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The hadronization into the octet of pseudoscalar mesons in terms of SU(N) gauge invariant Lagrangian

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By breaking the initial $SU(N)$ symmetry, we derive the Lagrangian governing the dynamics of the massive scalar particles, which can be treated as the octet of pseudoscalar mesons. The contribution of both the quark-gluon interaction and self-interaction gluon field into the masses of the octet particles is obtained in the explicit form in the considered approach.

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1. Introduction

The hadronization of the deconfined matter is a very important problem in the physics of strong interaction. It plays a key role in the description of the hadron spectra arising in various processes at high energy collisions. The mechanism of conversion of the exact freedom degrees of the deconfined particles into the approximate freedom degrees of observable hadrons is the central point the hadronization. Such conversion can be particularly carried out by integrating or combining the freedom degrees of gluons or quarks in the complete exact QCD Lagrangian by means of one or another way, that leads to some effective model Lagrangians governing the hadron dynamics.

The hadronization is a very complicate issue to be solved in the unified approach, starting from the initial QCD Lagrangian. Therefore, a lot of various models have been already developed, and continue to be developed, for describing the hadronization of the deconfined matter. They, in very conditional gradation, form two main approaches with respect to the hadronization problem. The first one is based on the parton duality of particles in the confined and deconfined matter. In the framework of this approach hadrons are created in the result of fragmentation of string or cluster (see, for exaple[1, 2, 3, 4]). The second approach can be named as dynamic (see, for instance [5, 6]. Studies of Green's function in the pure gluodynamics have shown that there is some running gluon mass which can be associated with the hadron mass. Another leg in the dynamics approach consists with the derivation effective Lagrangian based on the renormalizatin group calculation[7].

2. Gluodymanic Lagrangian

The Lagrangian governing the fermions interacting with the SU(N) gauge field is [8]

$$\mathscr{L} = \sum_f \left[\bar{\Psi}_f \gamma^{\mu} D_{\mu}\, \Psi_f - \bar{\Psi}_f\, m_f\, \Psi_f\right] - \frac{1}{4} F^a_{\mu\nu}\, F^{\mu\nu}_a + \mathscr{L}_{gh}, \tag{2.1}$$

where A^a_{μ} and Ψ_f are the gauge and fermion fields in the Minkowskii (3+1)-dimensional space-time with coordinates $x \equiv x^{\mu} = (x^0, \mathbf{x}) = (x^0, x^1, x^2, x^3)$, m_f is a fermion mass, g is the coupling constant, f means a quark flavor. In Eqs.(2.1) we introduce,

$$D_{\mu} \;=\; i\partial_{\mu} + g\; T_a A^a_{\mu}, \quad F^a_{\mu\nu} \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} + g\; f^a_{bc}\; A^b_{\mu}\; A^c_{\nu}, \tag{2.2}$$

where γ^{ν} are the Dirac matrices, T_a are the infinitesimal operators satisfying the standard commutative relations and the normalization condition[8], $a, b, c = 1 \ldots N^2 - 1$ are SU(N) group indices; $\mathscr{L}_{gh}$ is the ghost Lagrangian, $\partial_{\mu} = (\partial/\partial t, \nabla)$.

The Lagrangian (2.1) leads to the Dirac equation:

$$\left\{ i\gamma^{\mu} \left(\partial_{\mu} - ig \cdot A^a_{\mu}(x)\; T_a\right) - m_f \right\} \Psi_f(x) = 0, \tag{2.3}$$

which solution can written as follows[9]

$$\Psi_{c,f}(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \sum_{\sigma\lambda} \left[u_{\sigma}(P)\; a_f(P, \sigma, \lambda, c)\; \theta(P^0) \;+\; u_{\sigma}(-P)\; a_f(-P, \sigma, \lambda, c)\; \theta(-P^0) \right]$$
$$\times \frac{\exp(-iP_{\mu} x^{\mu})}{\sqrt{2\varepsilon(\vec{p})}} \{T_{l(x_0;x)} \exp\} \left\{ igT_a \int dx^{\mu} A^a_{\mu} \right\}\; v_{c,f}(x_0), \tag{2.4}$$

where P^μ is a 4-momentum, $u_\sigma(P)$ are the free Dirac bispinors, normalized by the doubled mass ($\bar{u}u = 2m_f$). The symbol $v_{c,f}(x_0)$ is a vector in the charged space, which also depends on a point in the Minkowskii space-time. We take $v_{c,f}(x_0)$ to be normalized by a condition:

$$(v^\dagger)_{c'f'}(x_0) v^{cf}(x_0) = \delta^c_{c'} \delta^f_{f'}. \tag{2.5}$$

Here, σ and c denote the spin and color variables. The symbol $\{T_{l(x_0;x)} \exp\}$ means that the integration is to be carried out along the line from the point x_0 to the point x such that the factors in exponent expansion are chronologically ordered from x_0 to x, whereas summation with respect to λ means summing over all the possible such trajectories.. The subscribes c and f enumerate the colors and flavor states, respectively while the coefficients $a_f(P,\sigma,\lambda,c)$ are related to particles or anti-particles, and satisfy the standard commutative relations for the Fermi operators under the quantization.

Substituting Eq.(2.4) into Eq.(2.1) and carrying out some mathematical manipulation[9], we go to the gluodynamic Lagrangian governing evolution of the mass non Abelian field

$$\mathscr{L} = M^2 A^\mu_a(x) \left(A^a_\mu(x) - \frac{\partial^\lambda \partial_\mu}{\partial^2} A^a_\lambda(x) \right) - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \mathscr{L}_{gh}, \tag{2.6}$$

where M, having been interpreted as the filed mass generated by interaction between fermions and a gauge field, is given by a formula

$$M^2 = \frac{g^2}{8} \int \frac{d^4P}{(2\pi)^3} \sum_{f,c,\lambda} \sum_\sigma \frac{\partial}{\partial P_\nu} \left\{ n_f(P,\sigma,\lambda,c) \frac{P^\nu \left[\delta(P^0 + \varepsilon(\vec{p})) + \delta(P^0 - \varepsilon(\vec{p})) \right]}{\varepsilon(\vec{p})} \right\}. \tag{2.7}$$

3. Scalar particle Lagrangian

To derive the Lagrangian governing observable particles we need to eliminate the unobservable degrees of a freedom, and to fix a gauge. We take the Lorenz gauge

$$\partial_\mu A^\mu_a = 0, \tag{3.1}$$

because of its relativistic invariance, and since there is no necessity in the ghost fields in this case.

Then, the Lagrangian (2.6) takes a form

$$\mathscr{L} = M^2 A^\mu_a(x) A^a_\mu(x) - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a. \tag{3.2}$$

In the case of the $SU(3)$ symmetry the Lagrangian (3.2) contains 8 independent fields. Therefore, it is reasonable to relate them with the octet of the pseudosalar mesons, arising in the result of the hadronization of the deconfined matter. This is some specific confinement situation since there are only gluons in the Lagrangian (3.2). As for the quark subsystem, the information about it appears to be incorporated into this Lagrangian by means of the mass term.

To derive the scalar particle Lagrangian we primarily have to go from vector fields to scalar ones. We carry out it by separating the variables in $A^\mu_a(x)$ which correspond to the Minkowskii

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and representation spaces. Let us assume that the hadronization occurs when the $SU(3)$ symmetry appears to be spontaneously broken so that the fields $A_a^\mu(x)$ take a form:

$$A_a^\mu(x) = \mathfrak{a}_a^\mu + e^\mu \varphi_a(x), \tag{3.3}$$

where $\varphi_a(x)$ are scalar functions, whereas the constant vectors $\mathfrak{a}_a^\mu$ are assumed to be orthogonal to both the unit vector e_μ, normalized by condition $e_\mu\ e^\mu = -1$, and the scalar field gradients $\partial_\mu \varphi_a(x)$

$$\mathfrak{a}_a^\mu\ e_\mu = 0, \quad \mathfrak{a}_\mu^a\ \partial^\mu \varphi_b(x) = 0. \tag{3.4}$$

We note, that the Lorentz gauge (3.1) results in the additional orthogonality condition:

$$e^\mu \partial_\mu \varphi_a(x) = 0. \tag{3.5}$$

The fields $A_a^\mu(x)$ governed by Eqs.(3.3)- (3.5) means physically, that the scalar fields $\varphi^a(x)$ can only propagate along the direction in the Minkowskii space-time, which is perpendicular to the plane fixed by the orthogonal vectors e^μ and $\mathfrak{a}_a^\mu$. Since these planes are different for the different a, the fields $\varphi_a(x)$ are independent in terms of their evolution in the Minkowskii space-time. We should also note that such kinematic restriction in the field propagation leads to arising the additional mass of the field $\varphi_a(x)$, as it will be shown below.

Substituting Eq.(3.3) into the Lagrangian (3.2), and taking into account Eqs.(3.1),(3.4)-(3.5), we derive after direct calculations:

$$\mathscr{L} = \frac{1}{2}\,(\partial^\mu \varphi^a(x))(\partial_\mu \varphi_a(x)) - \frac{1}{2}(\mathscr{M}^2)_b^a \varphi_a(x)\varphi^b(x), \tag{3.6}$$

where $(\mathscr{M}^2)_b^a$ is the matrix of the squared masses which is given by a formula

$$(\mathscr{M}^2)_b^a = 2M^2(N_c,N_f)\delta_b^a - g^2 \mathfrak{a}_c^\mu\ \mathfrak{a}_\mu^{c'} f_b^{\ cs} f_{c's}^{a}. \tag{3.7}$$

We should note the contribution of both gluon-gluon and quark-interaction interaction into masses of particles is taken into account in the mass term in Eq.(3.7).

Further, we follow Gell-Mann[10], and take the conservation of the isospin T and strangeness S, rather than supporting the exact $SU(3)$ symmetry, under hadronization into the octet of the colorless mesons. This means breaking the initial symmetry $SU(3)$ upto $SU_{S=0}(2) \bigotimes SU_{S=1}(2) \bigotimes U(1)$ one. The new symmetry implies that these 8 pseudoscalar mesons, which are $\pi^\pm, \pi^0, K^\pm, K^0, \tilde{K}^0, \eta$, including antiparticle (K^- and $\tilde{K}^0$), can be placed into the strangenessless pion triple, two kaon doublets at $S = \pm 1$, where $S = -1$ corresponds to antiparticles, and η meson having the zeroth isospin and strangeness. Moreover, such a symmetry violation affects on the mass $M = M(N_c, N_f)$ since its value depends explicitly on numbers of colors N_c and flavors N_f .

Therefore, let us establish relation of these pseudoscalar mesons to the fields φ_a by means of the complex subscribe $a \Rightarrow (T,S)$:

$$\begin{aligned}
&\varphi_1(x) = \varphi_{\pi^+}(x) = \varphi_{(1,0)}(x); \varphi_2(x) = \varphi_{\pi^-}(x) = \varphi_{(1,0)}(x); \varphi_3(x) = \varphi_{\pi^0}(x) = \varphi_{(1,0)}(x);\\
&\varphi_4(x) = \varphi_{K^+}(x) = \varphi_{(1/2,1)}(x); \varphi_5(x) = \varphi_{K^-}(x) = \varphi_{(1/2,-1)}(x);\\
&\varphi_6(x) = \varphi_{K^0}(x) = \varphi_{(1/2,1)}; \varphi_7(x) = \varphi_{\tilde{K}^0}(x) = \varphi_{(1/2,-1)}(x); \varphi_8(x) = \varphi_\eta(x) = \varphi_{(0,0)}(x)
\end{aligned} \tag{3.8}$$

where T and S are the isospin and strangeness, respectively.

Such defined fields φ_a do not correspond to the observable mesons since the mass term in the Lagrangian (3.6) has not diagonalized yet. Upon carrying out the diagonalization[9], we go to a new basis $\Phi_a(x)$, that the diagonalized Lagrangian of the meson octet takes a form:

$$\mathscr{L} = \frac{1}{2}\left(\partial^\mu \Phi^a(x)\right)\left(\partial_\mu \Phi_a(x)\right) - \frac{1}{2}(m^2_{oct})^a_b \Phi_a(x)\Phi^b(x), \tag{3.9}$$

where Φ_a is the component of the octet: $\pi^+, \pi^-, \pi^0, K^+K^-, K^0, \tilde{K}^0, \eta^0$, whereas the mass matrix $(m^2_{oct})^a_b$ only consists of the diagonal elements[**?**], which are the squared masses are

$$(m^2_{oct})^a_b = diag(m_{\pi^+}; m_{\pi^-}; m_{\pi^0}; m_{K^+}; m_{K^-}; m_{K^0}; m_{\tilde{K}^0}; m_\eta). \tag{3.10}$$

In this way, all of these masses in the Eq.(3.10) are directly related [9] with masses given by Eq.(3.7).

The Lagrangian (3.9) governs the octet of the massive scalar particles, while the elements of the mass matrix $(m^2_{oct})^a_b$ appear to be generated by both the quark-gluon interaction and the self-interacting gauge fields.

4. Conclusion

On basis of the self-consistent consideration of the dynamics of fermion and boson fields in the gauge $SU(N)$ model the pure gluodynamic Lagrangian ruling evolution of the massive non-Abelian fields is derived. Violating the initial $SU(3)$ symmetry in such a gluodynamics Lagrangian, we obtain the Lagrangian governing the octet of the pseudoscalar mesons. The contribution of the quark-gluon interaction and self-interacting gluon field into the meson masses has been taken into account, and has been calculated in the explicit form in the developed approach.

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