Production and mixing of scalar mesons in η_c and χ_{c1} decays

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We briefly discuss how the chiral unitary approach in coupled channels and $SU(3)$ symmetry can be used to describe the production of $f_0(500)$, $f_0(980)$ and $a_0(980)$ in the $\chi_{c1} \to \eta \pi^+ \pi^-$ reaction, recently measured by the BESIII collaboration. In this reaction a very strong peak for the $a_0(980)$ can be seen in the $\eta \pi$ invariant mass, while clear signals for the $f_0(500)$ and $f_0(980)$ appear in the one of $\pi^+ \pi^-$. Next, we show the predictions made with the same model for the analogous decay $\eta_c \to \eta \pi^+ \pi^-$, which could also be measured experimentally. We discuss the differences of these two reactions which are interesting to test the picture where these scalar mesons are dynamically generated from the interaction of pairs of pseudoscalars. Furthermore, we comment on a new recent work where the same model was used to study the $a_0(980) - f_0(980)$ mixing in the $\chi_{c1} \to \pi^0 \pi^0 \eta$ and $\chi_{c1} \to \pi^0 \pi^+ \pi^-$ reactions, showing that quantitative agreement with the experimental measurement of this mixing, also performed by BESIII, can be obtained, revealing interesting aspects of the dynamics of this process and the importance of coupled channels.

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1. Introduction

The experiment on the $\chi_{c1} \to \eta \pi^+ \pi^-$ decay performed with high statistics by the BESIII collaboration [1], and previously by the CLEO collaboration [2], presents an interesting opportunity to test the picture where the scalar mesons $f_0(500)$, $f_0(980)$ and $a_0(980)$ are dynamically generated from the final state interaction of meson pairs $\pi^+\pi^-$ and $\eta\pi^\pm$. Indeed, it is found that the most dominant two-body structure comes from $a_0(980)^\pm \pi^\mp$, with $a_0(980)^\pm \to \eta \pi^\pm$.

First we will briefly discuss the work of Refs. [3, 4] where the chiral unitary approach and $SU(3)$ symmetry were used to describe the production of these three scalars in the BESIII experiment and to make predictions for the analogous reaction with η_c instead of χ_{c1}. We will make a short discussion on $SU(3)$ scalars and compare the treatment of the amplitude and mass distribution used to describe each decay. In the end we also comment on the recent work of Ref. [5], where the same model was used to study the $a_0(980) - f_0(980)$ mixing in the $\chi_{c1} \to \pi^0 \pi^0 \eta$ and $\chi_{c1} \to \pi^0 \pi^+ \pi^-$ reactions, which was suggested in Ref. [6] and later measured by BESIII [7].

2. Common Formalism

As in Ref. [8] we start by considering that the charmonium states $c\bar{c}$ behave as a $SU(3)$ scalar, and use the following ϕ matrix to get the weight of every trio of pseudoscalar mesons created in the χ_{c1} or η_c decay

$$
\phi \equiv \begin{pmatrix}
\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\
K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta'
\end{pmatrix}. \tag{2.1}
$$

If we think of ϕ as a $q\bar{q}$ matrix, as discussed in Ref. [3], it is natural to build a $SU(3)$ scalar by taking $SU(3)[\text{scalar}] \equiv \text{Trace}(\phi\phi\phi)$, where

$$
\text{Trace}(\phi\phi\phi) = 2\sqrt{3}\eta\pi^+\pi^- + \sqrt{3}\eta\pi^0\pi^0 + \frac{\sqrt{3}}{9}\eta\eta\eta + 3\pi^+ K^0 K^- + 3\pi^- K^+ \bar{K}^0,
$$

where we have neglected the terms that cannot make a transition to the final state $\eta\pi^+\pi^-$, and also the terms containing η', which plays only a marginal role in the building of the $f_0(500)$, $f_0(980)$, $a_0(980)$ resonances, because of its large mass and small couplings.

In fact, there are four $SU(3)$ scalars: $\text{Trace}(\phi\phi\phi)$, $\text{Trace}(\phi)\text{Trace}(\phi\phi)$, $[\text{Trace}(\phi)]^3$ and $\text{Det}(\phi)$. But by the Cayley-Hamilton relation,

$$
2\text{Trace}(\phi\phi\phi) - 6\text{Det}(\phi) - 3\text{Trace}(\phi)\text{Trace}(\phi\phi) + [\text{Trace}(\phi)]^3 = 0, \tag{2.2}
$$

only three of them are independent. In Refs. [4, 8] we discussed other possibilities and concluded that the best choice is indeed $\text{Trace}(\phi\phi\phi)$, since it yields results in good agrement with the recent experiment of BESIII [1] on the $\chi_{c1} \to \eta\pi^+\pi^-$ decay. Indeed, in Ref. [5] we have also added that this is in fact expected from large N_c counting, since each time one takes a trace a factor $1/N_c$ is introduced [9, 10]. Besides, if one does not include the η_1 — which we do through the inclusion of $\eta - \eta'$ mixing, in order to relate the ϕ matrix with the $q\bar{q}$ matrix [3] — but instead take $\eta \to \eta_8$ and no η', then $\text{Trace}(\phi) = 0$ and we are left only with the structure $\text{Trace}(\phi\phi\phi)$.

Next, we use the chiral unitary approach to describe how the scalar mesons are dynamically generated from the interaction of pairs of pseudoscalars in coupled channels. We follow the framework of Ref. [11], using an effective chiral Lagrangian where mesons are the degrees of freedom

$$\mathscr{L}_2 = \frac{1}{12 f_\pi^2} \text{Trace}[\, (\partial_\mu \phi \, \phi - \phi \, \partial_\mu \phi)^2 + M \phi^4 \,], \tag{2.3}$$

where ϕ is the matrix in Eq. (2.1), f_π is pion decay constant and

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}. \tag{2.4}$$

From this Lagrangian we extract the kernel of each channel, which in charge basis are: 1) $\pi^+\pi^-$, 2) $\pi^0\pi^0$, 3) K^+K^-, 4) $K^0\bar{K}^0$, 5) $\eta\eta$, 6) $\pi^0\eta$ and can be found in Refs. [12, 13]. These kernels are used to build the V matrix which is then inserted into the Bethe-Salpeter equation, summing the contribution of every meson-meson loop.

$$T = (1 - VG)^{-1} V, \tag{2.5}$$

where G is the meson-meson loop function, which we regularize with a cutoff using $q_{max} \sim 600$ MeV. After the integration in q^0 and $\cos\theta$ we have

$$G = \int_0^{q_{max}} \frac{q^2 dq}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1 \omega_2 [(P^0)^2 - (\omega_1 + \omega_2) + i\varepsilon]}, \tag{2.6}$$

with $\omega_i = \sqrt{q^2 + m_i^2}$, $P^0 = s$. Each kernel is projected in S-wave and a normalization factor is included when identical particles are present, which later needs to be restored. Finally, the T matrix will give us the scattering and transition amplitudes between each channel, and isospin symmetry is used to obtain the amplitude of channels with different charges [3].

3. Theoretical description of $\chi_{c1} \to \eta \pi^+ \pi^-$

Following the assumption that $c\bar{c}$ behaves as a $SU(3)$ scalar, we look at the quantum numbers of the initial and final states, combining them in two cases: η leaves in P-wave while $\pi^+\pi^-$ go through final state interaction with $I = 0$ to form the $f_0(500)$ and $f_0(980)$ in S-wave; and π^- (or π^+) leaves in P-wave while $\eta\pi^+$ (or $\eta\pi^-$) go through final state interaction with $I = 1$ to form the $a_0^\pm(980)$ in S-wave.

To illustrate our method, we will describe the case where η leaves in P-wave and $\pi^+\pi^-$ interact. In this case we will consider the diagrams of Fig. 1. Then from the $SU(3)$ scalar in Eq. (2.2), we select the terms in which we can isolate one η and let the other pairs rescatter, since our coupled channels approach allows them to make a transition to $\pi^+\pi^-$ final state,

$$\eta \left(2\sqrt{3}\pi^+\pi^- + \sqrt{3}\pi^0\pi^0 + \frac{\sqrt{3}}{9}\eta\eta \right). \tag{3.1}$$
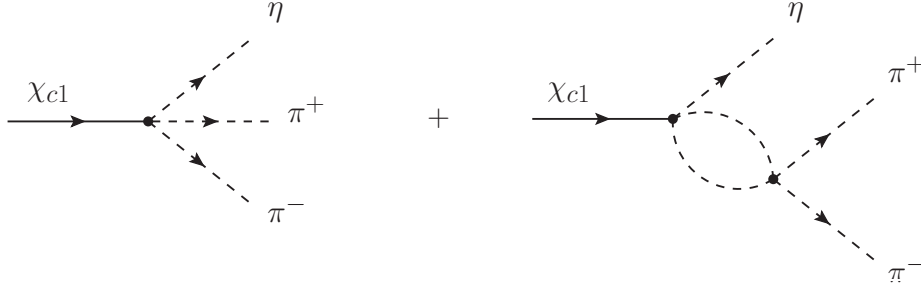
Figure 1: Diagrams considered in the description of $f_0(500)$ and $f_0(980)$ production in $\chi_{c1} \to \eta\pi^+\pi^-$ reaction: tree-level (left) and rescattering of $\pi^+\pi^-$ pair (right).

Then we will have the sum of tree-level and rescattering:

$$t_\eta = V_P\left(\vec{\varepsilon}_{\chi_{c1}} \cdot \vec{p}_\eta\right)\left(h_{\pi^+\pi^-} + \sum_i h_i S_i G_i[M_{\text{inv}}(\pi^+\pi^-)]t_{i,\pi^+\pi^-}[M_{\text{inv}}(\pi^+\pi^-)]\right), \qquad (3.2)$$

where h_i are the weights of Eq. (3.1), S_i are symmetry and combination factors for the identical particles and V_P provides a global normalization factor, which is adjusted to the data in the $a_0(980)$ peak. Finally, we can write the differential mass distribution for $\pi^+\pi^-$

$$\frac{d\Gamma}{dM_{\text{inv}}(\pi^+\pi^-)} = \frac{1}{(2\pi)^3}\frac{1}{4M_{\chi_{c1}}^2}\frac{1}{3}p_\eta^2 p_\eta \tilde{p}_\pi |t_\eta|^2, \qquad (3.3)$$

where p_η is the η momentum in the χ_{c1} rest frame and $\tilde{p}_\pi$ is the pion momentum in the $\pi^+\pi^-$ rest frame. Using this simple picture one can obtain a fair agreement with the experimental data of Ref. [1], as shown in Ref. [3] and further discussed in Refs. [4, 8].

4. Predictions for $\eta_c \to \eta\pi^+\pi^-$

In the analogous reaction $\eta_c \to \eta\pi^+\pi^-$ the dominant structure will be the one where every final state meson goes out in *S*-wave. Therefore one must consider the interference between each term in the amplitude, then

$$t = t_{tree} + t_\eta + t_{\pi^+} + t_{\pi^-}, \qquad t_{tree} = V_P h_{\eta\pi^+\pi^-}. \qquad (4.1)$$

Each of the later three terms is a function of an invariant mass, analogous to Eq. (3.2). We select $M_{\text{inv}}(\pi^+\pi^-)$ and $M_{\text{inv}}(\pi^+\eta)$ as variables and the third one is determined by the relation: $M_{13}^2 = M_{\eta_c}^2 + 2m_\pi^2 + m_\eta^2 - M_{12}^2 - M_{23}^2$. It is also necessary to consider the double differential mass distribution [14]

$$\frac{d^2\Gamma}{dM_{\text{inv}}(\pi^+\pi^-)dM_{\text{inv}}(\pi^+\eta)} = \frac{1}{(2\pi)^3}\frac{1}{8M_{\eta_c}^3}M_{\text{inv}}(\pi^+\pi^-)M_{\text{inv}}(\pi^+\eta)|t|^2, \qquad (4.2)$$

where we need to integrate in one of the invariant masses to get the distribution of the other one. This way the background of $\pi^+\eta$ appears naturally in the $\pi^+\pi^-$ mass distribution and vice-versa.

Since our approach is valid only for energies up to 1.2 GeV, we need to introduce a cut in each amplitude to perform the integration. To do that we evaluate $Gt(M_{inv})$ combinations up to $M_{inv} = M_{cut}$. From there on, we multiply Gt by a smooth factor to make it gradually decrease at large M_{inv},

$$Gt(M_{inv}) = Gt(M_{cut})e^{-\alpha(M_{inv}-M_{cut})}, \quad \text{for} \ M_{inv} > M_{cut}. \tag{4.3}$$
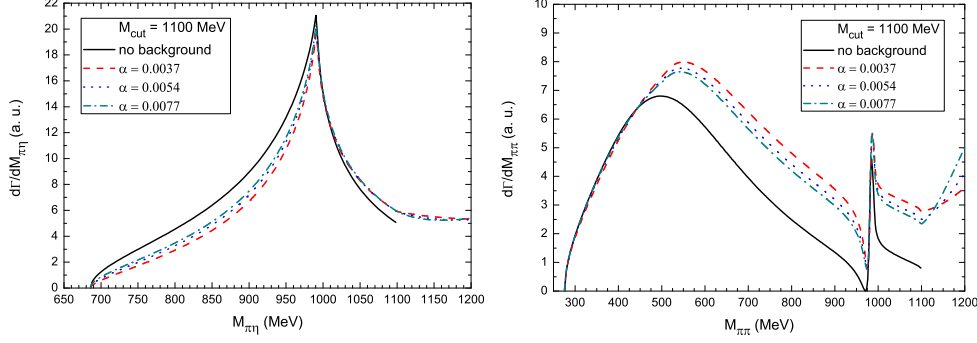
Figure 2: Predictions from Ref. [4] for the mass distribution of $\pi\eta$ (left) and $\pi^+\pi^-$ (right) in $\eta_c \to \eta\pi^+\pi^-$, using $M_{cut} = 1100$ MeV and $\alpha = 0.0037, 0.0054, 0.0077$ MeV^{-1}, which reduce Gt by a factor 3, 5 and 10, respectively, at $M_{cut} + 300$ MeV. The "no background" curve is obtained by keeping only the tree-level and the main rescattering amplitude.

In Fig. 2 we show the predictions for the production of $f_0(500)$, $f_0(980)$ and $a_0(980)$ in the $\eta_c \to \eta\pi^+\pi^-$ decay. To see the effect of the background and interference introduced by considering all the amplitudes in S-wave, we show with the solid curves, denoted by "no background", the results obtained by keeping only the tree-level and the main rescattering amplitude $t_{\pi^-}[M_{inv}(\pi^+\eta)]$ in the case of $a_0(980)$ and $t_\eta[M_{inv}(\pi^+\pi^-)]$ in the case of the $f_0(500)$ and $f_0(980)$.

5. $a_0(980) - f_0(980)$ **mixing in** $\chi_{c1} \to \pi^0\pi^0\eta(\pi^+\pi^-)$

This same model was recently used to study the $a_0(980) - f_0(980)$ mixing in the $\chi_{c1} \to \pi^0\pi^0\eta$ and $\chi_{c1} \to \pi^0\pi^+\pi^-$ reactions in Ref. [5], showing that quantitative agreement with the experimental measurement of this mixing, also performed by BESIII [7], can be obtained, revealing interesting aspects of the dynamics of this process and the importance of the coupled channels approach. It was shown that the neutral $a_0^0(980)$ can be produced in the isospin-allowed mode $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$ while the isospin-violating production of $f_0(980)$ can be seen in the $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$ mode, where the proximity of both scalar resonances to the $K\bar{K}$ threshold, and the fact that both couple to the $K\bar{K}$ channel is responsible for the mixing.

The difference in the mass of the charged and neutral kaons is the dominant cause of the isospin violation. The $f_0(980)$ production appears between the thresholds of $K^0\bar{K}^0$ and K^+K^-, and there are two important process, $K\bar{K} \to \pi^+\pi^-$ and $\pi^0\eta \to \pi^+\pi^-$, where the latter one appears due to the coupled channels approach, and both sum up constructively. This latter one is possible only when different masses for the kaons are also considered in the propagators that go inside the Bethe-Salpeter equation (2.5), and it was also found that the agreement with the experimental measurement of the mixing is much better when this is included.

Acknowledgments

We would like to thank N. Kaiser for information concerning $SU(3)$ invariants and Feng-Kun Guo for the information about the $1/N_c$ factor. V. R. Debastiani wishes to acknowledge the organizers of the event and the support from the Programa Santiago Grisolia of Generalitat Valenciana (GRISOLIA/2015/005). E. Oset wishes to acknowledge the support from the Chinese Academy of Science in the Program of Visiting Professorship for Senior International Scientists (Grant No. 2013T2J0012). This work is partly supported by the National Natural Science Foundation of China under Grants No. 11565007, No. 11547307 and No. 11475227. It is also supported by the Youth Innovation Promotion Association CAS (No. 2016367). This work is also partly supported by the Spanish Ministerio de Economia y Competitividad and European FEDER funds under the contract number FIS2014-57026-REDT, FIS2014-51948-C2-1-P and FIS2014-51948-C2-2-P, and the Generalitat Valenciana in the program Prometeo II-2014/068.

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