PROCEEDINGS OF SCIENCE

Constraints from the $1/N_{\mathrm{c}}$ Expansion on Properties of Exotic Tetraquark Mesons

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Scrutinizing the scattering of ordinary mesons in the limiting case of the number of colour degrees of freedom N_{c} of quantum chromodynamics approaching infinity, we formulate Feynman-diagram selection criteria and from these deduce rigorous self-consistency conditions for the manifestation of a tetraquark, a two-quark–two-antiquark bound state, as a pole in the corresponding amplitudes. Our constraints bear rather far-reaching consequences: In particular, all flavour-exotic tetraquarks, composed of four (anti)quarks of disparate flavour, must come in, at least, two variants differing in (and thus readily identifiable by) the large-N_{c} behaviour of their couplings to two ordinary mesons. Quite generally, irrespective of their flavour composition, all tetraquarks prove to be narrow. Their decay rates behave, for large N_{c}, like $1/N_{\mathrm{c}}^2$ and thus decrease faster than those of ordinary mesons.

XVII International Conference on Hadron Spectroscopy and Structure — Hadron2017
25–29 September, 2017
University of Salamanca, Salamanca, Spain

∗Speaker.

PoS(Hadron2017)233

1. Criteria for the Potential Existence of Polyquark Hadron States in Large-N_c QCD

Large-N_c QCD [1] is an element of a class of quantum field theories obtained as generalizations of quantum chromodynamics by allowing N_c, the number of the colour degrees of freedom of QCD, to differ from $N_\mathrm{c} = 3$. It is defined by considering the limit of N_c increasing beyond bounds, $N_\mathrm{c} \to \infty$, with correlative decrease of the strong coupling $\alpha_\mathrm{s} \equiv g_\mathrm{s}^2/4\pi$, $\alpha_\mathrm{s} \propto 1/N_\mathrm{c}$, and all quarks transforming according to the N_c-dimensional fundamental representation of its underlying gauge group $\mathrm{SU}(N_\mathrm{c})$.

Polyquarks are exotic hadrons that cannot be interpreted as just quark–antiquark or three-quark bound states; among these, tetraquarks are considered to be essentially composed of two quarks and two antiquarks. Within the framework of large-N_c QCD, only recently, after going through a kind of chequered history, the theoretical opinion about existence and observability of tetraquarks started to consolidate: At the leading order of their $1/N_\mathrm{c}$ expansion, all QCD Green functions are saturated by free mesons as intermediate states [2]. Tetraquarks may be formed only at N_c-subleading orders [3]; unlike ordinary mesons, tetraquarks were expected not to survive the limit $N_\mathrm{c} \to \infty$ as stable mesons. Mistakenly, the latter has been regarded as an argument for the tetraquarks' non-existence in nature.

From the point of view of experimental evidence, however, the relevant feature of tetraquarks is the large-N_c behaviour of their decay rates [4], which must not grow with N_c: observable tetraquarks should be narrow states. Tetraquarks with all quark flavours different exhibit decay rates decreasing like $1/N_\mathrm{c}^2$ [5]; so they are even narrower than ordinary mesons, whose decay rates are of order $1/N_\mathrm{c}$.

In that situation, we embarked on a systematic analysis [6,7] of the necessary conditions for the appearance of tetraquarks, T, with masses m_T assumed to remain finite for $N_\mathrm{c} \to \infty$, in form of poles corresponding to bound states of four (anti)quarks, with masses m_1, m_2, m_3, and m_4, in the s channel of scattering processes of two ordinary mesons into two ordinary mesons. To this end, neglecting all reference to spin or parity irrelevant in the present context, we study four-point correlation functions of operators $j_{ij} \equiv \bar{q}_i\, q_j$, bilinear in the quarks and interpolating the scattered mesons, M_{ij}, consisting of antiquark $\bar{q}_i$ and quark q_j (of flavour $i, j = 1,2,3,4$), that is, having nonvanishing matrix elements between vacuum and meson state, $\langle 0|j_{ij}|M_{ij}\rangle \equiv f_{M_{ij}} \neq 0$, which rise for large N_c like $f_{M_{ij}} \propto \sqrt{N_\mathrm{c}}$ [2].

With respect to the contributions to the expansion of the corresponding scattering amplitudes in powers of $1/N_\mathrm{c}$ and α_s, our foremost task is to provide the characterization of the "tetraquark-phile" Feynman diagrams (with subscripts T indicating their correlator contributions) that might develop a tetraquark pole at $s = m_T^2$ by two basic criteria, expressed in terms of meson momenta p_1 and p_2 [6]:

1. The Feynman diagram should depend in a nonpolynomial way on the variable $s \equiv (p_1 + p_2)^2$.
2. The Feynman diagram should admit adequate intermediate four-quark states with branch cuts starting at branch points $s = (m_1 + m_2 + m_3 + m_4)^2$. The potential presence or absence of such an intermediate-state threshold may be clearly decided by means of the Landau equations [8].

2. Rigorous Self-Consistency Conditions from Large-N_c Ordinary-Meson Scattering

In order to carve out characteristics of tetraquarks for large N_c, we identify our tetraquark-phile Feynman diagrams and derive the tetraquark–two-ordinary-meson amplitudes $A(T \leftrightarrow MM)$ that fix the tetraquarks' decay widths $\Gamma(T)$. Since we take into account all conceivable scattering processes, we expect to encounter "flavour-preserving" ones, where the flavour composition of initial and final ordinary mesons is identical, and "flavour-rearranging" ones, proceeding by a reshuffling of quarks.

2.1 Flavour-exotic tetraquarks: bound states of quarks involving four different flavour types

For genuinely flavour-exotic tetraquarks, the rightmost plots in Fig. 1 reveal that the N_c-leading terms in tetraquark-phile flavour-preserving and flavour-reshuffling four-point correlation functions exhibit a different large-N_c behaviour. In this case, the resulting constraints cannot be fulfilled under the premise of the existence of a single tetraquark; rather, they require pole contributions of, at least, two tetraquark states, called T_A and T_B, differing in their large-N_c couplings to two ordinary mesons. Expressed in terms of tetraquark–two-meson transition amplitudes A, generic decay constants f_M of ordinary mesons, and propagator poles at $p^2 = m^2_{T_{A,B}}$, the pole contributions then read, symbolically,

$$\langle j^\dagger_{12}\, j^\dagger_{34}\, j_{12}\, j_{34}\rangle_{\rm T} = f_M^4 \left(\frac{|A(M_{12}M_{34} \leftrightarrow T_A)|^2}{p^2 - m^2_{T_A}} + \frac{|A(M_{12}M_{34} \leftrightarrow T_B)|^2}{p^2 - m^2_{T_B}}\right) + \cdots = O(N_c^0)\ ,$$

$$\langle j^\dagger_{14}\, j^\dagger_{32}\, j_{14}\, j_{32}\rangle_{\rm T} = f_M^4 \left(\frac{|A(M_{14}M_{32} \leftrightarrow T_A)|^2}{p^2 - m^2_{T_A}} + \frac{|A(M_{14}M_{32} \leftrightarrow T_B)|^2}{p^2 - m^2_{T_B}}\right) + \cdots = O(N_c^0)\ ,$$

$$\langle j^\dagger_{14}\, j^\dagger_{32}\, j_{12}\, j_{34}\rangle_{\rm T} = f_M^4 \left(\frac{A(M_{12}M_{34} \leftrightarrow T_A)\,A(T_A \leftrightarrow M_{14}M_{32})}{p^2 - m^2_{T_A}} + \frac{A(M_{12}M_{34} \leftrightarrow T_B)\,A(T_B \leftrightarrow M_{14}M_{32})}{p^2 - m^2_{T_B}}\right) + \cdots = O(N_c^{-1})\ .$$

The amplitudes of leading order in N_c then govern the total decay rates of T_A and T_B; these, due to an inherent flavour symmetry of the above, prove to show a parametrically identical dependence on N_c:

$$A(T_A \leftrightarrow M_{12}M_{34}) = O(N_c^{-1})\ , \qquad A(T_A \leftrightarrow M_{14}M_{32}) = O(N_c^{-2}) \qquad \Longrightarrow \qquad \Gamma(T_A) = O(N_c^{-2})\ ,$$

$$A(T_B \leftrightarrow M_{12}M_{34}) = O(N_c^{-2})\ , \qquad A(T_B \leftrightarrow M_{14}M_{32}) = O(N_c^{-1}) \qquad \Longrightarrow \qquad \Gamma(T_B) = O(N_c^{-2})\ .$$

Of course, as bound states composed of the same set of constituents, T_A and T_B will undergo mixing.

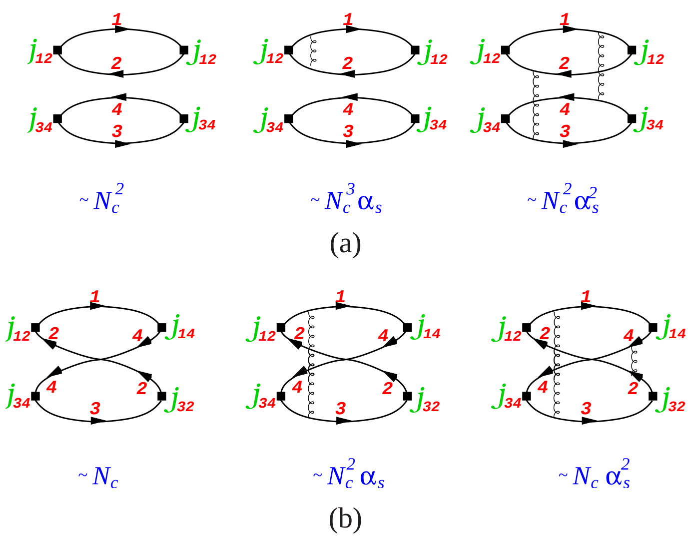

Figure 1: Flavour-preserving, $\langle j^\dagger_{12}\, j^\dagger_{34}\, j_{12}\, j_{34}\rangle$ (a), or -rearranging, $\langle j^\dagger_{14}\, j^\dagger_{32}\, j_{12}\, j_{34}\rangle$ (b), four-point correlators.

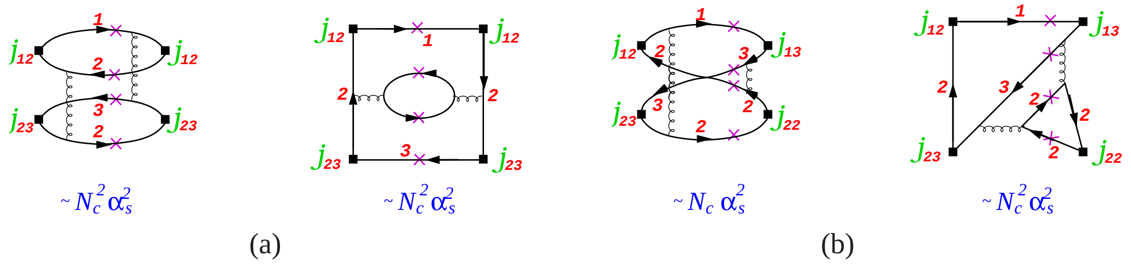

Figure 2: Flavour-preserving, $\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle$ (a), or -rearranging, $\langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle$ (b), four-point correlators; the four purple crosses at the quark propagators indicate the constituents of the intermediate-state tetraquarks.

2.2 Flavour-cryptoexotic tetraquarks: four-quark states exhibiting merely two open flavours

For a quark–antiquark pair with same flavour different from the other two, the relations (Fig. 2)

$$\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle_{\rm T} = O(N_c^0)\,, \qquad \langle j_{13}^\dagger\, j_{22}^\dagger\, j_{13}\, j_{22}\rangle_{\rm T} = O(N_c^0)\,, \qquad \langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle_{\rm T} = O(N_c^0)$$

are satisfied by a single tetraquark with two-meson couplings implying again a narrow decay width:

$$A(T \leftrightarrow M_{12}\, M_{23}) = O(N_c^{-1})\,, \qquad A(T \leftrightarrow M_{13}\, M_{22}) = O(N_c^{-1}) \qquad \Longrightarrow \qquad \Gamma(T) = O(N_c^{-2})\,.$$

Due to the same total flavour, such tetraquark may mix with an ordinary meson M_{13}. The pole terms

$$\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle_{\rm T} = f_M^4 \left(\frac{A(M_{12}\, M_{23} \to T)}{p^2 - m_T^2}\, g_{TM_{13}}\, \frac{A(M_{13} \to M_{12}\, M_{23})}{p^2 - m_{M_{13}}^2}\right) + \cdots = O(N_c^0)$$

then restrict the large-N_c behaviour of their associated mixing strength $g_{TM_{13}}$ to $g_{TM_{13}} \le O(1/\sqrt{N_c})$.

2.3 Resulting narrowness and number of tetraquarks: "always two there are, ... no less" [9]

In summary, at large N_c, we expect (1) any flavour-exotic tetraquarks to be formed pairwise and (2) all (compact) tetraquarks to be narrow, with decay widths decreasing not slower than $1/N_c^2$ [6,7].

Acknowledgement. D.M. was supported by the Austrian Science Fund (FWF), project P29028-N27.

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