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$SO(N)$ models and Higgs extensions

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We discuss previous studies on $SO(N)$ linear sigma models (LσM) and some limits of phenomenological interest. These models suffer a spontaneous symmetry breaking (SSB) down to $SO(N-1)$, with the appearance of an associated vacuum expectation value (vev) f, a heavy scalar degree of freedom (dof) with mass M and $N-1$ massless Nambu-Goldstone bosons (NGB). These models are of a high interest for beyond Standard Model extensions where the Higgs boson is identified with a pseudo Nambu-Goldstone boson (pNGB) that appears in the $SO(N)/SO(N-1)$ SSB. It gains a non-zero mass m due to soft explicit $SO(N)$ symmetry breaking (ExSB) terms in the Lagrangian. In particular, we will focus on the soft breaking pattern $SO(N) \stackrel{\text{ExSB}}{\longrightarrow} SO(4)\times SO(P) \stackrel{\text{SSB}}{\longrightarrow} SO(3)\times SO(P-1)$, with $4+P=N$, e.g., via new beyond Standard Model (BSM) gauge boson loops. The $SO(4)/SO(3)$ are the electroweak (EW) chiral/custodial groups and the associated SSB is exactly the Standard Model (SM) one, giving mass to the $W^\pm$ and Z gauge bosons while avoiding large corrections to the oblique T parameter. The comparison of this type of models with the current phenomenological situation, close to the SM ($m=0.125$ TeV, EW vev $v=0.246$ TeV, $M \gtrsim \mathcal{O}(\text{TeV})$, $g_{hWW}\approx g_{hWW}^{SM}$) sets important constraints on the LσM parameters: there is a very small mixing between the heavy and light LσM massive scalars and the pNGB h is essentially SM-like, the low-energy effective field theory (EFT) couplings are very close to the SM ones, and a large hierarchy $\xi = \frac{v^2}{f^2} \ll 1$ is needed in these LσM near the $SO(N)$ limit (and ξ much smaller than a certain ratio $\frac{\lambda_2}{\lambda_1}$ of quartic LσM couplings in the general case). Likewise, we note the existence of strongly coupled scenarios with a hierarchy $m^2\sim v^2 \ll f^2 \ll M^2$.

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1. BSM extensions of the SM scalar sector through an $SO(N)$LσM

In this proceedings we review and discuss some results on theories with the SSB [1, 2] [1]

$$\textbf{w/o ExSB:} \quad SO(N) \quad \overset{\text{SSB, vev f}}{\longrightarrow} \quad SO(N-1)\,, \tag{1.1}$$

$$\textbf{w/ ExSB:} \quad SO(N) \quad \overset{\text{ExSB}}{\longrightarrow} \quad SO(4)\times SO(P) \quad \overset{\text{SSB, vev (v,v}_\text{s})}{\longrightarrow} \quad SO(3)\times SO(P-1)\,, \tag{1.2}$$

The $SO(N)$ ExSB turns one of $N-1$ NGB from the symmetric case into a pNGB with mass proportional to the explicit breaking Lagrangian parameters; the other $N-2$ remain as NGB, being the three of them associated to the $SO(4)/SO(3)$ SSB the standard EW Goldstones. [2]

In order to study the implementation of this symmetry pattern in generic BSM scenarios, it is interesting to discuss its realization through an $SO(N)$ toy-model renormalizable LσM with a real scalar multiplet $\vec{\Sigma}^T = (\phi_1,\phi_2,\phi_3,\phi_4,S_1,S_2,...S_P)$ in the fundamental representation [2]:

$$\mathscr{L} = \frac{1}{2}(D_\mu\vec{\Sigma})^2 - \left[V_0(\Sigma)+V_1(\Sigma,\bar{\zeta})\right]\,, \tag{1.3}$$

$$V_0(\Sigma) = -\frac{\mu_1^2}{4}\Sigma^2+\frac{\lambda_1}{16}\Sigma^4\,, \qquad V_1(\Sigma,\bar{\zeta}) = \Sigma^4\left(\frac{\mu_1^2-\mu_2^2}{4\Sigma^2}\bar{\zeta}+\frac{\lambda_1+\lambda_2-2\lambda_3}{16}\bar{\zeta}^2+\frac{\lambda_3-\lambda_1}{8}\bar{\zeta}\right),$$

with $\Sigma\equiv|\vec{\Sigma}|$, $\phi\equiv|\vec{\phi}|$, $S\equiv|\vec{S}|$ and the field $\bar{\zeta}=1-\zeta=S^2/\Sigma^2\in[0,1]$. V_0+V_1 develops the vev $\langle\phi^2\rangle = v^2 = \frac{2(\lambda_2\mu_1^2-\lambda_3\mu_2^2)}{\lambda_2\lambda_1^{\text{eff}}}$ and $\langle S^2\rangle = v_s^2 = \frac{2(\lambda_1\mu_2^2-\lambda_3\mu_1^2)}{\lambda_2\lambda_1^{\text{eff}}}$, with $\langle\Sigma^2\rangle = f^2 = v^2+v_s^2$, $\xi=\langle\zeta\rangle = v^2/f^2$ and $\lambda_1^{\text{eff}}\equiv\lambda_1-\frac{\lambda_3^2}{\lambda_2}$. In the $SO(N)$ symmetric limit $V_1=0$ ($\lambda_j=\lambda$ and $\mu_j^2=\mu^2$), there is a massive dof with $M^2=\mu^2=\frac{\lambda f^2}{2}$, $N-1$ NGB and a continuum of $SO(N-1)$ invariant vacua with $f^2=\frac{2\mu^2}{\lambda}$ and different $\xi\in[0,1]$ related through $SO(N)$ transformations. When $V_1\neq 0$, the set of potential minima take a unique value $\xi=\frac{\delta_3-\Delta_1}{\delta_3(2+\Delta_1)-\delta_1}$, with $\lambda_j\equiv\lambda_2(1+\delta_j)$, $\mu_1^2\equiv\mu_2^2(1+\Delta_1)$; an arbitrarily small deviation from $SO(N)$ may lead to scenarios with either $v^2\ll v_s^2$, $v^2\sim v_s^2$ or $v^2\gg v_s^2$.

One may induce an ExSB in an $SO(N)$ invariant LσM through the gauging of just a few components of $\vec{\Sigma}$, e.g., the gauging of the $\vec{S}$ components under a BSM group: $SO(P)$ gauge bosons A_μ^* with coupling e_* explicitly break the $SO(N)$ symmetry and induce a one loop contribution to V_1 à la Coleman-Weinberg (CW) proportional to powers of e_*^2 while gaining a mass $M_{A^*}=e_* v_s$ [4]. For instance, the A_μ^* loops induce for $P=2$ an effective potential of the form $V_1^{\text{A}^*-\ell\text{oop}} = \frac{3e_*^4}{64\pi^2}S^4\ln\frac{S^2}{R^2}$ [4]: [3] the V_1 potential is no longer flat for the field ζ and one Goldstone h becomes a pNGB with mass proportional to powers of the ExSB parameter e^*.

[1] In the case $P=2$ one has a discrete parity Z_2 in the place of $SO(P-1)$. It is worthy to note that $SO(6)/SO(5)\sim SU(4)/Sp(4)$ provides the minimal coset of this type with an ultraviolet (UV) completion of fermions in a complex representation of the gauge group, and represents the minimal $SO(N)$ realization of an UV-complete pNGB composite Higgs model [3]. The often denoted as minimal coset $SO(5)/SO(4)$ lacks a four dimensional UV completion.

[2] The experimental absence of massless scalars implies that all the remaining $P-1=N-5$ NGB gain mass through some mechanism not discussed in these proceedings, such as Higgsing or some BSM non-perturbative dynamics.

[3] These results corresponds to the Landau gauge and R is the renormalization scale in an appropriate scheme. There are further corrections $\sim\frac{e_*^2\lambda_j}{(4\pi)^2}S^4$ and $\sim\frac{e_*^2\mu_j^2}{(4\pi)^2}S^2$ if one considers a different gauge. SM loops introduce further $SO(N)$ ExSB terms, as the SM only couples to the $\vec{\phi}$ components.

In the general broken case, the ϕ–S mixing parameter $\omega \in [0,1]$ [2] ($h \approx \phi$ for $\omega \approx 0$ and $h \approx S$ for $\omega \approx 1$) and the masses are related to the model parameters in the form [4]

$$M^2, m^2 = \frac{\bar{M}^2}{2}\left(1 \pm \sqrt{1-\frac{4\bar{m}^2}{\bar{M}^2}}\right), \quad \lambda_2 v_s^2, \lambda_1 v^2 = \bar{M}^2\left(1 \pm |1-2\omega|\sqrt{1-\frac{4\bar{m}^2}{\bar{M}^2}}\right), \tag{1.4}$$

$$|1-2\omega| = \left(1-\frac{4\bar{m}^2}{\bar{M}^2}\right)^{-\frac{1}{2}}\left(1-\frac{4\lambda_1\bar{m}^2}{\lambda_1^{\rm eff}\bar{M}^2}\right)^{\frac{1}{2}}, \quad \bar{M}^2 = \frac{1}{2}(\lambda_1 v^2 + \lambda_2 v_s^2), \quad \bar{m}^2 = \frac{\lambda_1^{\rm eff}\lambda_2 v^2 v_s^2}{4\bar{M}^2}.$$

2. Low energy limit and Effective Field Theory

In the limit of a large mass gap $m^2 \ll M^2$ –which we will assume from now on–, one has $m^2 \approx \bar{m}^2$, $M^2 \approx \bar{M}^2$ and $\lambda_1 v^2 + \lambda_2 v_s^2 \approx 2M^2$, up to corrections $\mathcal{O}(m^2/M^2)$. This alone does not imply a hierarchy between v^2 and v_s^2. However, in general, the hWW coupling is related to the mixing in the exact form $\omega = 1 - \left(\frac{g_{hWW}}{g_{hWW}^{SM}}\right)^2$ [2], leading to the relations

$$\begin{matrix}\text{SM} \approx \text{EFT} \\ \gamma \ll 1\end{matrix} \Leftrightarrow \text{Mixing } \omega \ll 1 \Longleftrightarrow \frac{4\lambda_1\bar{m}^2}{\lambda_1^{\rm eff}\bar{M}^2} \ll 1 \Longleftrightarrow \lambda_1 v^2 \approx \frac{2\lambda_1 m^2}{\lambda_1^{\rm eff}} \ll \lambda_2 v_s^2 \approx 2M^2, \tag{2.1}$$

with the positive parameter $\gamma = \frac{\lambda_3^2 v^2}{2\lambda_2\mu_2^2}$ [2], and up to $\mathcal{O}(\frac{m^2}{M^2})$ and $\mathcal{O}(\omega)$ corrections. Thus, there is a large $v^2 \ll v_s^2 \approx f^2$ hierarchy when $\lambda_1 \approx \lambda_2$. In the limit (2.1), the low-energy EFT is organized in powers of $\gamma \approx \frac{(\lambda_1 - \lambda_1^{\rm eff})m^2}{\lambda_1^{\rm eff} M^2} \ll 1$, such that, up to $\mathcal{O}(\gamma)$, one finds [2], e.g.,

$$V(h)^{\rm EFT} = \frac{m^2h^2}{2} + \left(1-\frac{3\gamma}{2}\right)\frac{m^2h^3}{2v} + \left(1-\frac{25\gamma}{3}\right)\frac{m^2h^4}{8v^2} - \gamma\frac{m^2h^5}{2v^3} - \gamma\frac{m^2h^6}{12v^4} \tag{2.2}$$

$$\mathcal{F}_C(h)^{\rm EFT} = 1 + \left(1-\frac{\gamma}{2}\right)\frac{2h}{v} + (1-2\gamma)\frac{h^2}{v^2} - \frac{4\gamma}{3}\frac{h^3}{v^3} - \frac{\gamma}{3}\frac{h^4}{v^4}, \tag{2.3}$$

with the low-energy potential $V(h)^{\rm EFT}$ and $\Delta\mathcal{L} = \left(\frac{g^2v^2}{4}W^\mu W_\mu^\dagger + \frac{(g^2+g'^2)v^2}{4}Z_\mu Z^\mu\right) \times \mathcal{F}_C(h)^{\rm EFT}$ the Lagrangian providing the interaction vertices $W^+W^-, ZZ \to h, hh\ldots$ (the SM corresponds to the value $\gamma = 0$). Experimentally $\lambda_1^{\rm eff} \approx \frac{2m^2}{v^2} \approx 0.5$ and $0 \le \gamma \approx \omega = 1 - \left(\frac{g_{hWW}}{g_{hWW}^{SM}}\right)^2 \le 0.2$ for an hWW coupling in the range $0.9 \le \frac{g_{hWW}}{g_{hWW}^{SM}} \le 1$.

In terms of $v^2, v_s^2 \neq 0$ and the $\lambda_{1,2,3}$, one approaches the $SO(N)$ invariant limit when $|\delta_j| \ll 1$. Thus, $\lambda \sim \lambda_1^{\rm eff}/\delta_j$ can become non-perturbative near the $SO(N)$ symmetric limit, for small enough δ_j: e.g., for $|\delta_j| \lesssim \frac{1}{(4\pi)^2} \ll 1$ one has $\lambda \gtrsim 8\pi^2$. We have performed a numerical analysis for the benchmark points (BP) of the form $\lambda_2 = \lambda$, $\delta_1 = \delta_3 = -\delta$ with $0 \le \delta \le 1/2$ and such that $\lambda_1^{\rm eff} = 0.5$. In order to have a solution for δ one needs $\lambda \ge 4\lambda_1^{\rm eff} = 2$. In Fig. 1, we have plotted $\frac{\lambda_1 v^2}{2M^2}$ vs. $\frac{m}{M}$ and $\frac{\lambda_1 v^2}{2M^2}$ vs. ξ for arbitrary values of v, v_s. We fix $\delta = 0.64 \times 10^{-2}$ (soft ExSB), 0.15 (moderate ExSB), $\frac{1}{2}$ (large ExSB) for the benchmark points A, B and C, respectively, which correspond to $\lambda = 8\pi^2, 4, 2$. It is illustrative to note that, in the strongly coupled case $\lambda = 8\pi^2$, one has $M \approx 3.6$ TeV ($M \approx 6.5$ TeV) for $\xi = 1/4$ ($\xi = 1/16$). The results are exact and no expansion is performed here.

[4] The relations in the second identity in (1.4) also admit the inverted hierarchy $\mp$.

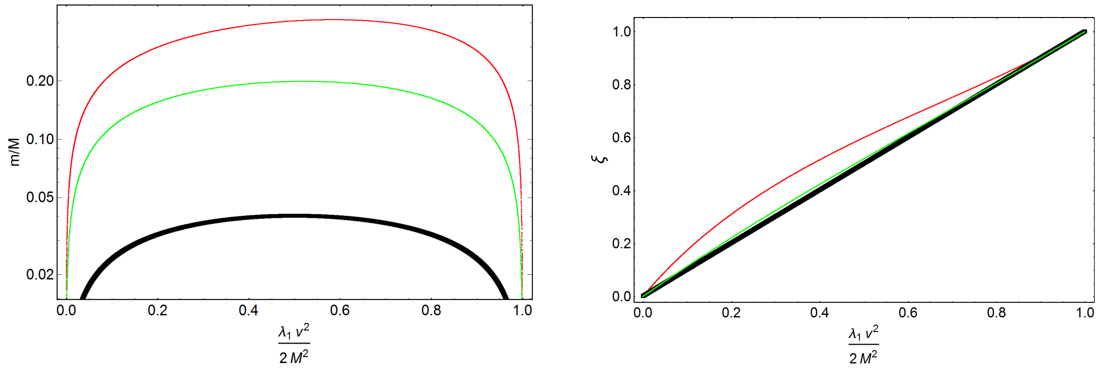

Figure 1: BP A (thick black), B (light green) and C (red), from bottom to top. For the plot for ξ, the lines for the BP A and B –and all the BP in between– are essentially superimposed and very approximately coincide with the straight line $\xi = \frac{\lambda_1 v^2}{2M^2}$. This linear relation is approximately fulfilled for any δ in this type of BP. We note that $\frac{m^2}{M^2} \to 0$ for either $\frac{\lambda_1 v^2}{2M^2} \to 0$ or 1, so the Higgs mass is linked to the EW SSB.

In conclusion, the symmetry pattern $SO(N) \xrightarrow{\text{ExSB}} SO(4)\times SO(P) \xrightarrow{SSB} SO(3)\times SO(P-1)$ naturally recovers the SM at low energies provided the ExSB potential V_1 generates a vev $\langle \zeta \rangle = \xi \ll 1$ (obviously, far from trivial). We would like to point out in these proceedings the existence of strongly interacting scenarios with a large coupling λ and a scale hierarchy of the type $m^2 \sim v^2 \ll f^2 \ll M^2 \approx \frac{\lambda f^2}{2}$ near the $SO(N)$ limit, and $\xi \ll \frac{\lambda_2}{\lambda_1}$ in general. Other works consider variants of this symmetry pattern with $N = 6$: $SO(6) \xrightarrow{SSB} SO(4)\times SO(2)$, which gives places to 8 NGB [5]; a non-linear realization of $SO(6) \xrightarrow{SSB} SO(5)$ where one of the 5 NGB is proposed as as a dark matter candidate [1]; lattice simulations of the $SU(4)/Sp(4)$ ($\sim SO(6)/SO(5)$) spectrum properties [3]; a non-linear realization of the latter [6], where a large deviation from the SM is found for g_{hhh}; variations of the ExSB V_1 based on fermion-loop estimates of the CW potential [7]. All of them point out $SO(N)$ models as appropriate BSM extensions which naturally generate a light pNGB h and reproduce the SM phenomenology and its $SO(4)/SO(3)$ chiral/custodial EW structure at low energies, deserving further studies in the future.

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