PROCEEDINGS OF SCIENCE

Dalitz decays of π^0, η and η' mesons

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The time-like transition form factors of the π^0, η and η' pseudoscalar mesons are predicted by means of different fits to existing experimental data on their space-like counterparts using Padé approximants. As a byproduct, we calculate the integrated branching ratios of the single Dalitz decays $\pi^0 \to e^+e^-\gamma$ and $\eta^{(\prime)} \to \ell^+\ell^-\gamma$, with $\ell = e$ or μ, and the double Dalitz decays $\pi^0 \to e^+e^-e^+e^-$ and $\eta^{(\prime)} \to \ell^+\ell^-\ell^+\ell^-$.

XVII International Conference on Hadron Spectroscopy and Structure - Hadron2017
25-29 September, 2017
University of Salamanca, Salamanca, Spain

[*]Speaker.

[†]I would like to express my gratitude to the Hadron2017 Organizing Committee for the opportunity of presenting this contribution, and for the pleasant and interesting workshop we have enjoyed.

1. Introduction and motivation

The transition form factors (TFFs) of the pseudoscalar mesons π^0, η and η' encode the strong interaction dynamics of the couplings of these mesons to pairs of photons. When the photons are off-shell the mathematical function characterising the corresponding TFF depend on both virtualities, $F_{P\gamma^*\gamma^*}(q_1^2,q_2^2)$. However, from the point of view of experiment, so far, only one virtuality is accessible, for instance in central production experiments such as $e^+e^-\to e^+e^-(\pi^0,\eta,\eta')$ where one final lepton is tagged but the other is not, thus becoming the function behind the TFF $F_{P\gamma^*\gamma^*}(-Q^2,0)\equiv F_{P\gamma\gamma^*}(Q^2)$ with $Q^2\equiv -q^2$.

The main goal of the analysis in which this contribution is based on was predict the time-like region of the π^0, η and η' TFFs from the corresponding fits to existing and precise experimental data on their space-like counterparts using Padé approximants (PAs) [1]. As a byproduct, we were able to calculate the invariant mass spectra and the integrated branching ratios of the so-called single Dalitz decays $\pi^0\to e^+e^-\gamma$ and $\eta^{(\prime)}\to \ell^+\ell^-\gamma$, with $\ell = e$ or μ, and the double Dalitz decays $\pi^0\to e^+e^-e^+e^-$ and $\eta^{(\prime)}\to \ell^+\ell^-\ell^+\ell^-$.

2. Transition form factors

For the parameterization of the TFFs when confronted with experimental data in the corresponding space-like regions we use PAs as the fitting functions. PAs to a given function are ratios of two polynomials (with degree L and M, respectively),

$$P_M^L(q^2)=\frac{a_0+a_1q^2+\cdots+a_L(q^2)^L}{1+b_1q^2+\cdots+b_M(q^2)^M}\,, \tag{2.1}$$

whose Taylor expansion around the origin is that of $f(q^2)$ up to $\mathcal{O}(q^2)^{L+M+1}$ [2]. For the TFFs under consideration we use two relevant sequences of PAs: *i)* $P_1^L(q^2)$ when the TFF is believed to be dominated by a single resonance, and *ii)* $P_N^N(q^2)$ when the TFF is supposed to fulfil the asymptotic behaviour, that is, $\lim_{Q^2\to\infty}Q^2|F_{P\gamma\gamma^*}(Q^2)| = \text{const}$. The highest element of a given Padé sequence will be fixed by experimental data. While the function behind a TFF of this kind is known to be analytical everywhere in the complex plane except for a branch cut starting at the two-pion threshold, PAs are rational functions including only poles as singularities. Therefore, one could think that they are invalid for describing the time-like region of the TFF. However, it was shown in Ref. [3] that the effect of the unitary cut is smooth and can be reasonably substituted by a single or multiple poles on the real axis, thus making the description of the TFFs in terms of PAs well grounded.

For the single Dalitz decays, where one photon is on-shell, the original double-virtual TFF is simplified to a single-virtual TFF, which indeed is the only one being measured so far, mainly in the space-like region. For the double Dalitz decays, we use two methods: *i)* the standard factorization approach, which in terms of normalized TFFs reads $\tilde{F}_{P\gamma^*\gamma^*}(q_1^2,q_2^2)=\tilde{F}_{P\gamma\gamma^*}(q_1^2,0)\tilde{F}_{P\gamma\gamma^*}(0,q_2^2)$, and *ii)* the lowest order bivariate approximant which consists of a generalization of the univariate PAs named Chisholm approximants. While the second method can fulfil the asymptotic space-like constraints, $\lim_{Q^2\to\infty}F_{P\gamma^*\gamma^*}(-Q^2,0)\propto 1/Q^2$ [4] and $\lim_{Q^2\to\infty}F_{P\gamma^*\gamma^*}(-Q^2,-Q^2)\propto 1/Q^2$ [5], the first cannot. The lack of experimental information for the double-virtual TFFs prevents to distinguish between the two methods.

3. Results

For the π^0 case, due to the small phase-space available, our prediction for its time-like TFF is based on a Taylor expansion built from an average of the low-energy constants (LECs), slope and curvature, obtained from several fits to space-like data using different PAs [6]. The values used in the analysis for the LECs and the plot for the predicted π^0 time-like TFF in comparison with the measurement performed by the A2 Coll. [7] are found in Ref. [1]. The predictions for the integrated branching ratios of $\pi^0 \to e^+e^-\gamma$ and $\pi^0 \to e^+e^-e^+e^-$ together with their experimental values are shown in Table 1.

For the η case, the highest elements for the $P_1^L(q^2)$ and $P_N^N(q^2)$ Padé sequences obtained from several fits to space-like data are $P_1^5(q^2)$ and $P_2^2(q^2)$, respectively, while for the η' case, they are $P_1^6(q^2)$ and $P_1^1(q^2)$ [8]. Needless to say, a Taylor expansion is not reasonable in these cases, more obvious for the η' where the phase-space allows for the on-shell presence of the lowest lying vector mesons. Our predictions for the η and η' time-like TFFs and the corresponding reported experimental measurements are displayed in Figure 1. A Vector Meson Dominance (VMD) description of the η' time-like TFF is also included for comparison. For the η' case, our approach is limited to some point in the neighbourhood of the pole (see details in Ref. [1]), thus making mandatory to use a model, for instance VMD, to describe the TFF in the resonance region. Again, the predictions for the integrated branching ratios of the η and η' single and double Dalitz decays together with their experimental values are found in Table 1.

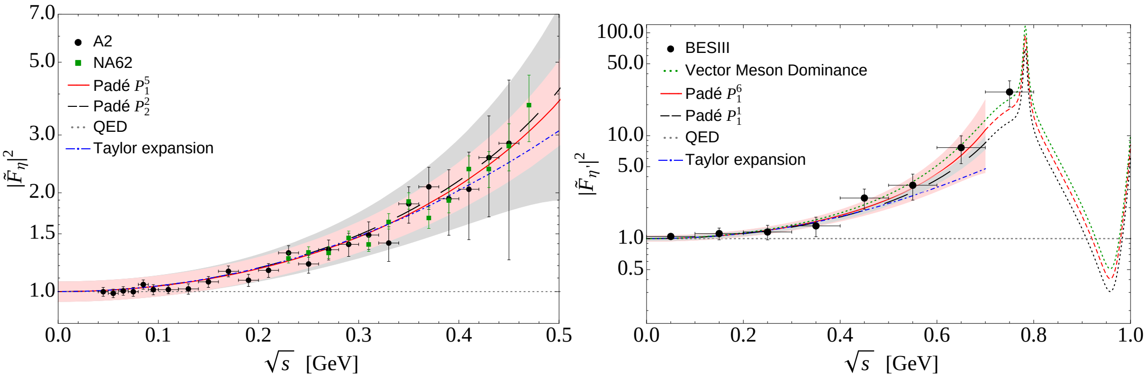

Figure 1: Our predictions for the modulus square normalised time-like TFFs $\widetilde{F}_{\eta\gamma\gamma^*}(q^2)$ (left) and $\widetilde{F}_{\eta'\gamma\gamma^*}(q^2)$ (right) as a function of the invariant dilepton mass, $\sqrt{s} \equiv m_{\ell\ell}$. For the η time-like TFF, the predictions coming from the $P_1^5(q^2)$ (red solid line) and $P_2^2(q^2)$ (black long-dashed line) PAs, and the Taylor expansion (blue dot-dashed line) are compared to the experimental data from $\eta \to e^+e^-\gamma$ [9] (black circles) and $\eta \to \mu^+\mu^-\gamma$ [10] (green squares). For the η' time-like TFF, the predictions up to the matching point located at $\sqrt{s} = 0.70$ GeV coming from the $P_1^6(q^2)$ (red solid line) and $P_1^1(q^2)$ (black long-dashed line) PAs, and the Taylor expansion (blue dot-dashed line) are compared to the experimental data from $\eta' \to e^+e^-\gamma$ [11] (black circles). From the matching point on, rescaled versions of the VMD description are used. The QED predictions (grey dotted lines) are also displayed.

Acknowledgments: This work was supported in part by the Ministerio de Economía, Industria y Competitividad under grants CICYT-FEDER-FPA 2014-55613-P and SEV-2016-0588. Sergi Gonzàlez-Solís also received support from the CAS President's International Fellowship Initiative for Young International Scientists (Grant No. 2017PM0031).

Decay	Predicted value	Experimental value [12]	n_σ
$\pi^0 \to e^+e^-\gamma$	$1.169(1)\%$	$1.174(35)\%$	0.15
$\eta \to e^+e^-\gamma$	$6.61(50)\times 10^{-3}$	$6.90(40)\times 10^{-3}$	0.45
$\eta \to \mu^+\mu^-\gamma$	$3.26(46)\times 10^{-4}$	$3.1(4)\times 10^{-4}$	0.26
$\eta' \to e^+e^-\gamma$	$4.38(32)\times 10^{-4}$	$4.69(20)(23)\times 10^{-4}$	0.70
$\eta' \to \mu^+\mu^-\gamma$	$0.75(6)\times 10^{-4}$	$1.08(27)\times 10^{-4}$	1.19
$\pi^0 \to e^+e^-e^+e^-$	$3.36689(5)\times 10^{-5}$	$3.34(16)\times 10^{-5}$	0.17
$\eta \to e^+e^-e^+e^-$	$2.71(2)\times 10^{-5}$	$2.4(2)(1)\times 10^{-5}$	1.38
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$3.98(15)\times 10^{-9}$	$< 3.6\times 10^{-4}$	
$\eta \to e^+e^-\mu^+\mu^-$	$2.39(7)\times 10^{-6}$	$< 1.6\times 10^{-4}$	
$\eta' \to e^+e^-e^+e^-$	$2.10(45)\times 10^{-6}$	not seen	
$\eta' \to \mu^+\mu^-\mu^+\mu^-$	$1.69(36)\times 10^{-8}$	not seen	
$\eta' \to e^+e^-\mu^+\mu^-$	$6.39(91)\times 10^{-7}$	not seen	

Table 1: Predicted values for the integrated branching ratios together with the experimental values. n_σ stands for the number of standard deviations the experimental measurements are from our theoretical predictions.

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