K can be suppressed by the crossed and uncrossed box diagrams when the gluino mass is heavier than the squark mass, while there is no such cancellation in a chargino box contribution to $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ which permits potentially large effects. We investigate the correlation between ε'_K and $\mathscr{B}(K \to \pi\nu\bar{\nu})$ varying the following parameters:

$$|\Delta_{Q,12}|, \ \theta, \ M_3, \ m_{\bar{U}}/m_{\bar{D}}, \tag{2.2}$$

with $0 < |\Delta_{Q,12}| < 1$ and $0 < \theta < 2\pi$. Here, defining the bilinear terms for the squarks as $\theta \equiv \arg(\Delta_{Q,12})$. We fix the slepton mass and the lightest squark mass close to the experimental limit ($m_L = 300\,\text{GeV}$ and $m_{\tilde{q}_1} = 1.5\,\text{TeV}$) and use GUT relations among all three gaugino masses.

The right panel of Fig. 2 shows the correlations between ε'_K and $\mathscr{B}(K \to \pi\nu\bar{\nu})$ in the $\mathscr{B}(K_L \to \pi^0\nu\bar{\nu})$–$\mathscr{B}(K^+ \to \pi^+\nu\bar{\nu})$ plane which is normalized by their SM predictions [26]. We find that the necessary amount of the tuning in the gluino mass and the *CP* violating phase in order to suppress contributions to ε_K determines deviations of $\mathscr{B}(K \to \pi\nu\bar{\nu})$ from the SM values. A quantity which parameterizes the fine-tuning parameter is defined in Ref. [26]. The current $\varepsilon'_K/\varepsilon_K$ discrepancy between Eq. (1.1) and Eq. (1.2) is resolved at 1σ (2σ) within the dark (light) green region. We used $m_{\bar{D}}/m_{\bar{U}} = 2$ with $m_{\bar{U}} = m_Q$ for $0 < \theta < \pi$, and $m_{\bar{U}}/m_{\bar{D}} = 2$ with $m_{\bar{D}} = m_Q$ for $\pi < \theta < 2\pi$. Numerically, we observe $\mathscr{B}(K_L \to \pi^0\nu\bar{\nu})/\mathscr{B}^{\text{SM}}(K_L \to \pi^0\nu\bar{\nu}) \lesssim 2\,(1.2)$ and $\mathscr{B}(K^+ \to \pi^+\nu\bar{\nu})/\mathscr{B}^{\text{SM}}(K^+ \to \pi^+\nu\bar{\nu}) \lesssim 1.4\,(1.1)$ in light of $\varepsilon'_K/\varepsilon_K$ discrepancy, if all squarks are heavier than 1.5 TeV and if a $1\,(10)\%$ fine-tuning is permitted.

We also observe a strict correlation between $\mathscr{B}(K_L \to \pi^0 \nu \bar{\nu})$ and $m_{\tilde{U}}/m_{\tilde{D}}$: $\text{sgn}\left[\mathscr{B}(K_L \to \pi^0 \nu \bar{\nu}) - \mathscr{B}^{\text{SM}}(K_L \to \pi^0 \nu \bar{\nu})\right] = \text{sgn}\left[m_{\tilde{U}} - m_{\tilde{D}}\right]$. Thus, $\mathscr{B}(K_L \to \pi^0 \nu \bar{\nu})$ can indirectly determine whether the right-handed up or down squark is the heavier one.

3. Z-penguin dominated (modified Z-coupling) scenario

Next we focus on the Z-penguin dominated scenario. The largest negative contribution to ε'_K comes from Z-penguin diagrams in the SM. Since in the SM there is a large numerical cancelation between QCD- and Z-penguin contributions to $\varepsilon'_K/\varepsilon_K$, a modified Z flavour-changing (s–d) interaction from NP can explain the current $\varepsilon'_K/\varepsilon_K$ easily [27, 28]. Then, the decay, $s \to d \nu \bar{\nu}$, proceeding through an intermediate Z boson, must be modified by the NP. Therefore, the branching ratios of $K \to \pi \nu \bar{\nu}$ are likely to deviate from the SM predictions once the $\varepsilon'_K/\varepsilon_K$ discrepancy between Eq. (1.1) and Eq. (1.2) is explained by the modified Z-coupling. They could be a signal to test the scenario. In the MSSM, such a scenario is also realized when the off-diagonal components of the trilinear SUSY-breaking couplings are large [29–31].

Such a signal is constrained by the ε_K. The modified Z couplings affect the ε_K via the so-called double penguin diagrams. Such a contribution is enhanced when there are both left-handed and right-handed couplings because of the chiral enhancement of the hadronic matrix elements. The important point is that since the left-handed coupling is already present in the SM, the right-handed coupling must be constrained even without NP contributions to the left-handed one. Such interference contributions between the NP and the SM have been overlooked in the literature. References [31–33] have revisited the modified Z-coupling scenario including the interference contributions using a framework of the SMEFT, and found the parameter regions allowed by the indirect CP violation change significantly.

We find that similar to the previous section, the deviations of $\mathscr{B}(K \to \pi \nu \bar{\nu})$ from the SM values are determined by the necessary amount of the tuning in NP contributions to ε_K. We parametrize it by ξ: A degree of the NP parameter tuning is represented by $1/\xi$, e.g., $\xi = 10$ means that the model parameters are tuned at the 10% level. The definition of ξ is given in Ref. [32].

In Fig. 3, contours of the tuning parameter ξ are shown for the simplified scenarios: LHS (all NP effects appear as left-handed), RHS (all NP effects appear as right-handed), ImZS (NP effects are purely imaginary), and LRS (left-right symmetric scenario) on the plane of the branching ratios of $K \to \pi \nu \bar{\nu}$ which are normalized by their SM predictions. We scanned the whole parameter space of the modified Z-coupling in each scenario, and selected the parameters where $\varepsilon'_K/\varepsilon_K$ is explained at the 1σ level. The experimental bounds from ε_K, ΔM_K, and $\mathscr{B}(K_L \to \mu^+ \mu^-)$ are satisfied. In most of the allowed parameter regions, $\xi = \mathscr{O}(1)$ is obtained. Thus, one does not require tight tunings in these simplified scenarios. In the figures, $\mathscr{B}(K_L \to \pi^0 \nu \bar{\nu})$ is always smaller than the SM value by more than 30%. In LHS, $\mathscr{B}(K_L \to \pi^0 \nu \bar{\nu})$ is much smaller and could be ruled out if a SM-like value is measured. On the other hand, $\mathscr{B}(K^+ \to \pi^+ \nu \bar{\nu})$ depends on the scenarios. In LHS, we obtain $0 < \mathscr{B}(K^+ \to \pi^+ \nu \bar{\nu})/\mathscr{B}(K^+ \to \pi^+ \nu \bar{\nu})_{\text{SM}} < 1.8$. In RHS, $\mathscr{B}(K^+ \to \pi^+ \nu \bar{\nu})$ is comparable to or larger than the SM value, but cannot be twice as large. In ImZS, the branching ratios are perfectly correlated and $\mathscr{B}(K^+ \to \pi^+ \nu \bar{\nu})$ does not deviate from the SM one. In LRS, $\mathscr{B}(K_L \to \pi^0 \nu \bar{\nu})$ does not exceed about a half of the SM value. The more general situation is discussed in Ref. [32].
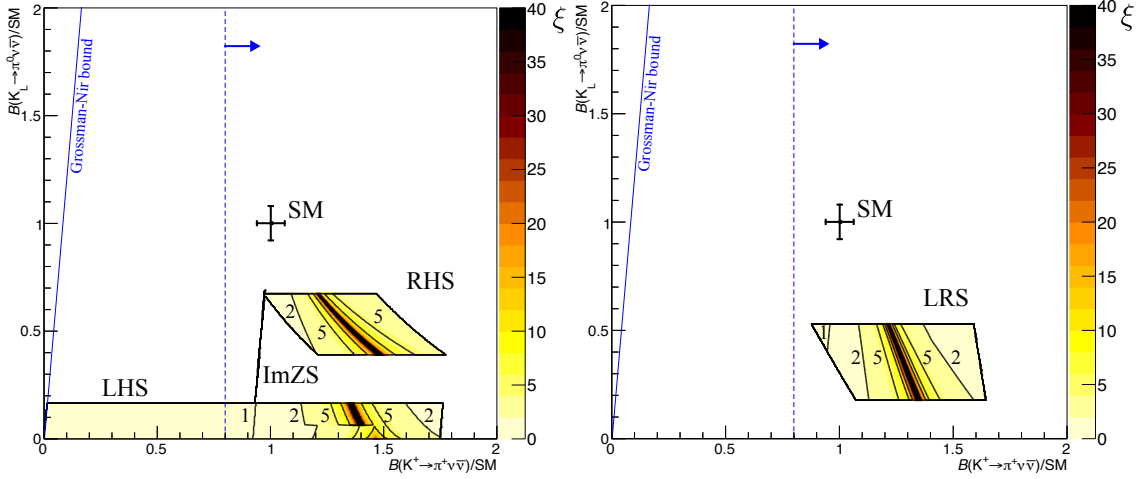
Figure 3: Contours of the tuning parameter ξ are shown in the simplified modified Z-coupling scenarios: LHS, RHS, and ImZS (left panel) and LRS (right). In the colored regions, $\varepsilon'_K/\varepsilon_K$ is explained at 1σ.

4. Discussion and conclusions

In this talk, we presented the current situation for $\varepsilon'_K/\varepsilon_K$ within the SM. The first lattice result and the improved perturbative calculations have shown the discrepancy between the predicted value and the data. Several NP models can explain the discrepancy of $\varepsilon'_K/\varepsilon_K$, and then $\mathscr{B}(K \to \pi\nu\bar{\nu})$ are predicted to deviate from the SM predictions. We have shown the correlations between $\varepsilon'_K/\varepsilon_K$, $\mathscr{B}(K_L \to \pi^0\nu\bar{\nu})$, and $\mathscr{B}(K^+ \to \pi^+\nu\bar{\nu})$ in two different NP scenarios; the box dominated scenario and the Z-penguin dominated one. It is found that the constraint from ε_K produces distinguishable correlations. In the future, measurements of $\mathscr{B}(K \to \pi\nu\bar{\nu})$ will be significantly improved. The NA62 experiment at CERN measuring $\mathscr{B}(K^+ \to \pi^+\nu\bar{\nu})$ is aiming to reach a precision of $10\,\%$ compared to the SM value [34]. Concerning $K_L \to \pi^0\nu\bar{\nu}$, the KOTO experiment at J-PARC is aiming in a first step to measure $\mathscr{B}(K_L \to \pi^0\nu\bar{\nu})$ around the SM sensitivity. Furthermore, the KOTO-step2 experiment will aim at 100 events for the SM branching ratio, implying a precision of $10\,\%$ of this measurement. Therefore, we conclude that when the $\varepsilon'_K/\varepsilon_K$ discrepancy is explained by a NP contribution, the NA62 experiment could probe whether a modified Z-coupling scenario is realized or not, and KOTO-step2 experiment can distinguish the box dominated scenario and the simplified modified Z-coupling scenario.

We should comment on $K_S \to \mu^+\mu^-$ decay which proceeds via long-distance *CP*-conserving and short-distance *CP*-violating processes. Since the decay rate is dominated by the former, whose uncertainty is large, the sensitivity to the short-distance contributions is diminished. However, it is pointed out that the short-distance contribution is significantly amplified through interference between the K_L and K_S states in the neutral kaon beam [35]. Therefore, one can also distinguish the NP scenarios using the correlation with $K_S \to \mu^+\mu^-$. Such a correlation has been investigated in the box dominated scenario (with large $\tan\beta$) [36] and the modified Z-coupling scenario [31, 35].

Acknowledgment

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