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Scalar and tensor meson contributions to the $\tau \to \pi\pi\pi\nu_\tau$ axial-vector form-factor

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In these proceedings we study the scalar ($J^{PC} = 0^{++}$, S) and tensor ($J^{PC} = 2^{++}$, T) resonance contributions to the $\pi\pi\pi$ axial-vector form-factor (AFF), relevant for phenomenological studies of tau decays. Chiral symmetry and its isospin subgroup are key ingredients of our construction, implemented via a chiral invariant Lagrangian which incorporates S, T and axial-vector (A) resonances and the light multiplet of pseudoscalars, the chiral Goldstones (pions, kaons and etas). Thus, one obtains the right isospin relation between the $\pi^0\pi^0\pi^-$ and $\pi^-\pi^-\pi^+$ production amplitudes. The chiral invariant construction ensures the recovery of the low-energy limit, provided by Chiral Perturbation Theory (χPT) and the transversality of the current in the chiral limit at all energies. The amplitudes are further constrained by imposing high-energy constraints, prescribed by Quantum Chromodynamics (QCD). We discuss the improvement of the Breit-Wigner and Flatté representations for the broad σ scalar resonance provided by the incorporation of the real logs required by analyticity, à la Gounaris-Sakurai. The aim of this work is to improve the description of these decay channels oriented to its implementation in the Tauola Monte Carlo and future Belle data analyses.

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1. Introduction

In this proceedings we discuss the $\tau \to \pi\pi\pi\nu_\tau$ decay mediated through intermediate T and S resonances [1], focused on the following four goals:

- **Chiral invariance and partial conservation of the axial-vector current (PCAC):** longitudinal corrections come naturally suppressed by m_q. In addition, as isospin is a subgroup of the chiral symmetry, our chiral invariant Lagrangian approach yields the right relation between the $\pi^0\pi^0\pi^-$ and $\pi^-\pi^-\pi^+$ tau decay form-factors, prescribed by isospin symmetry [2].
- **Low-energy limit:** the construction of a general chiral invariant Lagrangian ensures the right low-energy structure and the possibility of matching χPT [3].
- **On-shell description**: previous works have performed a fine work in describing the decays through axial-vector and tensor resonances when their intermediate momenta are near their mass shell [4, 5]. Our outcome reproduces these previous results when the intermediate resonance becomes on-shell.
- **High-energy QCD limit:** by imposing high-energy conditions and demanding the behaviour prescribed by QCD for the form-factors at large momentum transfer [6] we will constrain the resonance parameters.

Bose symmetry implies that the matrix element $H^\mu_{3\pi}\langle \pi(p_1)^{a_\pm}\pi(p_2)^{a_\pm}\pi^\pm(p_3)|\bar{d}\gamma^\mu\gamma_5 u|0\rangle$ (with $a_+ = -$ and $a_- = 0$) is determined in terms of a transverse form-factors $\mathscr{F}_1(s_1,s_2,q^2)$ and a longitudinal AFF $\mathscr{F}_P(s_1,s_2,q^2) = \mathscr{F}_P(s_2,s_1,q^2)$ in the form

$$H^\mu_{3\pi} = i P_T^{\mu\nu}(q)\left[\mathscr{F}_1(s_1,s_2,q^2)\,(p_1-p_3)_\nu \;+\; \mathscr{F}_1(s_2,s_1,q^2)\,(p_2-p_3)_\mu\right] \;+\; i q_\mu\, \mathscr{F}_P(s_1,s_2,q^2)\,. \quad (1.1)$$

We will use the definitions $q = p_1+p_2+p_3$, $k = p_1+p_2$, $\Delta p^\rho = p_1^\rho - p_2^\rho$, $P_T(q)^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu/q^2$, the scalar products $s_1 = (p_2+p_3)^2$, $s_2 = (p_3+p_1)^2$, $s_3 = (p_1+p_2)^2 = k^2$, $qp_j = (m_\pi^2 + q^2 - s_j)/2$, $qk = (q^2 - m_\pi^2 + s_3)$. The matrices $R = \sum_{a=0}^{8} \frac{\lambda^a}{\sqrt{2}} R^a$ contain the lightest $U(3)$ resonance nonets for $R = S, T_{\mu\nu}, A_{\mu\nu}$, with the axial-vector $A_{\mu\nu}$ described in the antisymmetric representation [7]. The $\mathscr{F}_P$ and next longitudinal AFF vanish in the chiral limit. All the results in our analysis [1] refer to $\pi^0\pi^0\pi^-$. Isospin symmetry relates the $\pi^0\pi^0\pi^-$ and $\pi^-\pi^-\pi^+$ AFF [1, 2]:

$$\begin{aligned}
\mathscr{F}_1^{\pi^-\pi^-\pi^+}(s_1,s_2,q^2) &= \mathscr{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_3,q^2) - \mathscr{F}_1^{\pi^0\pi^0\pi^-}(s_2,s_3,q^2) - \mathscr{F}_1^{\pi^0\pi^0\pi^-}(s_3,s_2,q^2)\,,\\
\mathscr{F}_P^{\pi^-\pi^-\pi^+}(s_1,s_2,q^2) &= \mathscr{F}_P^{\pi^0\pi^0\pi^-}(s_1,s_3,q^2) + \mathscr{F}_P^{\pi^0\pi^0\pi^-}(s_2,s_3,q^2)\,. \qquad (1.2)
\end{aligned}$$

We will consider interactions between chiral Goldstones and A, S and T resonances. The non-resonant and V contributions to the AFF are explicitly separated and can be found in [13, 14]. In order to implement these properties we make use of the relevant RχT Lagrangian for this observable [7]

$$\mathscr{L}_{\mathrm{R}\chi\mathrm{T}} = \mathscr{L}_{\mathrm{non-R}} + \sum_R \mathscr{L}_R + \sum_{R,R'} \mathscr{L}_{RR'}\,, \qquad (1.3)$$

$$\mathscr{L}_{\mathrm{non-R}} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + L_1^{\mathrm{T,SD}}\langle u^\mu u_\mu\rangle^2 + L_2^{\mathrm{T,SD}}\langle u^\mu u^\nu\rangle\langle u_\mu u_\nu\rangle + L_3^{\mathrm{T,SD}}\langle (u^\mu u_\mu)^2\rangle\,, \qquad (1.4)$$

$$\mathscr{L}_R = \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle + c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle + g_T\langle T_{\mu\nu}\{u^\mu, u^\nu\}\rangle\,, \tag{1.5}$$

$$\mathscr{L}_{RR'} = \lambda_1^{AS}\langle\{\nabla_\mu S, A^{\mu\nu}\}u_\nu\rangle + \lambda_1^{AT}\langle\{T_{\mu\nu}, A^{\nu\alpha}\}h^\mu_\alpha\rangle + \lambda_2^{AT}\langle\{A_{\alpha\beta}, \nabla^\alpha T^{\mu\beta}\}u_\mu\rangle\,, \tag{1.6}$$

with the covariant derivative ∇^α, u_μ and $h_{\mu\nu}$ containing one and two derivatives of the chiral Goldstones, $f_{-\,\mu\nu}$ providing the $W^\pm_{\mu\nu}$ field-strength tensors and the chiral tensor χ_+ introducing the chiral breaking due to the quark masses [1, 7]. The $\mathscr{O}(p^4)$ terms $L_2^{\rm T,SD} = 2L_1^{\rm T,SD} = -\frac{L_3^{\rm T,SD}}{2} = -\frac{g_T^2}{M_T^2}$ in $\mathscr{L}_{\rm non-R}$ are required to reproduce the correct short-distance behaviour for the forward $\pi\pi$ scattering in the presence of T resonances [8].

2. Scalar and tensor resonance contributions to $\pi\pi\pi$–AFF

2.1 $S\pi$ and $T\pi$ production

The $S\pi$ and $T\pi$ tree-level production is provided in RχT by the AFF [1]

$$\textbf{S}\pi\textbf{–AFF}: \quad \mathscr{F}^a_{S\pi}(q^2;s_3) = \frac{2c_d}{F_\pi}\frac{M_A^2}{M_A^2-q^2}\,, \quad \mathscr{H}^a_{S\pi}(q^2;s_3) = \frac{4}{F_\pi}\frac{m_\pi^2}{q^2(q^2-m_\pi^2)}\left[c_d(qp)+c_m q^2\right]\,,$$

$$\textbf{T}\pi\textbf{–AFF}: \quad \mathscr{F}^a_{T\pi}(q^2;s_3) = -\frac{8g_T}{F_\pi}\frac{M_A^2}{M_A^2-q^2}\,, \quad \mathscr{G}^a_{T\pi}(q^2;s_3) = \mathscr{H}^a_{T\pi}(q^2;s_3) = 0\,, \tag{2.1}$$

where a good high-energy vanishing behaviour have been imposed at $q^2\to\infty$ on the $S\pi$ and $T\pi$ AFF, in agreement with QCD [6], giving the constraints [1, 9]

$$\textbf{S}\pi\textbf{–AFF}: \quad \lambda_1^{AS} = \sqrt{2}c_d\,, \qquad \textbf{T}\pi\textbf{–AFF}: \quad F_A\lambda_2^{AT} = -2F_A\lambda_1^{AT} = 2\sqrt{2}g_T\,. \tag{2.2}$$

2.2 $\pi\pi\pi$–AFF via $S\pi$ and $T\pi$

Eq. (2.1) provides the S resonance contributions to the $\pi^0\pi^0\pi^-$ AFF's:

$$\mathscr{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_S = \frac{2}{3}\mathscr{F}^a_{S\pi}(q^2;s_3)\mathscr{G}_{S\pi\pi}(s_3)\,. \tag{2.3}$$

The propagation of S and its decay into $\pi\pi$ is given by $\mathscr{G}_{S\pi\pi}(s_3) = \frac{\sqrt{2}[c_d(s_3-2m_\pi^2)+2c_m m_\pi^2]}{F_\pi^2(M_S^2-s_3)}$.

The T resonance contribution to the $\pi^0\pi^0\pi^-$ transverse AFF is given by

$$\mathscr{F}_1^{\pi^0\pi^0\pi^-}(s_1,s_2,q^2)\Big|_T = \frac{8\sqrt{2}g_T^2}{3F_\pi^3 M_T^2}(2s_1-s_2+s_3-4m_\pi^2) \tag{2.4}$$
$$-\frac{8\sqrt{2}}{3F_\pi^3}\frac{g_T^2}{M_T^2}\frac{M_A^2}{M_A^2-q^2}\left[(kp_3)+\frac{s_3}{3}\left(1-\frac{2(kp_3)}{M_T^2}\right)-\frac{M_T^2}{M_T^2-s_3}\left(3(q\Delta p)+\frac{(\Delta p)^2}{3}+\frac{(kp_3)(\Delta p)^2}{3M_T^2}\right)\right].$$

The contributions to the longitudinal AFF $\mathscr{F}_P$ are suppressed by m_π^2 and are given in [1].

An important part of [1] was the study of parametrizations for the $\pi\pi$ final state interactions. For not-so-broad states such as the $a_1(1260)$, and $f_2(1270)$ we use Flatté widths. However, for the σ, analyticity implies that large real logarithms accompany the large imaginary part required by unitarity, suggesting a propagator modification à la Gounaris-Sakurai (GS) [1, 11, 10]. In addition, we consider a small $\sigma - f_0(980)$ mixing angle $\phi_S = -8^\circ$ [12].

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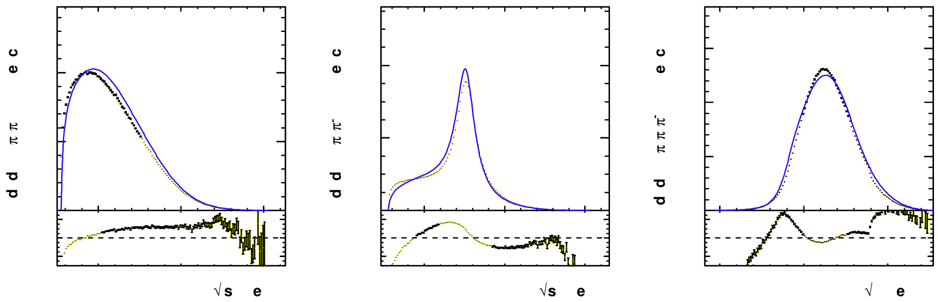

Figure 1: Comparison between the CLEO 'emulated' data and our prediction for the $\pi^0\pi^0\pi^-$ decay mode. A similar agreement is shown in [1] for $\pi^-\pi^-\pi^+$.

3. Phenomenology

Table 1: Numerical values of the parameters used to produce the theoretical spectra in Fig. 1. All the parameters are in GeV units except for c_σ and c_{f_0}, which are dimensionless. More details can be found in Ref. [1].

M_ρ	$M_{\rho'}$	$\Gamma_{\rho'}$	M_{a_1}	M_σ	M_{f_2}	Γ_{f_2}	F_π
0.772	1.35	0.448	1.10	0.8064	1.275	0.185	0.0922
F_V	F_A	β_ρ	g_T	c_d	c_σ	M_{f_0}	c_{f_0}
0.168	0.131	-0.32	0.028	0.026	76.12	1.024	17.7

Our S and T resonance amplitudes are combined with the vector resonance (V) contributions [13, 14], which are dominant. This provides the results in Fig. 1. [1] This is an illustration of our model, not a fit, where we have used the previous determinations of the parameters [8, 11, 15, 16] in Table 1. A proper determination is postponed to a future work and will probably need of the fitting of the Dalitz plot, not just one-variable distributions.

Here we show just the $\pi^0\pi^0\pi^-$ channel, as the various contributions are more neatly separated: V only resonates in the s_1 and s_2 spectra, and S and T tensors only resonate in the s_3 distribution. The S resonances (in particular the σ) serve to cure the slight discrepancies with respect to the data that appear in the low energy regions, $M_{\pi\pi} < M_\rho$ [16]. In Fig. 2 we show the ratio of our theoretical $M_{\pi^0\pi^0} = \sqrt{s_3}$ distribution including only the vector contribution V [16]) and its full result ($V+S+T$) in Fig. 1. Tensor produce a negligible effect except at $M_{\pi^0\pi^0} \sim 1.3$ GeV, where one observes a clear f_2 structure. However, it is at the end of the spectrum and will need a high integrated luminosity for the signal to become significant. For the $S\pi\pi$ coupling $c_d = 26$ MeV [11] we find small S corrections in the left-hand side (lhs) of Fig. 1. On its right-hand side (rhs) we obtain a large σ effect by increasing c_d a factor 3. Thus, large variations in the S parameters will be correlated and compensated in a fit to data by small modification of the V couplings.

[1] We thank J. Zaremba for providing the corresponding unnormalized CLEO distributions.

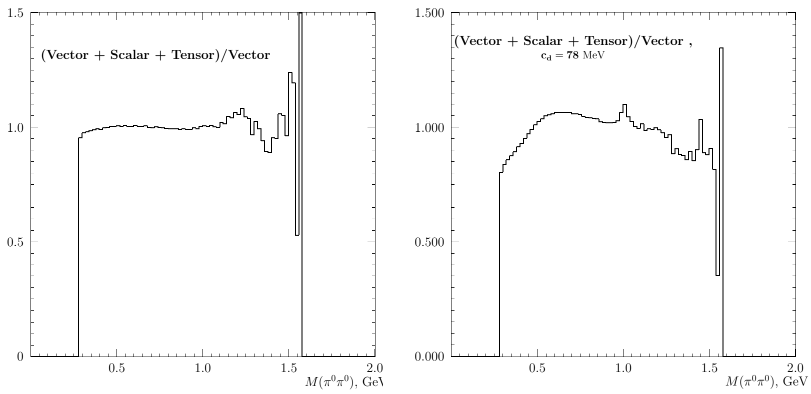

Figure 2: Ratio of the vector+tensor+scalar and only vector $\sqrt{s_3} = M_{\pi^0\pi^0}$ spectral function for $\tau \to \nu_\tau\pi^0\pi^0\pi^-$ for $c_d = 26$ MeV and $c_d = 78$ MeV (lhs and rhs, respectively).

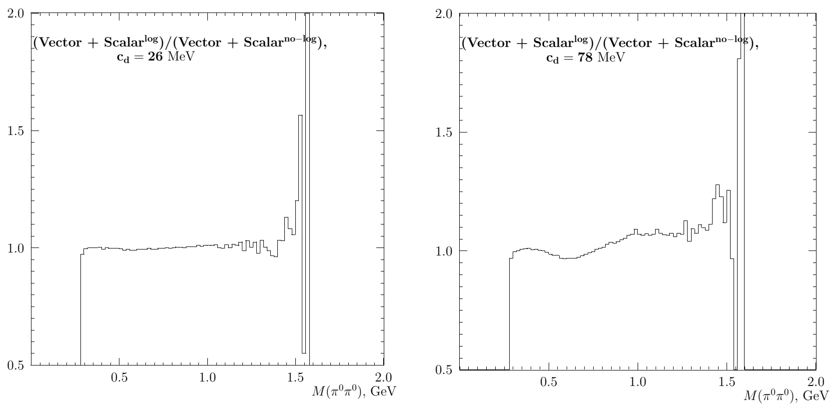

Figure 3: Plots for the ratios of the $\sqrt{s_3} = M_{\pi^0\pi^0}$ spectral functions for $\tau \to \nu_\tau\pi^0\pi^0\pi^-$: a) ratio of the full result and the spectral function without the real part of the logs in the σ propagator for $c_d = 26$ MeV; b) ratio of the full result and the spectral function without the real part of the logs in the σ propagator for $c_d = 78$ MeV. In order to better pin down the impact of the scalar propagator structure we only consider the $V+S$ contribution, dropping T resonances.

The importance of the real logs introduced in the σ propagator á la GS is studied in Fig. 3.a (Fig. 3.b) for $c_d = 26$ MeV ($c_d = 78$ MeV). For all the other inputs we use Table 1 and take only the $V+S$ contributions for sake of clarity. Since the scalar contribution is quite small, the impact of the real logs of the σ propagator in the full spectral distributions is quite suppressed for this τ decay. We want to emphasize that although a Breit-Wigner σ can provide an equally good description of the data [16], the aim of the present analysis of the σ à la GS is rather to improve the theoretical understanding of broad resonances within a Lagrangian formalism and its matching to χPT at low energies.

In summary, in this article we have computed the S and T contributions to the $\pi\pi\pi$ AFF. by means a chiral invariant Lagrangian including the relevant A, S, T and chiral Goldstones. This incorporates chiral and isospin symmetries, ensures the proper low-energy matching with χPT and

PCAC, improving previous descriptions [1, 4, 5]. We have also studied an alternative approach to the sigma description incorporating an analytical parametrization of the width à la GS [11, 10]: instead of just the imaginary part $i\rho_\pi(s)$ required by unitarity in the K-matrix formalism or the Breit-Wigner form [16], we considered the full complex logarithm $\overline{B}_0$ from the analytical Chew-Mandelstam dispersive integral [1, 10, 11]. Although it requires further refinements, we find the exploration of this approach for $\tau \to \pi\pi\pi\nu_\tau$ worthy, as it may help to understand whether it is possible or not to use a Lagrangian formalism for the description of broad resonances. We extend Ecker and Zauner's work on T resonances [8] and plan to include $V-T$ interactions in a similar way in a future paper dedicated to the study of the $e^+e^- \to a_2\pi$ process [17]. In order to obtain a good fit to the BaBar data, one will probably need not only the one-dimensional distributions but also the Dalitz plot. A proper tuning of the Monte Carlo parameters (e.g., the $S\pi\pi$ coupling c_d) should be ready before the beginning of the Belle-II data taking [18]. Its high luminosity will give us an opportunity to measure both $\pi^-\pi^-\pi^+$ and $\pi^0\pi^0\pi^-$ decays and study their intermediate production mechanisms like, e.g., the tiny contribution from the $f_2\pi^-$ channel.

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