PROCEEDINGS OF SCIENCE

Discrete Symmetries and Hairs: Top-down and Bottom-up Approaches

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We present our recent work on discrete hairs from the bottom-up approach, which utilizes subgroups of global symmetries. Along this line, we point out one global symmetry from string compactifications which can work as a rationale for an "invisible" axion at the intermediate scale.

EPS-HEP 2017, European Physical Society conference on High Energy Physics
5-12 July 2017
Venice, Italy

[*]Speaker.

[†]JEK is supported by the National Research Foundation (NRF) grant funded by the Korean Government (MEST) (NRF-2015R1D1A1A01058449) and the IBS (IBS-R017-D1-2016-a00).

PoS(EPS-HEP2017)040

Discrete gauge symmetries are defined to be subgroups of gauge symmetries in the top-down approach such that spontaneous breaking of the gauge symmetries to those discrete groups does not lead to any unsatisfactory gravitational effects [1]. Here, we argue that in the bottom-up approach discrete symmetries can also allow hairs and unsatisfactory gravitational effects do not arise [2]. The original worry, originating from the danger of introducing global symmetries together with gravitational interactions, led to considerations that some discrete symmetries might be required not to be subgroups of global symmetries. String theories also are not favorable to global symmetries [3].

The observed fermions in the low energy effective theory are better to be chiral fermions if they arise from gauge symmetry alone. The observed standard model(SM) is a chiral model. There is another hidden-sector chiral model discovered recently [4]. In these models 'chirality' is defined only from gauge symmetries, without resorting to any global or discrete symmetries. Except these gauge models with the chiral spectra, some global and discrete symmetries are still used in the low energy effective theories for the rationale of cold dark matter (CDM) [5]. For WIMPs, the exact or almost exact discrete symmetries are needed. For collective motion of light bosons [6], the QCD axion is the most popular one towards a CDM candidate.

After the Brout-Englert-Higgs-Guralnik-Hagen-Kibble mechanism has been accepted, we do not talk about approximate gauge symmetries, and we have a fundamental scalar boson h with energy dimension 1. Because of the low dimension 1 of h, it has strong influence in cosmology and particle physics under the name of Higgs portals. Even though global symmetries are not exact, we can consider "approximate global symmetries", and the important question is what is the breaking level of the global symmetries. If the discrete symmetries are subgroups of gauge groups, then we do not talk about approximate discrete symmetries. But, if a discrete symmetry is a subgroup of global symmetries, then we can talk about approximate discrete symmetries. With this caveat in mind, we will discuss the discrete hairs.

The most studied discrete symmetry in relation to CDM is $\mathbf{Z}_2$. If it is exact, cosmological domain walls are created in the evolving Universe [7]. The DFSZ axion model has the domain wall number 6. The KSVZ axion model with one heavy quark has the domain wall number 1. The model-independent (MI) axion from string theories allows the domain wall number 1 [8], which is a desired one in view of the cosmological constraints and from an ultra-violet completed theory. But, the decay constant of the MI axion is at the level of $10^{16}\,$GeV [9]. Fortunately, it is possible to transfer this MI axion to the longtudinal degree of an anomalous U(1) gauge boson, and leave a global Preecei–Quinn symmetry down to an intermediate scale relevant for an "invisible" axion [10]. So, for CDM with an "invisible" axion, there are good examples with the axionic domain wall number 1. Therefore, discrete hairs we consider below applies mainly to the WIMP CDM models.

Black holes are known to have hairs of gauge symmetries. These are field lines ($\mathbf{E}$ field) around the black holes. For example, consider a charged black hole with a positive charge inside as shown in Fig. 1 (a). The larger event horizon (out of two solutions) of the Reissner-Nordström black hole [11] occurs at $r_+ = \frac{1}{2}\left(r_S + \sqrt{r_S^2 - 4r_Q^2}\right)$ where r_S is the Schwarzschild radius without the charge and r_Q is the radius in case of carrying Q units of charge. The event horizon takes into account the energy inside it. It is the black hole of graviton fields, for the graviton fields to be impossible to go out of the horizon. It is because graviton couples only to positive energy, and hence graviton

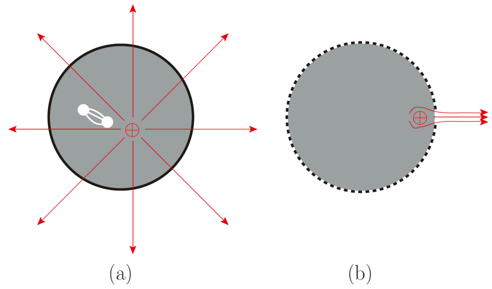

Figure 1: A charged black hole. (a) The solid circle represents the horizon surface. The red arrows represent gauge field lines and white lines represent gravity lines. Gravity lines cannot go out beyond the horizon surface while gauge lines can. (b) Collection of flux lines of (a) into a thin cylinder: The dashed cricle is the horizon of gravity fields. Both in (a) and (b) the gray part is inside the horizon.

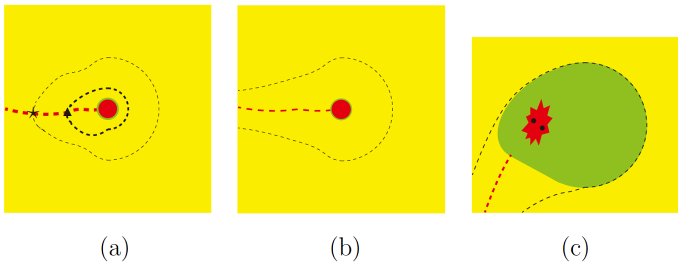

Figure 2: Closed surfaces around a tadpole, (a) two different surfaces, (b) hairs packed to a bundle, and (c) closing in the boundary near the tadpole.

fields leave and end at positive energy sources as shown in Fig. 1(a) with white lines. On the other hand, U(1) gauge field lines leave positive charges and end at negative charges. Within the closed boundary, enclosing a net positive charge shown in Fig. 1 (a), the electromagnetic field cannot end inside the blackhole horizon as shown with red arrows. The **E** field lines go out of the horizon of the gravity field, which looks like hairs from the black hole. Namely, when one considers the mass of a charged particle inside a black hole, it includes the outside field energy also. So, to consider the charged particle energy just inside the horizon, one should subtract the electromagnetic field energy permeating from the horizon to infinity. In Fig. 1 (b), one can move the **E** field lines to a small pinched hole such that at most of the surface of the sphere the horizon of graviton fields is also the horizon of the gauge fields except at the pinched hole. Through this hole, the **E** field lines can be connected to the outside from the black hole.[1]

Discrete symmetries allow non-equivalent vacua if they are distinguished by VEVs of scalar fields. In the evolving Universe, the discrete symmetries spontaneously broken by the VEVs of spin-0 fields create domain walls [7].[2] Consider $\mathbf{Z}_2$ and its two distinct vacua, which will be colored yellow ($q = 0$) and red ($q = 1$) in Figs. 2, 3, and 4. Between them, there is a domain

[1]Note added: The mass limit of graviton $< 0.77 \times 10^{-22}$ eV is from the black hole mass of GW170104 [12]. The MI axion with the known f_a [9] can be copiously produced if the black hole mass is $< 2 \times 10^{14}$ kg [13].

[2]The horizon of graviton is from the VEV of spin-2 $g_{\mu\nu}$, and the **E** field lines are from the VEV of spin-1 A_μ.

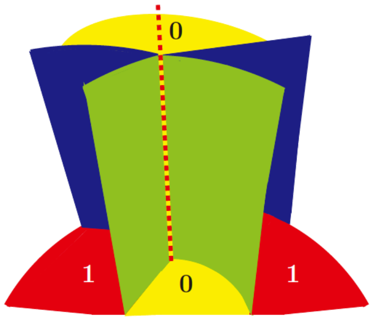

Figure 3: Vacua of a scalar field around a tadpole. The numbers 0 and 1 correspond to the $\mathbf{Z}_2$ quantum numbers of the vacua. The red dashed line is the intersection of two different walls of $\mathbf{Z}_2$.

wall as shown in Figs. 2 (a), (b), and (c). The domain wall is colored limegreen. One can consider discrete charges: sum of the charges of particles and vacuum. Suppose, the yellow vacuum carries 0 discrete charge. If we enclose the particles in the red vacuum by a closed surface, there can be a definite discrete quantum number. Let us consider the total discrete charge within the closed surface passing through the star in Fig. 2 (a). Moving the boundary of this closed surface to the closed surface passing through the triangle, the discrete charge inside the surface remains the same. Therefore, one can define a 'tadpole' of discrete charges, through the tail of which a discrete quantum number goes out, as shown in Fig. 2 (b). Even this closed surface can be moved to touch the domain wall as shown in Fig. 2 (c). The dashed lines end at the tadpole or at an infinity point.

For simplicity, let us consider a $\mathbf{Z}_2$ symmetry as in the above discussion. We obtain a two dimensional delta function at the surface of radius r in the spherical-polar coordinate system, (θ_0, φ_0). Consider an effective $\mathbf{Z}_2$ symmetric action,

$$\mathscr{L}_{\rm eff} = \frac{1}{M_{\rm eff}^2}\,(\partial^\mu\partial^\nu\psi^*)\,\left(\partial_\mu\partial_\nu\psi\right). \tag{1}$$

With this effective Lagrangian, it has been shown that the boundary of $\mathbf{Z}_2$ domain walls produces a delta function at black hole horizon at r, which is depicted in Fig. 3, with the current density outside the tadpole given by [2],

$$j^{\theta\varphi}(\theta,\varphi) = \frac{1}{r^2}\delta(\cos\theta - \cos\theta_0)\delta(\varphi-\varphi_0). \tag{2}$$

This shows that the black hole has a $\mathbf{Z}_2$ hair, as shown with the red dashed line in Fig. 3.

We can use this tadpole passing from the visible sector universe to a hidden sector universe as shown in Fig. 4 (b). The space marked with O is the visible universe with the SM and Einstein–Hilbert actions. The space marked with S is a shadow world with its own actions. The observer O notices the following. If the wormhole is cut along the gray plane of Fig. 4 (b), there results Fig. 4 (c). The tadpole heads connected to the O and S universes, respectively, resulting from this cutting, have the opposite discrete charges (or a non-zero charge 1 since we illustrated with $\mathbf{Z}_2$). Thus, two tadpole charges in the S universe in Fig. 4 (c) add to 0. Figure 4 (c) is consistent since the tadpoles created by cutting the tail satisfy their boundary conditions that they end at tadpoles.

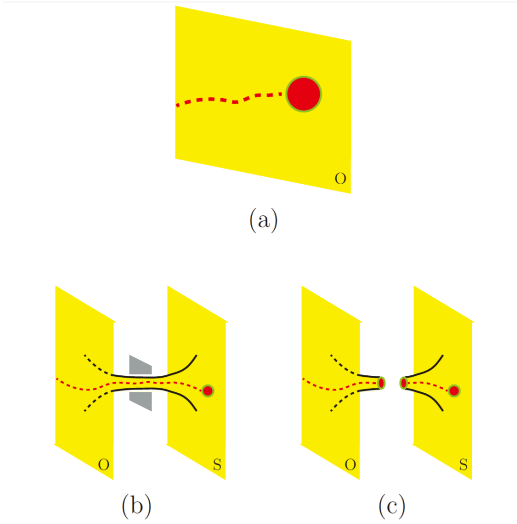

Figure 4: A $\mathbf{Z}_2$ hair attached to a tadpole: (a) a tadpole and its tail of Fig. 2 (b), (b) the tadpole passing through a wormhole, and (c) cutting the wormhole.

The discrete charges of two tadpoles separated to the hidden-sector carry zero discrete charge. The observer O in the visible sector notices that no discrete charge is lost from his universe O.

To summarize, we have shown that the existence of discrete hairs implies that discrete symmetries are not broken by gravitational effects of black holes and wormholes.

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