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The role of theory input for exclusive Vcb determinations

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Available form factor parametrizations for $B \to D^* l\nu$ imply different theoretical assumptions and different treatments of theoretical uncertainties. They give results for $|V_{cb}|$ whose central values are apart by up to 8%. The way the Caprini Lellouch Neubert (CLN) parametrization has been used in experimental analyses sets theoretical uncertainties of the Heavy Quark Effective Theory (HQET) results on slope and curvature of the form factor ratios R_1 and R_2 to zero. Furthermore, the relation of curvature and slope of the axial form factor A_1 is fixed to the HQET central value. In view of the current experimental precision these uncertainties cannot be neglected any more. Using the Boyd Grinstein Lebed (BGL) parametrization and taking into account theoretical uncertainties in a conservative way, we extract $|V_{cb}|$ from recent preliminary Belle data and the world average of the total branching ratio. We include an $\mathcal{O}(10\% - 20\%)$ theoretical uncertainty of HQET input due to unknown corrections beyond NLO which were neglected in all previous analyses. This is important for reliable extractions of $|V_{cb}|$ as well as precision tests of the Standard Model with robust predictions of the lepton flavor nonuniversality observable $R(D^*)$ and the τ polarization asymmetry P_τ. Including input from Light Cone Sum Rules (LCSRs) we find $|V_{cb}| = 40.6\left(^{+1.2}_{-1.3}\right) \cdot 10^{-3}$, $R(D^*) = 0.260(8)$ and $P_\tau = -0.47(4)$. Without LCSRs we find $|V_{cb}| = 41.5(1.3) \cdot 10^{-3}$ and the same results for $R(D^*)$ and P_τ. The $R(D^*)$ anomaly is persistent, but its statistical significance is slightly reduced to 2.6σ.

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1. Introduction

V_{cb} is an element of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix and as such a fundamental parameter of the Standard Model (SM). It plays an important role for the search for New Physics (NP) in global fits that overconstrain the Unitarity Triangle [1, 2]. The ratio $|V_{ub}/V_{cb}|$ directly constrains one side of the CKM triangle. Different methods for the extraction of V_{cb} show long-standing discrepancies. The Heavy Flavor Averaging Group (HFLAV) summarizes the current situation as [3]

$$|V_{cb}| = (42.19 \pm 0.78) \cdot 10^{-3} \qquad \text{from} \quad B \to X_c l \nu_l \,, \tag{1.1}$$

$$|V_{cb}| = (39.05 \pm 0.47_{\text{exp}} \pm 0.58_{\text{th}}) \cdot 10^{-3} \qquad \text{from} \quad B \to D^* l \nu_l \,, \tag{1.2}$$

$$|V_{cb}| = (39.18 \pm 0.94_{\text{exp}} \pm 0.36_{\text{th}}) \cdot 10^{-3} \qquad \text{from} \quad B \to D l \nu_l \,, \tag{1.3}$$

where $l = e, \mu$. For a discussion of $B \to D l \nu$ see also Ref. [4]. A key issue in the extraction of $|V_{cb}|$ is that we have only a limited knowledge of the hadronic form factors. Recently, Belle published new preliminary $B \to D^* l \nu_l$ data which is independent of a certain form factor parametrization [5]. This triggered a lot of new theoretical studies [6, 7, 8, 9, 10, 11]. Here, we present the results of our recent work Refs. [7, 9]. We discuss the available theoretical form factor constraints and parametrizations in Sec. 2. Results for $|V_{cb}|$ are shown in Sec. 3. After that, in Sec. 4 we give predictions for the observables $R(D^*)$ and P_τ, which can be used for precision tests of the SM. To conclude, we briefly summarize our results.

2. Theory Constraints on Form Factors

In the limit of massless leptons the decay $B \to D^* l \nu$ depends on two axial and one vector form factor, which are denoted as $A_{1,5}$ and V_4, respectively. In order to treat $B \to D^* \tau \nu_\tau$ decays, one needs the additional pseudoscalar form factor P_1. We adopt here the notation of Ref [12], see Ref. [9] for a translation table to the notation of Ref. [13]. The form factors can be written as functions of the dimensionless kinematical quantity $w = (m_B^2 + m_{D^*}^2 - q^2)/(2 m_B m_{D^*})$, where $q^2 \equiv (p_B - p_{D^*})^2$. Dispersion relations allow to relate the semileptonic region $m_l^2 \leq q^2 \leq (m_B - m_{D^*})^2$ to the pair-production region beyond threshold $q^2 \geq (m_B + m_{D^*})^2$. Using perturbative QCD [14], one can constrain the form factors in the pair-production region. Then, one can translate this constraint back to the semileptonic region using analyticity. This motivates the model independent Boyd Grinstein Lebed (BGL) parametrization [13, 15, 16], which performs the form factor expansion

$$F(z) = \frac{1}{P_F(z) \phi_F(z)} \sum_{n=0}^{\infty} a_n^F z^n \,, \qquad z \equiv \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}} \,, \tag{2.1}$$

with the outer function $\phi(z)$, the Blaschke factor $P(z)$ and the expansion coefficients a_n^F, for details see Ref. [13]. The BGL parameters a_n^F are bounded by the unitarity conditions [13]

$$\sum_{n=0}^{\infty} \left(a_n^{V_4}\right)^2 \leq 1 \,, \qquad \sum_{n=0}^{\infty} \left(\left(a_n^{A_1}\right)^2 + \left(a_n^{A_5}\right)^2 \right) \leq 1 \,. \tag{2.2}$$

The unitarity bounds Eq. (2.2) are the (weak) special case of the general (strong) unitarity conditions which include also the contributions from the BGL parameters of all other $b \to c$ channels, such as $B \to D$, $B^* \to D$, and $B^* \to D^*$ [13]. For $B \to D^* l\nu$ the expansion parameter z lies in the range $0 < z < 0.056$. This and the unitarity bounds Eqs. (2.2) imply that the expansions Eq. (2.1) converge very fast. We have already $z^3 \sim 10^{-4}$, so that in practice taking into account exponents up to the power of two is already enough.

Additional information on the form factors is provided by Lattice QCD (LQCD), Heavy Quark Effective Theory (HQET) and Light Cone Sum Rules (LCSRs). LQCD provides the normalization for the $|V_{cb}|$ extraction with the form factor value $A_1(w=1) = 0.902(12)$ (our average from Refs. [17, 18]). LCSRs give values at the other end of the kinematic spectrum: $A_1(w_{max}) = 0.65(18)$, $R_1(w_{max}) = 1.32(4)$, and $R_2(w_{max}) = 0.91(17)$ [19]. We will show fit results with and without including LCSR input. HQET and QCD sum rules [6, 12, 20, 21, 22, 23, 24, 25] give strong constraints for all the $B^{(*)} \to D^{(*)}$ form factors. In the heavy quark limit $m_{c,b} \gg \Lambda_{\rm QCD}$ all of them can be written using a single Isgur-Wise function. NLO corrections at $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b}, \alpha_s)$ are known and can be written in terms of three subleading Isgur-Wise functions. Following the calculation of Ref. [6] we updated all $B^{(*)} \to D^{(*)}$ form factor ratios, see Table II in Ref. [9], which updates Table A.1 in Ref. [12]. The parametric error of the NLO contributions can be taken into account by varying the corresponding subleading parameters as given in Ref. [6]. We also take into account the theoretical uncertainty due to the unknown corrections beyond NLO, which are parametrically $\mathcal{O}(\alpha_s^2, \Lambda_{\rm QCD}^2/m_{c,b}^2, \alpha_s\Lambda_{\rm QCD}/m_{c,b})$. A reliable estimate of their size is complicated by the fact that at zero recoil several form factors are protected from NLO power corrections through Luke's theorem [20], which does not apply to the N^{2}LO corrections. The form factors which are not protected by Luke's theorem do have NLO corrections up to 60%. Actually, we have [9]

$$\frac{V_6(w)}{V_1(w)} = 1.0\,, \qquad \text{(LO)} \tag{2.3}$$

$$\frac{V_6(w)}{V_1(w)} = 1.58(1-0.18(w-1)+\ldots)\,. \qquad \text{(NLO)} \tag{2.4}$$

For an estimate it is also instructive to compare LQCD and HQET results:

$$\left.\frac{S_1(w)}{V_1(w)}\right|_{\rm LQCD} = 0.975(6)+0.055(18)w_1+\ldots, \qquad \left.\frac{S_1(w)}{V_1(w)}\right|_{\rm HQET} = 1.021(30)-0.044(64)w_1+\ldots$$

$$\left.\frac{A_1(1)}{V_1(1)}\right|_{\rm LQCD} = 0.857(15), \qquad \left.\frac{A_1(1)}{V_1(1)}\right|_{\rm HQET} = 0.966(28)$$

$$\left.\frac{S_1(1)}{A_1(1)}\right|_{\rm LQCD} = 1.137(21), \qquad \left.\frac{S_1(1)}{A_1(1)}\right|_{\rm HQET} = 1.055(2),$$

where $w_1 = w-1$. Between the LQCD and HQET results there are deviations of $5\% - 13\%$ which must come from higher order corrections beyond NLO.

Taking everything into account, N^{2}LO corrections as large as $\mathcal{O}(10\% - 20\%)$ cannot be excluded for robust tests of the SM and reliable extractions of V_{cb}.

Using the results from HQET, it is possible to relate the BGL parameters of the other $B^{(*)} \to D^{(*)}$ modes to the ones of $B \to D^*$. In this way we derive the strong version of the unitarity constraints Eq. (2.2), one for each Lorentz structure. In order to be conservative, in the derivation of these strong unitarity constraints we allow for deviations from the central value of the HQET result by $\pm 25\%$ ($\pm 30\%$) at zero (maximal) recoil, which includes both NLO and N^{2}LO corrections.

Fit	BGL weak	BGL weak	BGL strong	BGL strong	CLN	CLN
LCSR	$\times$	✓	$\times$	✓	$\times$	✓
χ^2/dof	28.2/33	32.0/36	29.6/33	33.1/36	35.4/37	35.9/40
$\vert V_{cb}\vert$	0.0424 (18)	0.0413 (14)	0.0415 (13)	$0.0406\,\left(^{+12}_{-13}\right)$	0.0393(12)	0.0392(12)

Table 1: Extractions of $|V_{cb}|$ using BGL and CLN parametrizations with and without LCSR input. For BGL we also show fit results with and without strong unitarity constraints. Table adapted and extended from Refs. [7, 9].

We use these constraints as a side condition in the fit. A different method to utilize the strong unitarity relations is to eliminate directly some of the form factor parameters and to obtain in this way a simplified form factor parametrization. Of course, the theoretical uncertainty of this operation has to be taken into account. This is the strategy of the Caprini Lellouch Neubert (CLN) parametrization [12]. A form of this parametrization which is traditionally used in experimental analyses is (see *e.g.* Ref. [5])

$$h_{A_1}(w) = h_{A_1}(1)\left(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right), \tag{2.5}$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2, \tag{2.6}$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2, \tag{2.7}$$

where $R_1 \equiv V_4/A_1$ and R_2 is related to the form factor ratio A_5/A_1. Note that in the experimental analyses the slope and curvature of R_1 and R_2 as well as the relation of slope and curvature of h_{A_1} are kept fixed, *i.e.*, parts of the theoretical uncertainties of HQET which were noted in Ref. [12] are neglected [6, 7, 8].

3. Extraction of V_{cb}

Summarizing the discussions of Sec. 2, we distinguish three ways to treat the $B \to D^*$ form factors: (1) BGL using only weak unitarity, (2) BGL using strong unitarity as an additional constraint in the fit, and (3) CLN, which uses strong unitarity to obtain a simplified parametrization. We fit the BGL and CLN parameters to the global HFLAV average of the total branching ratio $\mathcal{B}(\bar{B}^0 \to D^{*+} l^- \bar{\nu}_l) = 0.0488 \pm 0.0010$ [3] and the recent preliminary Belle data [5]. We use the CLN parametrization in the traditional form of Eqs. (2.5)–(2.7). The results are shown in Table 1. Our fit scenario (1) agrees very well with Ref. [8]. Note that while all of the fits are compatible with each other, the central values for $|V_{cb}|$ extracted using the BGL and the CLN parametrization differ by up to 7.9%, and 5.4% including the LCSR input. Including the strong unitarity bounds, the deviation is reduced to 5.6% and 3.6% (with LCSR). The main reason for the deviation is that the CLN parametrization Eqs. (2.5)–(2.7) does not take into account any theoretical uncertainties for the slope of R_1 and R_2, but fixes them to the central value of the HQET result. Consequently, the BGL parametrization is more flexible, especially near $w = 1$. This is the most important kinematical region of the data, because here we have LQCD information on the normalization of the form factor A_1, which is essential for the extraction of V_{cb}. Relaxing the bound on R_1 and R_2 in the CLN parametrization leads to a result which is quite comparable to the BGL fits, see Refs. [7, 11].

Ref.	Our result [9]	[26]	[27]	[6]	[10]	[28, 29]
$R(D^*)$	0.260(8)	0.252(3)	0.252(2)(3)	0.257(3)	0.257(5)	0.252(4)
P_τ	$-0.47(4)$		$-0.502(^{+5}_{-6})(17)$			$-0.497(13)$
Deviation	2.6σ	3.5σ	3.4σ	3.1σ	3.0σ	3.4σ

Table 2: Our results for $R(D^*)$ and P_τ compared with other theoretical results available in the literature. The experimental measurements are $R(D^*)^{\text{exp}} = 0.304(13)(7)$ [3] and $P_\tau^{\text{exp}} = -0.38(51)(^{+21}_{-16})$ [30, 31]. The last line shows the respective deviation from the measurement $R(D^*)^{\text{exp}}$.

4. SM Predictions for $R(D^*)$ and P_τ

In order to predict $R(D^*) \equiv \mathscr{B}(B \to D^*\tau\nu)/\mathscr{B}(B \to D^* l\nu)$ and the τ polarization asymmetry $P_\tau \equiv (\Gamma^+ - \Gamma^-)/(\Gamma^+ + \Gamma^-)$, where $\Gamma^\pm$ are the integrated polarized decay rates, we need theory input for the pseudoscalar form factor P_1. This form factor is not constrained by $B \to D^* l\nu$ data. Employing its BGL parametrization we use the three constraints (1) strong unitarity, (2) the kinematical endpoint relation $P_1(w_{\text{max}}) = A_5(w_{\text{max}})$ with our fit result for $A_5(w_{\text{max}})$, and (3) the HQET result $P_1(1) = 1.21 \pm 0.06 \pm 0.18$, where the first error is the parametric NLO error and the second error is our estimate of the N^{2}LO uncertainty as 15% of the central value, see our discussion in Sec. 2. These conditions determine the three P_1 BGL parameters. Our results are shown in Table 2. They are consistent with the results in the literature, however the value of $R(D^*)$ and the uncertainties that we obtain are larger. We find a 2.6σ deviation from the experimental measurement of $R(D^*)^{\text{exp}}$.

5. Conclusions

We use recent preliminary Belle data on $B \to D^* l\nu$ which is independent of a particular form factor parametrization in order to reappraise the methodology of exclusive V_{cb} extractions and predictions of $R(D^*)$ and P_τ. It turns out that the BGL parametrization and the form of the CLN parametrization as used in experimental analyses give different results for V_{cb}. This stays true when one includes strong unitarity constraints as external constraints on the BGL parameters. The reason is that the used form of the CLN parametrization neglects important theoretical uncertainties which leads to less flexibility of the parametrization near $w = 1$, where LQCD gives information on the form factor normalization, which is essential for the determination of $|V_{cb}|$. We advocate therefore to use the BGL parametrization with the strong unitarity constraints as side conditions, taking into account the input from HQET in a conservative way. We estimate the theoretical uncertainty of the HQET input due to unknown higher order corrections to be $\mathscr{O}(10\% - 20\%)$ on top of the parametric NLO error. Our results for $|V_{cb}|$, $R(D^*)$ and P_τ are given in Tables 1 and 2. The $R(D^*)$ anomaly is persistent, but slightly reduced to 2.6σ.

As we rely except for the world average of the total branching ratio on recent, preliminary data, we have to be patient: the V_{cb} puzzle is not yet solved. A reanalysis of previous BaBar and Belle data is necessary, taking properly into account theoretical uncertainties. Together with future lattice data which determines the slope of the form factors this will conclusively settle the case.

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