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Measurement of the leading hadronic contribution to the muon g-2 via space-like data

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A new experiment is proposed to determine the leading-order hadronic contribution to the muon g-2 based on the measurement of the effective electromagnetic coupling in the space-like region at low-momentum transfer. This can be achieved by scattering a 150 GeV muon beam, available at CERN, on a low-Z fixed target and measuring with very high precision the differential cross section of the elastic μe scattering. This novel approach could provide an independent determination, competitive with the usual time-like dispersive approach, thus consolidating the theoretical prediction for the muon g-2 in the Standard Model.

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1. Introduction

The muon anomalous magnetic moment $a_\mu = (g-2)/2$ is an important test bench for the Standard Model, as it can be both precisely predicted and measured. On the experimental side the final result of the E821 experiment at BNL was $a_\mu^{\text{Exp}} = (11659208.9 \pm 6.3) \times 10^{-10}$ [1], with a precision of 0.5 ppm, determined by the available statistics. The Standard Model prediction $a_\mu^{\text{SM}} = (11659180.2 \pm 4.9) \times 10^{-10}$ has a similar relative uncertainty of 0.4 ppm, dominated by the hadronic leading-order contribution, estimated to be $a_\mu^{\text{HLO}} = (692.3 \pm 4.2) \times 10^{-10}$ [2]. Pure QED contributions, calculated through five loops, and electroweak contributions are known with negligible uncertainty. The (3-4)σ discrepancy between prediction and observation, $\Delta a_\mu(\text{Exp} - \text{SM}) \sim (28 \pm 8) \times 10^{-10}$, has been considered as a possible indication of physics beyond the Standard Model for over a decade (for a short review see [3]). A recent update to the LO hadronic contribution [4], with $\sim 20\%$ improved precision, does not change significantly the compatibility with the experimental result. New experiments at Fermilab and J-PARC [5, 6] are targeting a reduction of the experimental error by a factor of four and are about to start. To cope with this improved precision it is highly desirable to reduce the theoretical uncertainty on the SM prediction, in particular its largest contribution, on the LO hadronic correction.

The difficulty about the hadronic correction comes from the fact that it cannot be computed perturbatively given the low energy scale of the process. The LO contribution is usually evaluated via a dispersion integral of the hadron production cross section in e^+e^- annihilation [2, 4]. The low-energy region, with the many resonances and threshold effects, is enhanced in the integral, which constitutes the main difficulty of the method. Alternative evaluations by lattice QCD are not yet competitive, though they are expected to improve in the next few years.

Differently from the standard dispersive approach, one could determine a_μ^{HLO} from a measurement of the effective electromagnetic coupling α in the space-like region, where the vacuum polarization is a smooth function of the squared momentum transfer [7]. A method to determine the running of α in the space-like region was proposed in [8], by using small-angle Bhabha scattering. There exist few such measurements, the most precise one from the OPAL experiment at LEP [9]. Recently a new dedicated experiment has been proposed, MUonE [10], based on the elastic μe scattering. Here the main ideas are summarized.

2. Theoretical framework

For the calculation of a_μ^{HLO}, an alternative formula can be exploited [11, 7]:

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx\,(1-x)\,\Delta\alpha_{\text{had}}[t(x)]\,, \tag{2.1}$$

where $\Delta\alpha_{\text{had}}(t)$ is the hadronic contribution to the running of the QED coupling, evaluated at

$$t(x) = -\frac{x^2 m_\mu^2}{1-x} < 0, \tag{2.2}$$

the space-like (negative) squared four-momentum transfer. In contrast with the traditional s-channel approach, the integrand in Eq. (2.1) is smooth and free of resonances.

By measuring the running of α,

$$\alpha(t) = \frac{\alpha(0)}{1-\Delta\alpha(t)}, \tag{2.3}$$

where $t = q^2 < 0$ and $\alpha(0) = \alpha$ is the fine-structure constant, the hadronic contribution $\Delta\alpha_{\rm had}(t)$ can be extracted by subtracting from $\Delta\alpha(t)$ the purely leptonic part $\Delta\alpha_{\rm lep}(t)$, which can be calculated to very high precision in QED.

Fig. 1 (left) shows $\Delta\alpha_{\rm lep}$ and $\Delta\alpha_{\rm had}$ as a function of the variables x and t. The range $x \in (0,1)$ corresponds to $t \in (-\infty, 0)$, with $x = 0$ for $t = 0$. The integrand of Eq. (2.1), calculated with the routine `hadr5n12` [12, 13], which uses time-like hadroproduction data and perturbative QCD, is plotted in Fig. 1 (right). The peak of the integrand occurs at $x_{\rm peak} \simeq 0.914$ (corresponding to $t_{\rm peak} \simeq -0.108$ GeV^2) and $\Delta\alpha_{\rm had}(t_{\rm peak}) \simeq 7.86 \times 10^{-4}$.

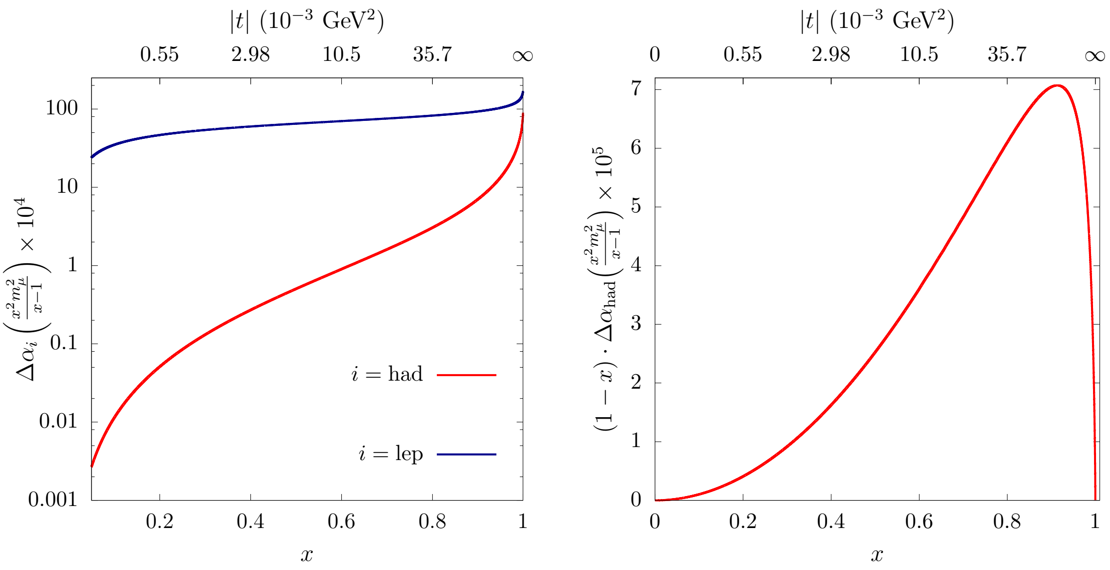

Figure 1: Left: hadronic and leptonic contribution to $\Delta\alpha$ as a function of x and t (upper scale). Right: the integrand $(1-x)\Delta\alpha_{\rm had}[t(x)]$ as a function of x and t.

3. Experimental proposal

Based on the t-channel approach expressed by Eq. 2.1, one could determine $a_\mu^{\rm HLO}$ from a measurement of the running of α in the space-like region. The elastic μe scattering with a muon beam of $E_\mu = 150$ GeV on atomic electrons of a low Z target (like *Be* or *C*) has been proposed in [10]. The technique is similar to the one used for the measurement of the pion form factor [14, 15]. The $\Delta\alpha_{\rm had}[t(x)]$ could be determined from a very precise measurement of the differential cross section. This approach is appealing for several reasons:

(i) At the tree level $\mu e \to \mu e$ is a pure t-channel process, making the dependence on t of the differential cross section proportional to $|\alpha(t)/\alpha(0)|^2$:

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha(0)}\right|^2, \tag{3.1}$$

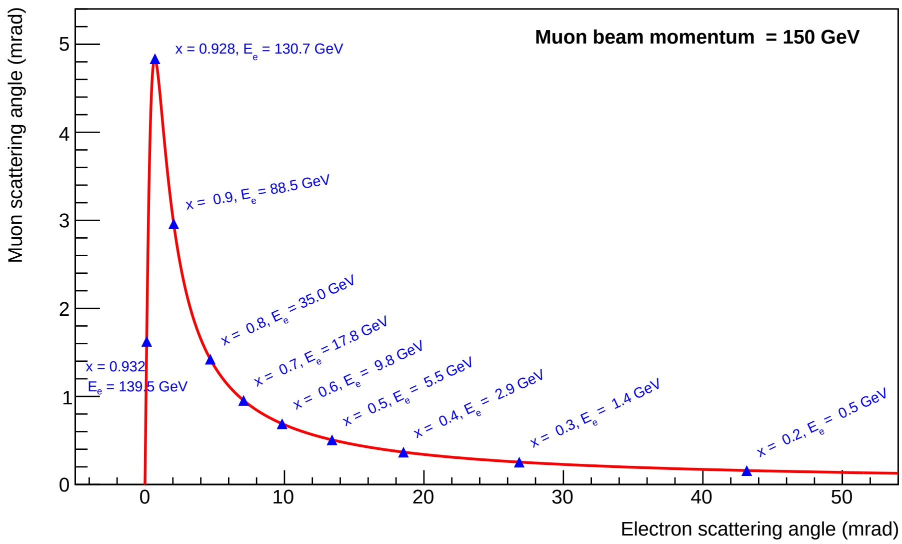

Figure 2: The relation between the muon and electron scattering angles for 150 GeV incident muon beam momentum. Blue triangles indicate reference values of the x variable and the corresponding electron energy.

where $d\sigma_0/dt$ is the effective Born cross section, including virtual and soft photons. The vacuum polarization effect, in the leading photon t-channel exchange, is incorporated in the running of α and gives rise to the factor $|\alpha(t)/\alpha(0)|^2$. It is understood that for a high precision measurement also higher-order radiative corrections must be included.

(ii) Given the incoming muon energy E_μ, in a fixed-target experiment the t variable is related to the energy of the scattered electron E_e or its angle θ_e. The angle θ_e spans the range (0–31.85) mrad for the electron energy E_e in the range (1–139.8) GeV (the low-energy cut at 1 GeV is arbitrary).

(iii) For $E_\mu = 150$ GeV the squared centre-of-mass energy is $s \simeq 0.164$ GeV^2 and -0.143 GeV$^2 < t < 0$ GeV^2. The accessible region of x extends up to 0.93, corresponding to 87% of the integral in Eq. (2.1). The remaining 13% can be calculated from time-like data plus perturbative QCD or by Lattice QCD. The peak of the integrand function is at $x_{peak} = 0.914$, corresponding to an electron scattering angle of 1.5 mrad, as visible in Fig. 1 (right).

(iv) The angles of the scattered electron and muon are correlated as shown in Fig. 2. This constraint is extremely important to select elastic scattering events, rejecting background events from radiative or inelastic processes and to minimize systematic effects in the determination of t. Note that for scattering angles of (2–3) mrad there is an ambiguity between the outgoing electron and muon, as their angles and momenta are similar, to be resolved by means of μ/e discrimination.

(v) The boosted kinematics allows the same detector to cover the whole acceptance. Many systematic errors, *e.g.* on the efficiency, will cancel out (at least at first order) in the relative ratios of event counts in the high and low q^2 regions (signal and normalization regions).

In addition to the nice features listed above, a muon beam with the needed characteristics already exists at CERN, it is the beam M2 in the North Area, which has been used by the COMPASS experiment [16]. It provides an high average intensity of $\sim 1.3 \times 10^7$ muons/s, for a muon energy up to 160 GeV. Considering a Beryllium target with total thickness of 60 cm (actually resulting from an array of thin targets as described in the next section), and two years of data taking with a running time of 2×10^7 s/yr, one can reach an integrated luminosity of about 1.5×10^7 nb^{-1}. Taking into account the elastic μe cross section ($\sigma = 245\mu$b at leading order) one would get $\sim 4 \times 10^{12}$ events after a cut on the minimum electron energy $E_e > 1$ GeV. With these inputs, from a simplified simulation of the experiment a statistical sensitivity of roughly 0.3% is estimated for the determination of $a_\mu^{\rm HLO}$.

4. Considerations on the detector

In order to perform the measurement to the required precision, a dedicated detector is necessary. There are two contrasting needs. On one hand a large statistics is needed, in particular at the peak of the integrand function (Fig. 1/right), which implies having enough target material. On the other side one wants to have minimal distorsions of the e/μ trajectories within the target and small rate of background processes (as bremsstrahlung and pair production). The proposed solution is a modular apparatus, a sequence of identical modules with relatively thin targets ($\sim$ 1–3 cm thick) made of a low-Z material (like Be or C). Depending on the chosen thickness for the targets the number of modules range from 20 (for 3 cm) to 60 (for 1 cm). Each module operates as an independent tracking system, with three silicon strip stations providing two orthogonal coordinates, spaced by intermediate air gaps and instrumenting a region of about 0.5–1 m, as shown in Fig. 3. Space resolution on individual hits of $\sim 10\mu$m and angular resolution of ~ 0.02 mrad on the electron and muon tracks can be obtained using nowadays available silicon strip detectors. The whole acceptance is contained within 10×10 cm^2 sensors.

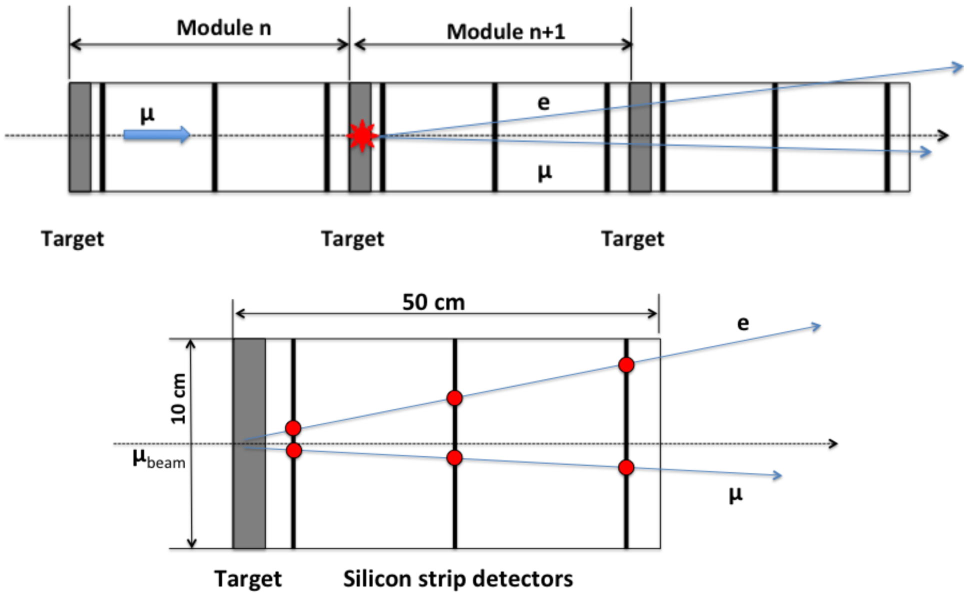

Figure 3: Scheme of a possible detector layout (not to scale). Top: a portion showing few consecutive modules; Bottom: one detector module.

The modular layout is almost transparent to non-interacting muons. For a scattering event occurring at a given module, the direction of the incoming muon can be measured by the previous module. Measuring both the electron and muon scattering angles also allows for a precise calibration of the average beam momentum, given the over-constrained kinematics of the elastic scattering. In addition the incoming muon momentum can also be measured event-by-event by a spectrometer similar to that used by COMPASS [16], located upstream of the first module.

The muon-electron ambiguity, visible in Fig. 2 for scattering angles of (2–3) mrad, has to be solved by a downstream electromagnetic calorimeter, placed after the last module, followed by a muon detector (a filter plus active planes). The calorimeter would also be useful to precisely measure the energy of electrons scattered on the last module and to identify additional electromagnetic showers.

The effect of the hadronic contribution to the running QED coupling, $\Delta\alpha_{\rm had}$, is to increase the differential cross section by a few per mille. Therefore a precise determination of $a_\mu^{\rm HLO}$ requires not only high statistics, but also high systematic accuracy, as the final goal of the experiment is equivalent to a determination of the differential cross section with ~ 10 ppm systematic uncertainty at the peak of the integrand function (Fig. 1). From the experimental point of view one challenging aspect is to keep multiple scattering under control, in particular for scattered electrons with energy as low as 1 GeV. Multiple scattering breaks the muon-electron two-body angular correlation, moving events out of the kinematic line in the 2D plot of Fig. 2. In addition, it causes acoplanarity, while two-body events are planar, within the resolution. These effects need to be modelled and measured using the data themselves.

In order to demonstrate that the required high precision can be realistic, a very detailed optimization of the experimental apparatus is necessary. Tests with beams (electrons and muons) and with one or two modules of the detector will be necessary. They will provide a proof-of-concept of the proposed method. As of today, a first test beam has been conducted at CERN (27 Sep–3 Oct 2017) in the H8 beam line. It has profited from an existing high resolution beam telescope [17], in use by the UA9 collaboration, conveniently adapted to the relevant kinematic region with a rearrangement of its detection planes. Multiple scattering has been tested with an electron beam with energy of 12-20 GeV impinging on carbon targets with different thickness (0.2-2 cm), as well as with targets made of other materials (Be, Al). Muon scattering events have also been recorded with a muon beam energy of 160 GeV. The analysis of the test beam data is now on-going. A test with one or two entire modules exploiting the CERN muon beam facility, including the identification setup (electromagnetic calorimeter and muon detector), is foreseen for 2018.

5. Theoretical uncertainties

A fully exclusive Monte Carlo generator describing the μe scattering with all the relevant radiative corrections is a mandatory tool. This will be obtained in steps. NLO QED corrections, which were already available, have been implemented in a MC code, whose results have been recently presented at the Padova workshop [18]. Multiple photon emission will then be included by matching the NLO calculation to leading-logarithmic corrections resummed to all orders, following a similar approach as the BabaYaga event generator [19] for Bhabha scattering. An additional issue for the μe scattering is the need to account for effects of the muon mass. Concerning the NNLO corrections, a complete calculations for μe scattering is not yet available, but a first important milestone has been recently achieved [20]. The full NNLO calculation will have to be finally matched with resummation of higher orders in the leading-logarithmic approximation. Some more details are discussed in [10].

6. Conclusions

A novel approach to determine $a_\mu^{\rm HLO}$, the leading hadronic contribution to the muon g-2, has been proposed. It is based on a precise measurement of the running of the effective QED coupling α in the space-like region. This can be extracted from the differential cross section of the elastic $\mu e \to \mu e$ scattering, probed with a muon beam with $E_\mu = 150$ GeV, available at CERN, on a low-Z fixed target.

The experiment relies on a precise measurement of the scattering angles of the two outgoing particles, which can efficiently select elastic scattering events, determining their squared momentum transfer q^2 and suppressing at the same time backgrounds and radiative events. The normalization of the cross section is provided by the very same $\mu e \to \mu e$ process in the low-q^2 region, where the effect of the hadronic corrections on α is negligible. Such a simple and robust technique has the potential to keep systematic effects under control, aiming to reach a systematic uncertainty of the same order as the statistical one. For this purpose a preliminary detector layout has been described. By considering the intensity of the CERN M2 beam, a statistical uncertainty of $\sim 0.3\%$ on $a_\mu^{\rm HLO}$ has been estimated after two years of data taking. A test performed using a single detector module, exploiting the muon beam facility, could provide a validation of the proposed method. Progress in the activities is regularly followed up within the "Physics Beyond Colliders" CERN study group, which will deliver a final report by the end of 2018.

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