PROCEEDINGS OF SCIENCE

Orbital angular momentum distributions at small-x

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I study the x-distribution of the orbital angular momentum (OAM) of quarks and gluons in the nucleon with particular emphasis on the small-x region. I argue, in two different ways, that the gluon OAM significantly cancels the gluon helicity distribution at small-x. A similar cancellation occurs also in the quark sector.

XXVI International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS2018)
16-20 April 2018
Kobe, Japan

*Speaker.

https://pos.sissa.it/

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1. Introduction

The nucleon spin decomposition continues to be an important subject of QCD spin physics. It has been known for a long time that quarks' helicity $\Delta\Sigma$ accounts for only about a quarter of the nucleon spin. Recent NLO global QCD analyses have found a nonzero contribution from the helicity of gluons ΔG [1]. When combined, these two contributions still fall short of the expected value of $\frac{1}{2}$. One might expect that the remaining discrepancy could be resolved by a precise future measurement of the gluon helicity distribution $\Delta G(x)$ in the small-x region where the current theoretical uncertainties are very large.

However, *a priori* there is no reason to expect that the nucleon spin entirely originates from partons' helicity. As the Jaffe-Manohar sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g, \tag{1.1}$$

shows, the resolution of the spin puzzle requires a full understanding of the orbital angular momentum of quarks L_q and gluons L_g. Unfortunately, at the moment very little is known about the actual value of $L_{q,g}$, and the community still has a long way to go in extracting them from experiments. The recent proposals of observables for $L_{q,g}$ [2, 3, 4, 5] as well as the first lattice QCD computation of L_q [6] are encouraging progress in this direction.

The four terms in (1.1) actually depend on the renormalization scale Q^2. Moreover, they can be written as the integral over Bjorken-x of the corresponding partonic distributions. For $\Delta\Sigma$ and ΔG, these are the usual polarized parton distributions $\Delta\Sigma(x)$ and $\Delta G(x)$. A less known fact is that the x-distributions for $L_{q,g}$ can also be defined [7, 8, 9]

$$L_{q,g}(Q^2) = \int_0^1 dx L_{q,g}(x,Q^2). \tag{1.2}$$

$L_{q,g}(x)$ are not the usual, twist-two parton distributions. It consists of the 'Wandzura-Wilczek' part and the genuine twist-three parts [9]. Introduction of such x-distributions is crucial for the measurability of the orbital angular momentum. In this contribution to the proceedings I present our recent work on the small-x behavior of $L_g(x)$. I argue, via two independent methods, that $L_g(x) \approx -\Delta G(x)$ at small-x, meaning that there is a significant cancellation between the two distributions.

2. Operator analysis

A particularly useful representation of $L_g(x)$ is in terms of the gluon Wigner distribution

$$L_g(x) = \int db_\perp dk_\perp (b_\perp \times k_\perp) W_g(x, b_\perp, k_\perp)\,. \tag{2.1}$$

The small-x behavior of $L_g(x)$ can thus be studied by analyzing the Wigner distribution at small-x. The latter contains the phase $e^{-ixP^+z^-}$ associated with the light-cone correlation function. At small-x, one may approximate $e^{-ixP^+z^-} \approx 1$. After this approximation, the Wigner distribution can be written entirely in terms of infinite Wilson lines along the light-cone $U_{\infty,-\infty}$. In this 'eikonal' approximation, one actually loses all information about the longitudinal spin. Therefore, one has to go to the next order $e^{-ixP^+z^-} \approx 1 - ixP^+z^-$. In this order, one finds a new class of operators

$$\mathrm{Tr}\left[U_{\infty,-\infty}(x_\perp)U_{-\infty,z^-}(y_\perp)D_i U_{z^-,\infty}(y_\perp)\right]. \tag{2.2}$$

The covariant derivative acting on the Wilson line represents the deviation from the eikonal trajectory. While the operator (2.2) looks unfamiliar, remarkably, exactly the same operator shows up in the analysis of the TMD polarized gluon distribution $\Delta G(x,k_\perp)$ at small-x. In other words, $L_g(x)$ and $\Delta G(x)$ are related at the operator level [3], and there exists a linear relation between $L_g(x)$ and $\Delta G(x)$[1]

$$L_g(x) = -\Delta G(x) + \cdots \,. \tag{2.3}$$

In Ref. [3], it has been argued that the neglected terms in (2.3) is small. (2.3) has significant implications on the nucleon spin sum rule. Currently there are huge uncertainties in the gluon helicity $\Delta G = \int_0^1 dx \Delta G(x)$ from the small-x region. But this is likely to be canceled by the orbital angular momentum in the same x-region so that the net contribution from the small-x region might be small.

3. Q^2-evolution equation

The second argument concerns the Q^2-evolution of $L_{q,g}(x,Q^2)$. The relevant evolution equation was essentially derived in [8] and reads

$$\frac{d}{d\ln Q^2}\begin{pmatrix} L_q(x) \\ L_g(x)\end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix} \hat{P}_{qq}(z) & \hat{P}_{qg}(z) & \Delta\hat{P}_{qq}(z) & \Delta\hat{P}_{qg}(z) \\ \hat{P}_{gq}(z) & \hat{P}_{gg}(z) & \Delta\hat{P}_{gq}(z) & \Delta\hat{P}_{gg}(z)\end{pmatrix}\begin{pmatrix} L_q(x/z) \\ L_g(x/z) \\ \Delta\Sigma(x/z) \\ \Delta G(x/z)\end{pmatrix}, \tag{3.1}$$

where

$$\hat{P}_{qq}(z) = C_F\left(\frac{z(1+z^2)}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right), \tag{3.2}$$
$$\hat{P}_{qg}(z) = n_f z(z^2+(1-z)^2), \tag{3.3}$$
$$\hat{P}_{gq}(z) = C_F(1+(1-z)^2), \tag{3.4}$$
$$\hat{P}_{gg}(z) = 6\frac{(z^2-z+1)^2}{(1-z)_+} + \frac{\beta_0}{2}\delta(z-1), \tag{3.5}$$
$$\Delta\hat{P}_{qq}(z) = C_F(z^2-1), \tag{3.6}$$
$$\Delta\hat{P}_{qg}(z) = n_f(1-z)(1-2z+2z^2), \tag{3.7}$$
$$\Delta\hat{P}_{gq}(z) = C_F(z-1)(-z+2), \tag{3.8}$$
$$\Delta\hat{P}_{gg}(z) = 6(z-1)(z^2-z+2). \tag{3.9}$$

The unusual structure of the equation is because $L_{q,g}(x)$ have a twist-two (Wandzura-Wilczek) component, so they mix with $\Delta q(x)$ and $\Delta G(x)$ under renormalization.

It is straightforward to solve these equations numerically once the initial condition is set [10]. Here I consider a very simple model

$$\Delta\Sigma(x,Q_0^2) = A_q x^{-0.3}(1-x)^3, \qquad \Delta G(x,Q_0^2) = A_g x^{-0.3}(1-x)^3,$$
$$L_q(x,Q_0^2) = L_g(x,Q_0^2) = 0, \tag{3.10}$$

[1] In [3], there was a mistake by a factor of 2 in the relation (2.3).

that is, the orbital angular momentum is zero at the initial scale $Q_0 = 1$ GeV. A_q and A_g are fixed by the conditions $\Delta\Sigma(Q_0^2) = \frac{1}{4}$ and $\Delta G(Q_0^2) = \frac{3}{8}$. The result with the one-loop running coupling is shown in Fig. 1 as a function of the rapidity $Y = \ln 1/x$.[2]

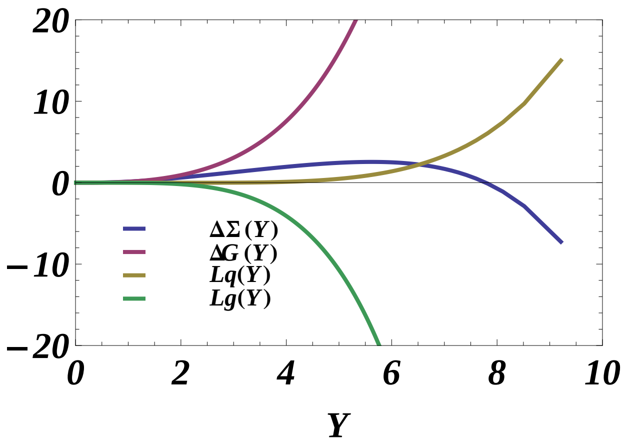

Figure 1: The four distributions at $Q^2 = 10$ GeV^2 as a function of $Y = \ln 1/x$.

We clearly see that $L_g(x)$ becomes large and negative, although initially zero, and mostly cancels the strong positive rise of $\Delta G(x)$. A similar cancellation occurs also between $\Delta q(x)$ and $L_q(x)$. Actually one can understand this cancellation analytically. Using the ansatz

$$L_g(x,Q^2) \approx A(Q^2)\frac{1}{x^c}, \qquad \Delta G(x,Q^2) \approx B(Q^2)\frac{1}{x^c}. \tag{3.11}$$

with $c > 0$ and keeping only the leading singularity, one obtains the following asymptotic relations

$$\frac{L_g(x)}{\Delta G(x)} \approx -\frac{2}{c+1}, \tag{3.12}$$

$$\frac{\Delta\Sigma(x)}{\Delta G(x)} \approx -n_f \frac{1-c}{c(1+c)\left[6\left(-H_{c-1} + \frac{1}{c} - \frac{1}{1+c}\right) + \frac{\beta_0}{2}\right]}. \tag{3.13}$$

$$L_q(x) \approx \approx -\frac{\Delta\Sigma(x)}{1+c}, \tag{3.14}$$

where $H_x = x\sum_{k=1}^{\infty} \frac{1}{k(x+k)}$ is the harmonic number. We see that the relative signs are correctly reproduced and the degree of cancellation is controlled by the Regge intercept c.

4. Conclusions

I have argued, via two independent arguments, that the helicity and orbital angular momentum significantly cancel at small-x. A similar phenomenon has been recently obtained in a model calculation of $L_{q,g}(x)$ [11]. This finding has an important implications for phenomenology. On one hand, the precise value of ΔG is of intrinsic interest in QCD, and it is certainly imperative to

[2] I thank my collaborator D. J. Yang for providing this plot.

reduce the uncertainties of $\Delta G(x)$ in the small-x region in future experiments such as at the planned Electron-Ion Collider (EIC). On the other hand, this is not sufficient to solve the nucleon spin puzzle because a good fraction of the would-be spin from $\Delta G(x)$ at small-x is canceled by the orbital angular momentum in the same x-region. This suggests that the resolution of the spin puzzle resides in the orbital angular momentum in the large-x region. Proposals of experimental observables aimed at this region are now available [**?**, 4, 5].

Finally, the DGLAP-type evolution equation considered here eventually breaks down and should be superseded by the small-x evolution equation which resums double logarithmic contributions $(\alpha_s \ln^2 1/x)^n$. This problem has been recently revisited in [12]. Furthermore, there may be a regime where nonlinear evolution equations come into play, as is the case for the unpolarized distributions. Unfortunately, at the moment very little is known about the small-x resummation for the orbital angular momentum distributions. This issue deserves further study.

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