PROCEEDINGS OF SCIENCE

Recent measurements of exclusive hadronic cross sections at *BABAR* and the implications for the muon $g-2$ calculation.

Fabio Anulli[*†]
Istituto Nazionale di Fisica Nucleare, Sezione di Roma
E-mail: fabio.anulli@roma1.infn.it

The *BABAR* Collaboration has an intensive program studying hadronic cross sections in low-energy e^+e^- annihilations, which are accessible with data taken near the $\Upsilon(4S)$ via initial-state radiation. Our measurements allow significant improvements in the precision of the predicted value of the muon anomalous magnetic moment. These improvements are necessary for shedding light on the current $\sim 3\sigma$ difference between the predicted and the experimental values. We have previously published results on a number of processes with two to six hadrons in the final state. We report here on several recent measurements of exclusive $e^+e^- \to$ hadrons cross sections.

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[*] Speaker.
[†] On behalf of the *BABAR* Collaboration.

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1. Introduction

The comparison between the experimental and the theory-predicted values of the anomalous magnetic moment of the muon $a_\mu = (g_\mu - 2)/2$ is a powerful test of the Standard Model validity.

To precisely calculate the value of a_μ, quantum fluctuation corrections at all orders, from all knwon interactions, must be considered. These are dominated by the Quantum Electrodynamics (QED) corrections, which are calculated with great precision in perturbation theory, as well as the tiny contribution from Weak-interaction corrections. Instead, the whole uncertainty is essentially produced by the small Quantum Chromodynamics (QCD) corrections, mainly vacuum polarization (VP) with hadrons, as low-energy hadronic processes cannot be calculated in perturbation theory. The leading-order VP contribution to a_μ is however reliably determined via dispersion relations connecting it to the cross section for $e^+e^- \to$ hadrons [1]:

$$a_\mu^{LO,VP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{R(s) \times \hat{K}(s)}{s^2} ds, \tag{1.1}$$

where $R(s)$ is the the cross section of e^+e^- to hadrons at center-of-mass (CM) energy squared s, normalized to the pointlike muon pair cross section, and $\hat{K}(s)$ is a known function that is of order unity on the s range $[(2m_\pi c^2)^2, \infty]$. Technically, the low energy part of the integral is obtained from experimental data (up to a value often chosen to be $E_{CM}^{cut} = 1.8\,\mathrm{GeV}$), while the high-energy part is computed from perturbative QCD. Due to the presence of the s^2 factor at the denominator of the integrand of Eq.(1.1), the precision of the prediction of $a_\mu^{LO,VP}$ relies on the measurements of exclusive hadronic cross sections at the lowest energies. Recent theroretical determinations [2, 3, 4] of a_μ give similar results (*e.g.* $a_\mu^{th} = (1659180.2 \pm 4.9)10^{-10}$, with $a_\mu^{LO,VP} = (692.3 \pm 4.2)10^{-10}$ [2]).

The experimentally measured value a_μ [5] exceeds the theoretical one by more than three standard deviations, with $\Delta a_\mu = a_\mu^{epx} - a_\mu^{th} = (28.7 \pm 8.0) \cdot 10^{-10}$, possibly hinting at new physics. The direct measurement and the theory calculation of a_μ have presently similar precisions, but new and more precise experimental determinations are expected within a few years from experiments at Fermilab and J-PARC. Therefore, more precise measurements of the hadronic cross sections would be desirable to have a similar improvement on the theoretical side.

2. Cross section measurements with the ISR method

The *BABAR* Collaboration [6, 7], at the SLAC National Accelerator Laboratory, has an intensive program to study exclusive e^+e^- cross sections at low energies making use of the initial-state radiation (ISR) process [8], with e^+e^- collisions at a nominal CM energy near the peak of the $\Upsilon(4S)$ resonance, that is $\sqrt{s} \sim 10.6\,\mathrm{GeV}$. In this experimental approach, the $e^+e^- \to f$ cross section is deduced from a measurement of the radiative process $e^+e^- \to f + \gamma_{ISR}$, where the photon is emitted by the e^+or the $e^-$with a CM energy $E_\gamma^* = x\sqrt{s}/2$, and f can be any final state produced at the reduced CM energy squared $E_{CM}^2 = s(1-x)$:

$$\frac{d\sigma_{e^+e^-\to f\gamma_{ISR}}}{dm}(s,m) = \frac{2m}{s} W(s,m)\sigma_{e^+e^-\to f}(m)\,, \tag{2.1}$$

where $m = E_{CM}$ is experimentally obtained from the reconstructed mass of the hadronic system. The so-called radiator function $W(s,m)$, is the probability density to radiate a photon with a fraction x of the beam energy and is known with a precision better than 1%.

In contrast with the energy scans that provided most of the earlier experimental information on the variations of the ratio R (see [9] for a summary and reference therein), the ISR method offers significant advantages, in particular it allows the measurement of the exclusive cross sections from the production threshold up to more than 4 GeV, in the same detector and accelerator conditions, as the data are taken simultaneously at all CM energies E_{CM} of the hadronic final state. In addition, for these analyses the detection of the ISR photon is required and the hadronic system is boosted in the opposite direction. As a consequence, the reconstruction efficiency is sizable down to the production threshold.

By using the ISR method *BABAR* has measured for the first time the cross section of many final states, and improved the precision of the contribution to a_μ of most of the relevant channels by a large factor, typically close to a factor of three. We report in the following the *BABAR* most recent results.

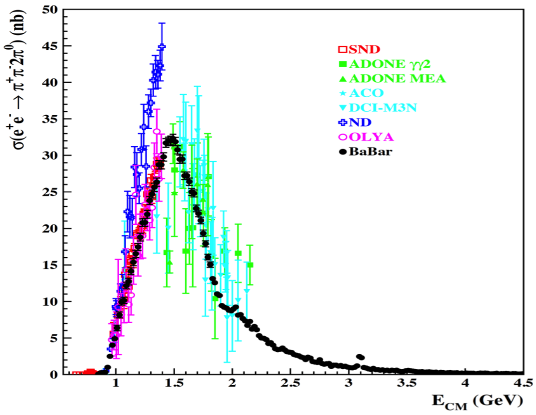
3. $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$

The $\pi^+\pi^-\pi^0\pi^0$ final state has been previosuly measured with poor precision, resulting in the main contribution to the $a_\mu^{LO,VP}$ uncertainty in the energy region between 1 and 2 GeV. A more precise measurement of the cross section is therefore particularly important.

The event selection makes use of the good solid angle coverage of the detector and the well known beam energies. The event is fully reconstructed requiring two tracks and at least five photons detected. The highest-energy photon is assumed to be the ISR photon and is required to have an energy of at least 3 GeV, while the other photons must be compatible with being produced in π^0 decays. The candidate events are subjected to a six-constraints kinematic fit (with constraints from energy-momentum conservation and π^0 masses), under the hypothesis $e^+e^- \to \gamma_{ISR}\pi^+\pi^-\pi^0\pi^0$ with $\pi^0 \to \gamma\gamma$ for both π^0's. The χ^2 of the fit is used for selecting the signal events. The main background comes from the ISR process $e^+e^- \to \gamma_{ISR}\pi^+\pi^-3\pi^0$, whose estimate has been obtained with a dedicated study.

The resulting cross section for $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ [10] is shown in Fig. 1(left), together with previous data. The systematic uncertainties, not reported in the plot, are about 3% in the peak region and $\sim 6-7\%$ above 2.7 GeV. *BABAR* is the only experiment able to cover the mass spectrum from 0.85 GeV (it results compatible with zero below this value) up to 4.5 GeV, and its data are by far the most precise, providing a major improving in the determination of the contribution to the hadronic cross section from this process. In the energy range between 0.85 and 1.8 GeV, by using only *BABAR* data, it results $a_\mu^{\pi^+\pi^-\pi^0\pi^0} = (17.9 \pm 0.1_{stat} \pm 0.6_{syst}) \cdot 10^{-10}$, where the first error is statistical and the second systematic, improving the uncertainty by more than a factor 2.5 wth respect to the combination of all previous data.

The internal structure of this process has also been investigated, studying the production of intermediate resonances. Clear signals of $\omega \to \pi^+\pi^-\pi^0$ and of correlated production of $\rho^0 f_0(980) \to (\pi^+\pi^-)(\pi^0\pi^0)$ and $\rho^+\rho^- \to (\pi^+\pi^0)(\pi^-\pi^0)$ are observed. As an example, the cross section measured for the process $e^+e^- \to \omega\pi^0$, with $\omega \to \pi^+\pi^-\pi^0$ is shown in Fig. 1(right).



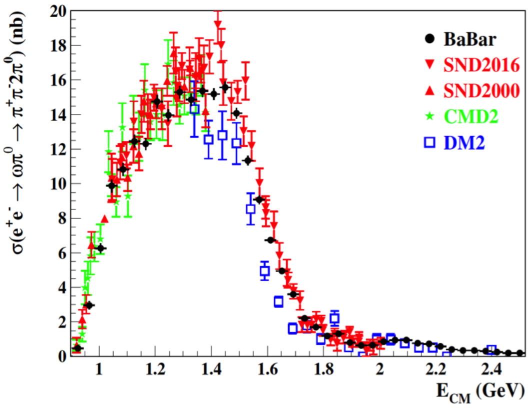


Figure 1: (left) The $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ cross section as a function of CM energy measured by *BaBar* (black dots), compared to previously published data [11]. (right) The cross section for $e^+e^- \to \omega\pi^0 \to (\pi^+\pi^-\pi^0)\pi^0$ as a function of CM energy, measured by *BaBar* (black dots), compared with previous experiments. Data measured in other decays than $\omega \to \pi^+\pi^-\pi^0$ are scaled by the appropriate branching ratios.

4. $e^+e^- \to \pi^+\pi^-\eta$

The final state is reconstructed requiring $\eta \to \gamma\gamma$ decays, and applying a selection strategy analogous to the four-pion case. The resulting cross section is shown in Fig.2 [12]. The results of this measurement are compared to previous data, including a *BaBar* measurement obtained in 2007 by using about half of the data set and reconstructing $\eta \to \pi^+\pi^-\pi^0$ decays [13]. The distribution of $m(\pi^+\pi^-)$ shows a pronounced $\rho^0(770)$ peak, indicating that this final state proceeds dominantly through $e^+e^- \to \rho\eta$. A slight shift to higher masses is observed which may indicate a $\rho(770)-\rho(1450)$ interference. Vector meson dominance is tested via fits to the E_{CM} distribution, using four models based on the production of ρ-like resonances. The model with $\rho(770)$, $\rho(1450)$, $\rho(1700)$ and $\rho(2150)$ fits well the data up to 2.2 GeV.

5. $e^+e^- \to KK\pi\pi$ final states

BaBar has recently measured the final states with two neutral kaons: $K^0_S K^0_L \pi^0$, $K^0_S K^0_L \pi^0\pi^0$, and $K^0_S K^0_L \eta$ [16], and the final states $K^0_S K^+\pi^-\pi^0$ and $K^0_S K^+\pi^-\eta$ [17]. These measurements, together with previous measurements of processes with charged kaons and/or pions, complete the program of measuring the e^+e^- hadronic cross sections for all possible $KK\pi\pi$ and $KK\pi$ final states. Therefore, the total cross sections for $e^+e^- \to KK\pi\pi$ and $e^+e^- \to KK\pi$ can be calculated without relying on isospin symmetry, significantly reducing the uncertainties on the contribution to a_μ from these channels. This is particularly true for the $KK\pi\pi$ final states for which a value of $a_\mu^{KK\pi\pi} = (0.85 \pm 0.05) \cdot 10^{-10}$ is obtained to be compared with the previous $(1.35 \pm 0.39) \cdot 10^{-10}$.

The $e^+e^- \to K^0_S K^+\pi^-\pi^0$cross section, reported in Fig. 3(left) is measured for the first time. The systematic uncertainties amount to $\sim 6-7\%$ below 2 GeV. The intermediate-state composition is very reach, showing a dominant contribution from $K^*(892)K\pi$, but also large fractions of $K^0_S K^+\rho(770)$ and $K^{*+}(892)K^{*-}(892)$. A significantly smaller cross section is measured for the

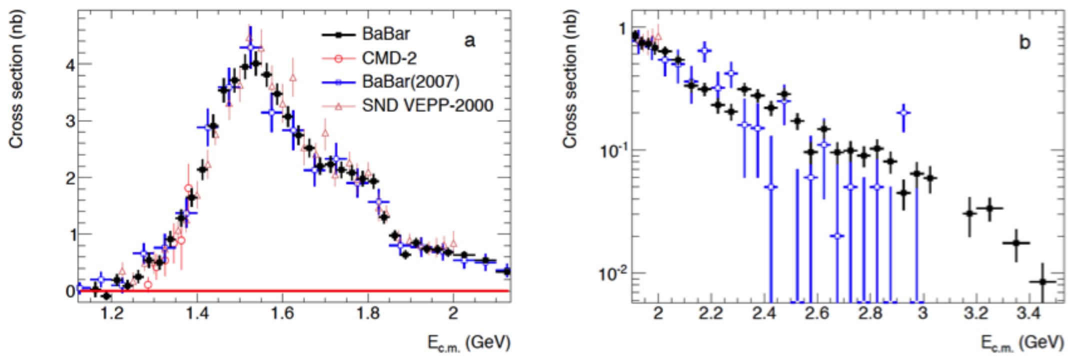


Figure 2: The $e^+e^- \to \pi^+\pi^-\eta$ cross section as a function of CM energy measured by *BABAR* (black dots), by using $\eta \to \gamma\gamma$ decays, compared to previously published data [14, 15, 13]. *BABAR*(2007) indicates the cross section data obtained by using $\eta \to \pi^+\pi^-\pi^0$ decays. The main contributions are below 2.2 GeV(left plot), while we also measured the tails up to 3.5 GeV, shown in a logarithmic scale (rght).

first time for $e^+e^- \to K^0_S K^0_L \pi^0\pi^0$, as shown in Fig. 3(right). The dominat contribution is from the $K^*(892)K\pi$ intermediate state.

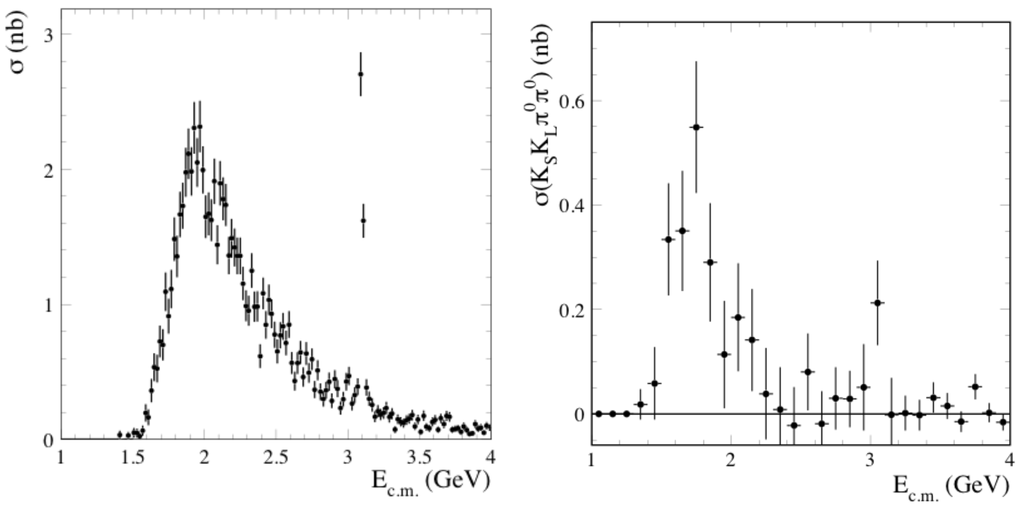


Figure 3: The cross section for the $KK\pi\pi$ final states $K^0_S K^+\pi^-\pi^0$ and $K^0_S K^0_L \pi^0\pi^0$ as a function of CM energy, measured by *BABAR*.

The cross sections for the other channels $e^+e^- \to K^0_S K^0_L \pi^0$, $e^+e^- \to K^0_S K^0_L \eta$, and $e^+e^- \to K^0_S K^+\pi^-\eta$ are also measured for the first time and are reported in Fig.4.

6. Conlcusions

The recent measurement performed by *BABAR* on exclusive hadronic cross sections provides a significant improvement in the precision of the hadronic contribution to a_μ. In particular the uncertainty on the contribution of $e^+e^- \to \pi^+\pi^-\pi^0\pi^0$ is reduced from 7% to about 3%. All $KK\pi\pi$

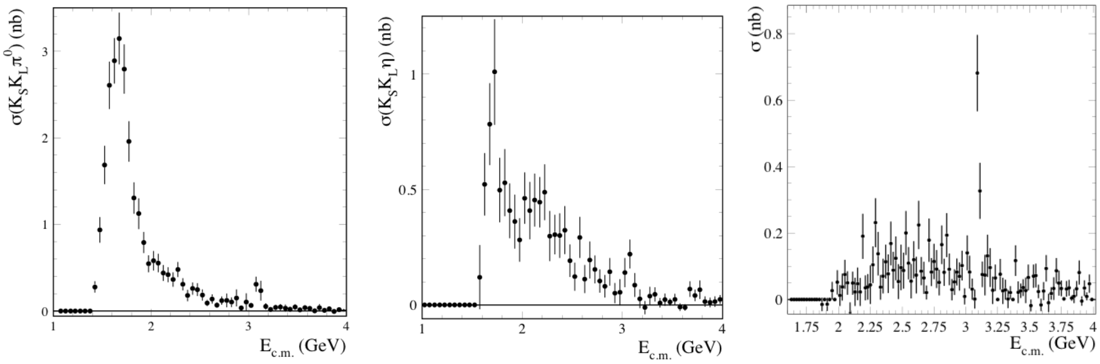


Figure 4: The cross section for the final states $K_S^0 K_L^0 \pi^0$, $K_S^0 K_L^0 \eta$ and $K_S^0 K^+ \pi^- \eta$ as a function of CM energy measured by BABAR.

and $KK\pi$ final states have been now measured and the corresponding contributions can be summed without any assumption on isospin symmetry.

New cross section measurements are expected from several experiments in the near future. These should further reduce the uncertainty on the theoretical determination of a_μ^{th}, while waiting for the new improved direct measurements of a_μ by the g-2 and E34 experiments at Fermilab and J-PARC, respectively.

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