

Decay Constants of the Heavy–Light Mesons $D^{(*)}$ and $B^{(*)}$: Isospin Breaking

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QCD sum rules are analytic predictions of hadron observables from quantum chromodynamics. In the so-called local-duality limit of their Borel-transformed form, all nonperturbative contributions are encompassed by just a single quantity, an effective threshold beyond which, by assumption, all hadronic contributions are reliably described by perturbative QCD. Reconstructing its quark-mass dependence enables us to investigate the differences of the leptonic decay constants of heavy–light mesons effectuated by the tiny isospin-violating discrepancy of the masses of up and down quarks. By and large, our findings enjoy excellent agreement with the outcomes of earlier studies, the only exception being a lattice-QCD prediction for the B mesons, where we find dramatic disagreement.

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QCD sum rules [1] relate hadron properties to the fundamental parameters of the quantum field theory of strong interactions, quantum chromodynamics; they are found from correlators of suitable interpolating operators by insertion of complete sets of hadron states and use of the operator product expansion. Applying Borel transformations removes all subtraction terms and suppresses all hadron contributions above the ground states. Any ignorance about heavier hadron states is swept under the carpet by invoking quark–hadron duality which assumes QCD and hadron contributions to mutually cancel above effective thresholds. The accuracy [2] of extracted predictions is considerably raised if taking into account such thresholds’ dependence [3] on the parameters of the Borel transformations. This advanced QCD sum-rule technique led us to revisit the heavy–light-meson decay constants [4], specifically, their relative magnitude [5]. Recently, we embarked on the analysis of isospin breaking induced in heavy–light-meson decay constants by the tiny up and down quark mass difference [6,7].

In the local-duality limit of vanishing Borel variables, the QCD sum rule for the decay constant f_{H_q} of a meson H_q formed by a heavy quark $Q = c, b$ of mass m_Q and a light quark $q = u, d, s$ of mass m_q reduces to a dispersion integral of a spectral density ρ found perturbatively from QCD in form of a series expansion in powers of the strong fine-structure constant α_s , with all nonperturbative effects subsumed by its effective threshold s_{eff} ; the inevitable truncation of the perturbative series entails an unphysical dependence of the spectral density and the effective threshold on renormalization scales:

$$f_{H_q}^2 = \int_{(m_Q+m_q)^2}^{s_{\text{eff}}(m_Q, m_q)} ds \rho(s, m_Q, m_q, \alpha_s).$$

Utilizing spectral densities up to $O(\alpha_s m_q^1)$ and $O(\alpha_s^2 m_q^0)$ [8–10], we derive the decay constants f_P and f_V of pseudoscalar mesons P and vector mesons V , with masses M_P and M_V , defined in terms of the axialvector and vector currents $A_\mu(x) = \bar{q}(x) \gamma_\mu \gamma_5 Q(x)$ and $V_\mu(x) = \bar{q}(x) \gamma_\mu Q(x)$ according to $\langle 0 | A_\mu(0) | P(p) \rangle = i f_P p_\mu$ and $\langle 0 | V_\mu(0) | V(p) \rangle = f_V M_V \varepsilon_\mu(p)$. Using modified minimal subtraction renormalization for the definition of parameters optimizes [9] perturbative convergence, cf. Table 1.

Table 1: Numerical values of input parameters in the modified minimal-subtraction renormalization scheme.

Quantity	Numerical input value	Reference
$\alpha_s(M_Z)$	0.1182 ± 0.0012	[11]
$\frac{1}{2}(m_u + m_d) \equiv m_{ud}(2 \text{ GeV})$	$(3.70 \pm 0.17) \text{ MeV}$	[11]
$(m_d - m_u)(2 \text{ GeV})$	$(2.67 \pm 0.22) \text{ MeV}$	[11]
$m_s(2 \text{ GeV})$	$(93.9 \pm 1.1) \text{ MeV}$	[11]
$m_c(m_c)$	$(1.275 \pm 0.025) \text{ GeV}$	[12]
$m_b(m_b)$	$(4.247 \pm 0.034) \text{ GeV}$	[13]

We fathom the discrepancy $f_{H_d} - f_{H_u}$ of our heavy–light meson decay constants by tracking the response $f_H(m_q)$ of the local-duality QCD sum rule to continuous variations of the light-quark mass m_q in its interval $[0, m_s]$. Given the perturbative spectral density at appropriately high order, the only required ingredient is the effective threshold’s quark-mass dependence $s_{\text{eff}} = s_{\text{eff}}(m_Q, m_q)$. Shuffling together available knowledge on heavy-quark expansion and chiral logarithmic corrections [14], we distil parametrizations of $z_{\text{eff}} \equiv \sqrt{s_{\text{eff}}} - m_Q - m_q$ [15] the coefficients of which are fixed by matching

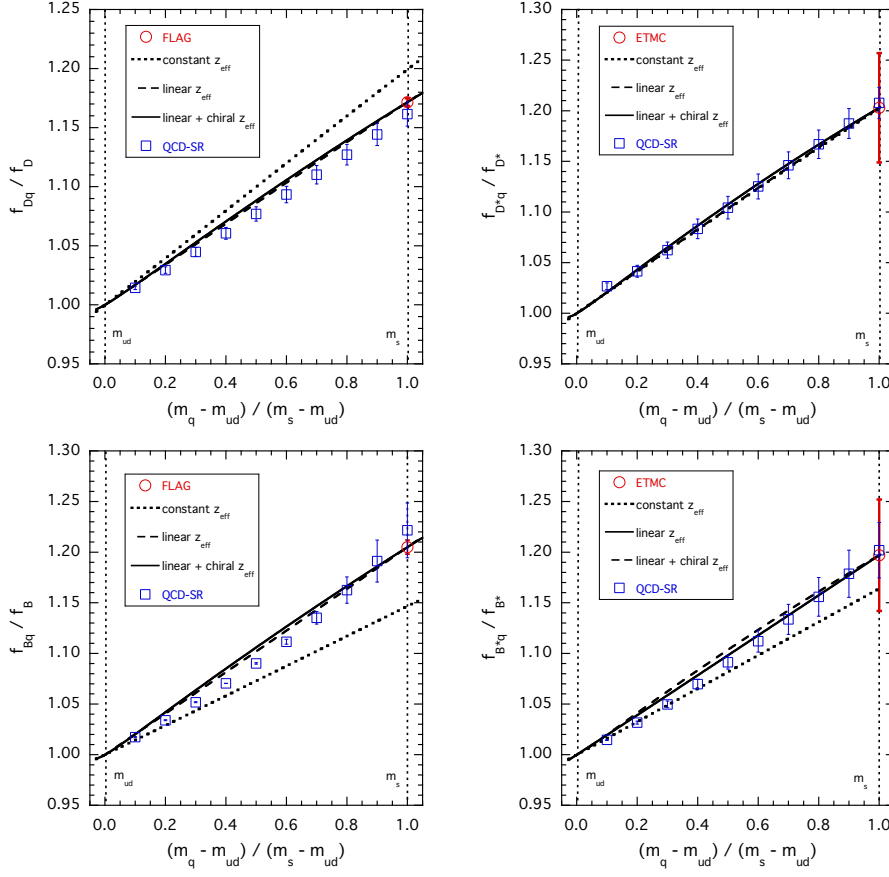


Figure 1: Behaviour of $f_{H_q}/f_H(m_{ud})$ with redefined m_q [15], compared with the results of Ref. [7] (squares).

$f_H(m_q)$ for $m_q = m_{ud} \equiv \frac{1}{2}(m_u + m_d)$ and $m_q = m_s$ to lattice-QCD results [11,16]. With respect to the m_q dependence, we consider three ansätze for z_{eff} : constant, linear and linear plus chiral logarithms. Averaging over the two latter variants within the renormalization-scale ranges 1–3 GeV for charmed mesons and 3–6 GeV for bottom mesons converts the resulting functional behaviour $f_H(m_q)$ (Fig. 1) to our prediction for the isospin-breaking differences of the heavy-meson decay constants (Table 2).

Table 2: Local-duality isospin breaking results for heavy-meson decay constants, exhibiting good agreement with Ref. [7] (apart from the $D^{\pm,0}$) and lattice-QCD outcomes for the $D^{\pm,0}$ but a notable tension for the $B^{0,\pm}$.

Mesons H_q	$f_{H_d} - f_{H_u}$ [MeV]		
	Borelized threshold [7]	Local-duality limit [15]	Lattice QCD
$D^{\pm,0}$	0.97 ± 0.13	0.96 ± 0.09	$0.94^{+0.50}_{-0.12}$ [17]
$D^{*\pm,0}$	1.73 ± 0.27	1.18 ± 0.35	—
$B^{0,\pm}$	0.90 ± 0.13	1.01 ± 0.10	3.8 ± 1.0 [18]
$B^{*0,\pm}$	0.81 ± 0.11	0.89 ± 0.30	—

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