

Does $X(3872)$ count?

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The question on whether or not weakly bound states should be effectively incorporated in a hadronic representation of the QCD partition function is addressed by analyzing the example of the $X(3872)$, a resonance close to the $D\bar{D}^*$ threshold which has been suggested as an example of a loosely bound molecule. This can be decided by studying the $D\bar{D}^*$ scattering phase-shifts in the $J^{PC} = 1^{++}$ channel and their contribution to the level density in the continuum, which also gives information on its abundance in a hot medium. In this work, it is shown that, in a purely molecular picture, the bound state contribution cancels the continuum, resulting in a null occupation number density at finite temperature, which implies the $X(3872)$ does not count below the Quark-Gluon Plasma crossover ($T \sim 150\text{MeV}$). However, if a non-zero $c\bar{c}$ component is present in the $X(3872)$ wave function such cancellation does not occur for temperatures above $T \gtrsim 250\text{MeV}$.

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QCD thermodynamics in a finite box is closely related to counting hadronic states. Within Hadron Resonance Gas (HRG) approximation, QCD thermodynamics in the confined phase is reduced due to the role of narrow resonances [1] and effective elementarity [2]. Also, if we build the cumulative number of all the states listed in the Particle Data Group (PDG), $N_{\text{PDG}}(M)$, we can check that it fits the trace anomaly, $\varepsilon - 3P = T^5 \partial_T (\log Z/T^3)/V$, on the lattice at temperatures $T \lesssim 200\text{MeV}$ below the crossover to the Quark-Gluon Plasma (QGP) phase [3] (see also Ref. [4] and references therein).

These results point to the need to count all the states listed in the PDG as genuine contributions to the QCD partition function and, thus, also included in the HRG. However, under special circumstances such strong conclusion is incorrect, as loosely bound states may become fluctuations in a mass-spectrum coarse grained sense [5]. These authors concluded that certain interactions do not create new states but just reorder the already existing ones. For the deuteron, a $J^{PC} = 1^{++}$ np composite, the nearby np continuum compensates the weak binding effect and the overall contribution of the deuteron is zero as depicted in [6].

The recent discovery of the so-called X,Y,Z states arises the question if these states must be included in the PDG and whether or not they add redundancy when building the hadron spectrum [4, 6]. In this work we study this effect for the $X(3872)$ resonance, a 1^{++} state discovered in 2003 by the Belle Collaboration [7], whose properties point to a dominant $D\bar{D}^*$ -molecular structure. Analyzing $D\bar{D}^*$ scattering we will show that the answer to this question depends on the particular dynamics of the system [8].

According to the quantum virial expansion [9], the average density of a composite particle with g -degrees of freedom and mass m in a medium with temperature T can be written as

$$n(T) = \int \frac{d^3p}{(2\pi)^3} dm \frac{g}{e^{\sqrt{p^2+m^2}/T} + \eta} \rho(m), \text{ where } \rho(m) = \frac{1}{\pi} \frac{d\delta}{dm}, \quad (1)$$

and with δ the scattering phase shifts. This equation implicitly includes the contribution of elementary particles, since the phase-shift of a narrow resonance with mass m_R and width $\Gamma_R \rightarrow 0$ can be written as $\delta(m) = \tan^{-1}[(m - m_R)/\Gamma_R]$, so that $\delta'(m) \rightarrow \pi\delta(m - m_R)$ [1].

Now, for a certain type of weakly bound states their contribution may in fact vanish, as was shown by Dashen and Kane [5]. The cumulative number in a given channel in the continuum with threshold M_{th} is

$$N(M) = \sum_n \theta(M - M_n^B) + \frac{1}{\pi} \sum_{\alpha=1}^K [\delta_\alpha(M) - \delta_\alpha(M_{\text{th}})], \quad (2)$$

where we have explicitly separated the bound state M_n^B contributions from the continuum, written in terms of the eigen phase-shifts of the coupled channel S-matrix. This equation satisfies $N(0) = 0$ and, in the single channel case, when $M \rightarrow \infty$ it becomes $N(\infty) = n_B + [\delta(\infty) - \delta(M_{\text{th}})]/\pi = 0$. This is a consequence of Levinson's theorem, which states that the total number of states does not depend on the interaction.

Since its discovery, the weak binding of the $X(3872)$ has suggested a purely molecular nature with no reference to underlying quarkdynamics (see e.g. [10]). Within this molecular picture, in the $D\bar{D}^*$ channel the appearance of the $X(3872)$ rapidly shifts $M = M_{\text{th}} - B_X$ by one unit so that

$N(M_{\text{th}} - B_X + 0^+) - N(M_{\text{th}} - B_X - 0^+) = 1$. However, this number decreases slowly to zero at about $\Delta M_{D\bar{D}^*} \sim 200$ MeV, so $N(M_{\text{th}} + \Delta M_{D\bar{D}^*}) - N(M_{\text{th}} - B_X - 0^+) \sim 0$ [8]. This illustrates the point made by Dashen and Kane [5], showing that, in the purely molecular picture, the $X(3872)$ does not count in the $D\bar{D}^*$ continuum on coarse mass scales of about 200 MeV.

This situation may change if the inner structure of the $X(3872)$ includes a non-vanishing $c\bar{c}$ component. The multichannel scattering problem with confined intermediate states was initiated after the first charmonium evidences [11, 12] based on the decomposition of the Hilbert space as $\mathcal{H} = \mathcal{H}_{c\bar{c}} \oplus \mathcal{H}_{D\bar{D}^*}$. Such decomposition was implemented in Ref. [13, 14] in the framework of a widely-used constituent quark model (CQM) [15]. There, a coupled-channels calculation for the $J^{PC} = 1^{++}$ states was addressed, and the $X(3872)$ was described as a mostly $D\bar{D}^* + h.c.$ molecule with a sizable amount of $c\bar{c}(2^3P_1)$ state, while an additional resonance was found with more than 60% of $c\bar{c}$ structure, assigned to the $X(3940)$. The meson-meson interaction includes the exchange of pseudo-Goldstone bosons at $q\bar{q}$ level [15] and the coupling with two- and four-quark configurations through the 3P_0 model. From the latter transition mechanism, an effective potential $V_{\beta'\beta}^{\text{eff}}$ arises, encoding the coupling with the $c\bar{c}$ bare spectrum (see further details in Ref. [14]). The intensity of the 3P_0 -model is controlled by a dimensionless parameter, dubbed γ , originally constrained via strong decays in the charmonium spectrum. Here, the effect of adiabatically connect the $c\bar{c}$ spectrum and the $D\bar{D}^*$ channel is analyzed, thus the γ will be varied from zero to the value employed in Ref. [13], fixing the mass of the bound state $X(3872)$ at its experimental value of 3871.7 MeV.

The cumulative number of states is shown in Fig. 1(a). We check that the conclusion of Dashen and Kane [5] is maintained only when the $X(3872)$ meson is a pure molecule, so in this case the $X(3872)$ does not count. On the contrary, there is an outstanding turnover of the cumulative number as soon as we couple with the $c\bar{c}$ structures, even for small couplings. Such steep rise in the phase shifts points to a resonance located at a mass of $M \sim 3945$ MeV, which was identified with the $X(3940)$ resonance in Ref. [14]. In the purely molecular picture the $c\bar{c}$ structures decouple from the $D\bar{D}^*$ states and such resonance disappears. So, we can conclude that such raise is not a

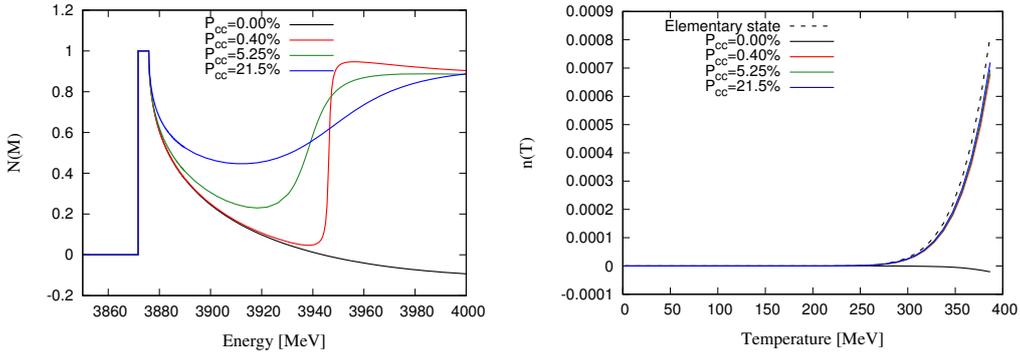


Figure 1: Left panel: Cumulative number in the $X(3872)$ channel as a function of the $D\bar{D}^*$ mass. Right panel: Occupation number $n(T)$ of the $D\bar{D}^*$ in the $J^{PC} = 1^{++}$ channel as a function of the temperature T (in MeV). The dashed line represents the contribution of the $X(3872)$ assuming it is an elementary particle and no continuum contribution.

consequence of the $X(3872)$ but it is due to the onset of the $X(3940)$ resonance.

Finally, we discuss the implications for finite temperature calculations. The occupation number is shown in Fig. 1(b), where we can appreciate the cancellation between the continuum and the bound state only happens for zero $c\bar{c}$ content, when both two- and four-quark sectors are decoupled. However, we want to remark that the non-vanishing occupation number is basically due to the resonant reaction $D\bar{D}^* \rightarrow X(3940) \rightarrow D\bar{D}^*$.

Concluding, our study shows that the signal for $X(3872)$ abundance may in fact be erroneously confused with the $X(3940)$ as a non-vanishing occupation number of the $D\bar{D}^*$ spectrum in the 1^{++} channel at temperatures, $T \gtrsim 250$ MeV, above the crossover to the QGP phase. Below this temperature, the $X(3872)$ does not count and should not be included in the Hadron Resonance Gas.

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