Towards the N^{3}LO evolution of parton distributions

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We present a brief status report on calculations for the four-loop splitting functions in perturbative QCD, which will pave the way to future determination of N^{3}LO parton distribution functions of hadrons. In the large-N_c limit, the exact four-loop contribution to the flavour non-singlet splitting functions has been obtained. For the remaining large-N_c suppressed terms, we provide approximate expressions that are sufficient for phenomenological applications.

XXVI International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS2018)
16-20 April 2018
Kobe, Japan

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PoS(DIS2018)034

1. Introduction

Despite the great success in the discovery of an elusive Higgs boson, no clear signal from new physics beyond the Standard Model has hitherto been detected at the LHC. Because pushing the high-energy frontier further requires a more gigantic accelerator and the construction could take decades even if the budget allows it, one may expect that the precision frontier would be the key to the next breakthrough in particle physics.

For hadron colliders, a hard scattering observable (e.g., the total cross section for the production of the Higgs boson) can be schematically written as

$$\sigma^{\text{hadronic}} = \sum_{i,j} f_i \otimes f_j \otimes \sigma_{ij}^{\text{partonic}}, \tag{1.1}$$

where f_i and f_j are the parton distribution functions (PDFs) in the initial hadrons and $\sigma_{ij}^{\text{partonic}}$ is the corresponding parton-level cross section. PDFs are intrinsically non-perturbative objects, but their scale dependence is governed by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation [1, 2, 3]:

$$\frac{d}{d\ln\mu_F^2} f_i(x,\mu_F^2) = \sum_j \Big[P_{ij}\big(\alpha_s(\mu_F^2)\big) \otimes f_j(\mu_F^2) \Big](x), \tag{1.2}$$

with perturbatively calculable splitting functions P_{ij}. (The evolution equation in the Mellin N-space goes back to refs. [4, 5, 6, 7].) In perturbative QCD, the partonic cross sections and splitting functions are expanded in the strong coupling α_s as

$$\sigma = \alpha_s^{n_0}\big[\sigma^{(0)} + \alpha_s\sigma^{(1)} + \alpha_s^2\sigma^{(2)} + \alpha_s^3\sigma^{(3)} + \dots\big], \tag{1.3}$$

$$P = \quad \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \alpha_s^4 P^{(3)} + \dots\big]. \tag{1.4}$$

Due to a high demand for the LHC physics, many calculations have been performed in the next-to-next-to-to-leading order (NNLO, or N^{2}LO). The splitting functions are known up to the same order [8, 9, 10, 11]. Recently, results for partonic cross sections even in the next order, N^{3}LO, are starting to appear: Higgs boson production via the gluon fusion [12, 13, 14] and the vector-boson fusion [15] in proton-proton collisions. N^{3}LO results are also available for inclusive lepton-hadron deep inelastic scattering (DIS) [16, 17, 18] and jet production in DIS [19]. However, such N^{3}LO analyses in principle require the four-loop splitting, which are still missing.

Here we briefly report the progress in computing partial results on the four-loop splitting functions. From basic symmetry arguments, the system (1.2) for quarks, antiquarks and gluon, i.e., $(2n_f+1)$ partons, can be decomposed into $(2n_f-1)$ equations for the flavour non-singlet part and a 2×2 flavour singlet system. The non-singlet part consists of $2(n_f-1)$ flavour asymmetries of quark-antiquark sums and differences and the total valence distribution. In the following, we mainly discuss the non-singlet part, for which 3 non-singlet combinations $P_{\text{ns}}^{\pm}$ and P_{ns}^{v} appear as the splitting functions. See refs. [20, 21, 22] for more details.

2. Calculation

We consider the Mellin transform of the splitting functions:

$$\gamma_{ij}(N) = -\int_0^1 dx\, x^{N-1} P_{ij}(x). \tag{2.1}$$

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The extra minus sign is for a standard convention. If one could compute $\gamma_{ij}(N)$ with all the N-dependence, one would perform the inverse Mellin transform to obtain the splitting functions $P_{ij}(x)$ as functions of x. At the four-loop level, this is a formidable task and beyond the current computing technology. Instead of computing the full N-dependence, one can compute fixed Mellin moments for low-integer N-values, and then try to construct approximate expressions in the x-space. Such approximations had been constructed at the three-loop level [23, 24, 25] from fixed Mellin moments [26, 27, 28] before the exact expressions became available.

We employed two methods for computing the Mellin moments. The first one is via the optical theorem and the operator product expansion of partonic forward scattering processes. The second one uses the fact that $\gamma_{ij}(N)$ are indeed the anomalous dimensions of the corresponding spin-N twist-two composite operators and computes the matrix elements of the operators sandwiched between off-shell partonic states. In both methods, the necessary computations reduce to evaluation of four-loop massless propagator-type Feynman integrals. This was handled by using the FORCER program [29], which can be considered as a four-loop extension of MINCER [30, 31].

The first method is relatively straightforward to set up, but the complexity of the computations rapidly grows as one goes for higher N. Indeed, the first motivation of developing FORCER was performing these computations efficiently. For general cases in both singlet and non-singlet parts, so far only $N \leq 6$ has been completed. We have checked that our non-singlet results up to $N \leq 4$ agree with previously known results [32, 33, 34, 35]. High-n_f parts have simpler diagram structures and values up to $N > 40$ have been computed, which are enough to determine the complete n_f^2 and n_f^3 parts of the non-singlet splitting functions and n_f^3 parts of the singlet contributions [18].

To obtain higher-N moments, we need to switch to the second method. Our code for the non-singlet parts was debugged with using the results up to $N \leq 6$ obtained by the first one, and have reached to $N = 16$. In the limit of large number of colours $N_c \to \infty$, the moments of $P_{\rm ns}^{(3),\pm}$ were computed up to $N \leq 20$, which is sufficient to reconstruct the full-N result for the large-N_c part. For the remaining large-N_c suppressed terms, approximate expressions were obtained from the low-N moments. The details are in ref. [21].

3. Numerical results

The approximated four-loop coefficients $P_{\rm ns}^{(3)\pm}(x)$ of the non-singlet splitting functions, obtained from the results in the N-space computed as in section 2, are shown for $n_f = 4$ in figure 1. For a comparison purpose, the corresponding three-loop coefficients $P_{\rm ns}^{(2)\pm}(x)$ are also shown. In the four-loop approximations, the lower and upper bounds are plotted, which deviate in the small-x region.

In figure 2 we plot the NLO, NNLO and N^{3}LO predictions for the scale evolution of the parton density $d\ln q_{\rm ns}^{+}/d\ln\mu_f^2$, with varying the ratio of the renormalisation scale μ_r to the factorisation scale μ_f over the range $\frac{1}{8}\mu_f^2 \leq \mu_r^2 \leq 8\mu_f^2$, at six typical values of x. The initial conditions are taken as $xq_{\rm ns}^{+}(x,\mu_0^2) = x^{0.5}(1-x)^3$ and $\alpha_s(\mu_0^2) = 0.2$. One can see that the N^{3}LO result is relatively stable under the variation of the renormalisation scale. The uncertainty due to the approximation from a finite number of Mellin moments is almost invisible except the last panel.

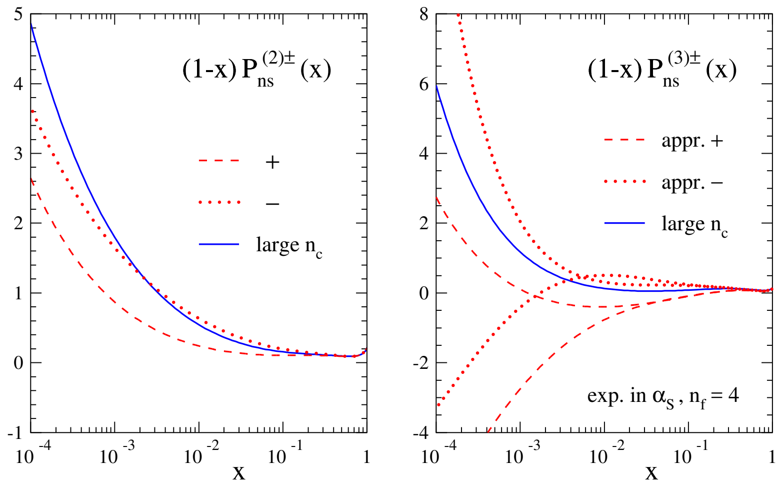

Figure 1: Left panel: the three-loop coefficients of the non-singlet splitting functions $P_{\rm ns}^{(2)\pm}(x)$ for QCD with $n_f = 4$. Right panel: the approximated four-loop coefficients $P_{\rm ns}^{(3)\pm}(x)$. The approximations are represented by two lines indicating the upper and lower bounds.

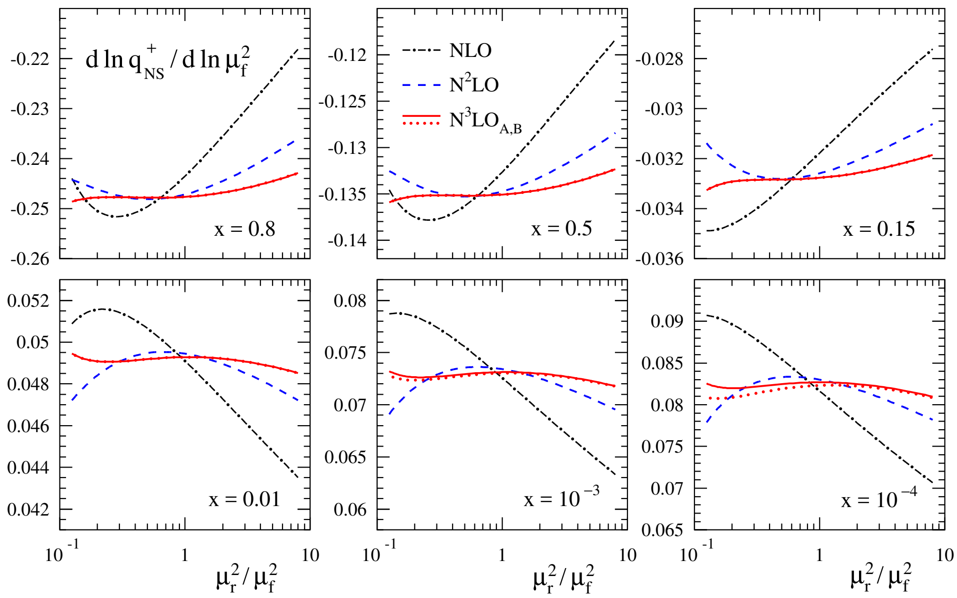

Figure 2: The dependence of the NLO, NNLO and N^{3}LO predictions for the scale evolution of the parton density $d\ln q_{\rm ns}^+/d\ln\mu_f^2$ on the renormalisation scale μ_r at six typical values of x for a sample initial condition.

4. Summary and outlook

Approximations to the QCD four-loop non-singlet splitting functions have been obtained, which are sufficiently accurate for collider physics applications. In practice, one also needs the singlet splitting functions for phenomenological studies. To obtain approximations for the singlet parts, we need to go for the method of computing operator matrix elements with off-shell states. Then renormalisation of the matrix elements of the gluon operator in four-loops is a theoretical challenge, which can be circumvented only for some colour factors [22]. We hope to solve this issue in a future publication.

Acknowledgments

The author thanks J. Davies, S. Moch, B. Ruijl, J.A.M. Vermaseren and A. Vogt for the fruitful collaboration. He also thanks S. Moch and J.A.M. Vermaseren for useful comments on this manuscript.

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