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Heavy meson potential from a modified Schwinger-Dyson strong coupling.

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We build a heavy quark-antiquark potential from a running coupling whose non perturbative momentum dependence comes from a dynamically generated gluon effective mass as the one obtained from a solution of the Schwinger-Dyson equations for QCD. The resulting potential has only one free parameter, Λ, the scale parameter in QCD. An excellent fit of the bottomonium (and charmonium) spectrum below the open flavor meson-meson thresholds is attained.

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1. Introduction

The Cornell potential [1, 2]

$$V^C(r) = \sigma r - \frac{\zeta}{r}$$

where σ and ζ are constants has been very successful in explaining heavy quarkonia (bottomonium and charmonium) spectra below the open flavor meson-meson threshold energies.

The coulombic term $-\frac{\zeta}{r}$ can be justified from perturbative QCD by approaching the heavy quark-antiquark static potential by the One Gluon Exchange (OGE) interaction. This OGE reads in momentum space (henceforth we shall use the static three momentum squared $\overrightarrow{q}^2$ instead of the four momentum squared Q^2)

$$V^P\left(\overrightarrow{q}^2\right) = -\frac{4}{3}\frac{\alpha^P\left(\overrightarrow{q}^2\right)}{\overrightarrow{q}^2}$$

where $\alpha^P\left(\overrightarrow{q}^2\right)$ is the perturbative QCD running coupling whose expression is known. To 1-loop it reads

$$\alpha^P\left(\overrightarrow{q}^2\right) = \frac{4\pi}{\beta_0 \ln\left(\frac{\overrightarrow{q}^2}{\Lambda^2}\right)}$$

with $\beta_0 = 11 - 2/3\ n_f$ being the first β-function coefficient for QCD, n_f the number of active quarks and Λ the scale parameter in QCD.

Then, by substituting $\alpha^P\left(\overrightarrow{q}^2\right)$ by an average constant value appropriate for the spectral study of heavy quarkonia one obtains through Fourier transform the coulombic term $-\frac{\zeta}{r}$.

Regarding the confining term σr Richardson proposed in the late 70's [3] a static potential form

$$V^R\left(\overrightarrow{q}^2\right) = -\frac{4}{3}\frac{\alpha^R\left(\overrightarrow{q}^2\right)}{\overrightarrow{q}^2}$$

with the running coupling ansatz

$$\alpha^R\left(\overrightarrow{q}^2\right) = \frac{4\pi}{\beta_0 \ln\left(1 + \frac{\overrightarrow{q}^2}{\Lambda^2}\right)}$$

so that $\alpha^R\left(\overrightarrow{q}^2\right) \to \alpha^P\left(\overrightarrow{q}^2\right)$ in the asymptotic limit $\left(\overrightarrow{q}^2 \gg \Lambda^2\right)$. By taking the Fourier transform he obtained a confining (plus coulombic) potential

$$V^R(r) = \frac{8\pi}{3\beta_0}\left(\Lambda^2 r - \frac{f(\Lambda r)}{r}\right)$$

that provided a good fit to the heavy quarkonia spectra.

The Richardson ansatz found years later some justification from non perturbative QCD (for a recent review of the possible definitions of the non perturbative coupling see [4]). Using the Schwinger-Dyson equations for QCD, Cornwall [5] obtained a non perturbative quenched (no light quark-antiquark pairs) solution for the gluon propagator in terms of a dynamically generated gluon effective mass which was momentum dependent and vanished at large momenta. This mass $m_g(\overrightarrow{q}^2)$ has been parametrized as [6]

$$m_g^2(\overrightarrow{q}^2) = \frac{m_0^2}{1+(\overrightarrow{q}^2/\mathscr{M}^2)^{1+p}}$$

with $m_0 = m_g(\overrightarrow{q}^2 = 0)$, $\mathscr{M}$ and $p > 0$ constants.

Then, from the gluon self-energy the following form for the running coupling was derived (see for example [7])

$$\alpha_s^{(\mathrm{SD})}(\overrightarrow{q}^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{\overrightarrow{q}^2+\rho m_g^2(\overrightarrow{q}^2)}{\Lambda^2}\right)}$$

where ρ is a constant.

It is then easy to check that substituting the gluon effective mass by a constant average

$$m_g^2(\overrightarrow{q}^2) = \frac{\Lambda^2}{\rho}$$

one gets the Richardson ansatz.

(Let us add for the sake of completeness that the consideration of the OGE interaction with the coupling $\alpha_s^{(\mathrm{SD})}(\overrightarrow{q}^2)$ and the non perturbative gluon propagator instead of $\frac{1}{\overrightarrow{q}^2}$ does not give rise to a linearly rising potential [8].)

2. Improved quark-antiquark potential

The Richardson potential lacks the detailed $\overrightarrow{q}^2$ gluon mass dependence. In order to incorporate this dependence in the definition of the potential one may impose, instead of the constant average mass at all $\overrightarrow{q}^2$, the condition [9]

$$m_g^2(0) = \frac{\Lambda^2}{\rho}$$

so that

$$\rho m_g^2(\overrightarrow{q}^2) = \frac{\Lambda^2}{1+(\overrightarrow{q}^2/\mathscr{M}^2)^{1+p}}$$

and

$$\alpha_s(\overrightarrow{q}^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{1}{1+(\overrightarrow{q}^2/\mathscr{M}^2)^{1+p}} + \frac{\overrightarrow{q}^2)}{\Lambda^2}\right)}$$

This running has the correct asymptotic $\left(\overrightarrow{q}^2 \gg \Lambda^2\right)$ behavior $\alpha_s(\overrightarrow{q}^2) \to \alpha^P\left(\overrightarrow{q}^2\right)$ whereas for $\overrightarrow{q}^2 \to 0$ it reproduces the Richardson ansatz $\alpha_s(\overrightarrow{q}^2) \to \alpha^R\left(\overrightarrow{q}^2\right)$. In this manner we implement at intermediate $\overrightarrow{q}^2$ the gluon mass dependence coming out from the quenched solution of the Schwinger-Dyson equations. More precisely the potential in momentum space reads

$$V\left(\overrightarrow{q}^2\right) = -\frac{4}{3}\frac{\alpha_s\left(\overrightarrow{q}^2\right)}{\overrightarrow{q}^2}$$

whose Fourier transform is

$$
\begin{aligned}
& -\frac{8}{3\pi}\int_0^\infty \alpha_s(\overrightarrow{q}^2)\frac{\sin(|\overrightarrow{q}|r)}{|\overrightarrow{q}|r}d|\overrightarrow{q}| \\
& = -\frac{32}{3\beta_0}\int_0^\infty \frac{1}{\ln\left(\frac{\overrightarrow{q}^2+\frac{\Lambda^2}{1+(\overrightarrow{q}^2/\mathscr{M}^2)^{1+p}}}{\Lambda^2}\right)}\frac{\sin(|\overrightarrow{q}|r)}{|\overrightarrow{q}|r}d|\overrightarrow{q}|
\end{aligned}
$$

This integral diverges for $|\overrightarrow{q}|\to 0$. For it to represent a potential it has to be regularized. The physical argument underlying the regularization procedure is that the origin of the potential is arbitrary so that the physics does not depend on its choice. From the technical point of view we introduce a regulator γ to write the integral as

$$
-\frac{32}{3\beta_0}\lim_{\gamma\to 0}\int_\gamma^\infty \frac{1}{\ln\left(\frac{\overrightarrow{q}^2+\frac{\Lambda^2}{1+(\overrightarrow{q}^2/\mathscr{M}^2)^{1+p}}}{\Lambda^2}\right)}\frac{\sin(|\overrightarrow{q}|r)}{|\overrightarrow{q}|r}d|\overrightarrow{q}|
$$

and we expand the integrand around $|\overrightarrow{q}|\to 0$ to identify after integration the terms giving rise to the divergent behavior when $\gamma\to 0$. It turns out that these terms (we group them as $I_s(\gamma)$) do not depend on r, say they are constants that can be removed by changing the origin of the potential. Then, we subtract them to have a perfectly well defined quenched potential

$$
V(r) = -\frac{32}{3\beta_0}\lim_{\gamma\to 0}\left(\int_\gamma^\infty \frac{1}{\ln\left(\frac{q^2+\frac{\Lambda^2}{1+(q^2/\mathscr{M}^2)^{1+p}}}{\Lambda^2}\right)}\frac{\sin(qr)}{qr}dq - I_s(\gamma)\right)
$$

which is evaluated numerically.

3. Results

The resulting potential [9] depends on three parameters $\mathscr{M}$, p and Λ. As for the first two we keep them fixed to their Schwinger-Dyson values $\mathscr{M}=436$ MeV and $p=0.15$ [7]. In order to fix Λ, the only free parameter, we require $V(r)$ to provide a reasonable description of the heavy quarkonia (bottomonium and charmonium) spectra (for bottomonium we use $n_f=4$ and for charmonium $n_f=3$; to calculate the spectrum we solve the Schrödinger equation). For any value of Λ, we choose the quark masses, m_b and m_c, to get the best spectral fit. In this regard, as $V(r)$ represents a quenched potential we restrict the comparison with data to energies below the corresponding open flavor meson-meson thresholds.

It turns out that only for a quite restricted range of values of Λ around 320 MeV a quite good spectral description for bottomonium and charmonium is obtained. The corresponding potentials for $\Lambda = 320$ MeV are drawn in Fig. 1.

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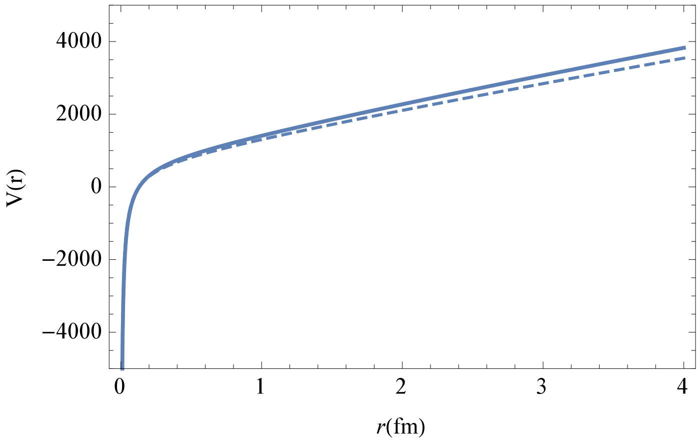

Figure 1: Bottomonium (solid) and charmonium (dashed) potentials for $\Lambda = 320$ MeV.

J^{PC}	nl	$M_{V(r)_{\Lambda=320\mathrm{MeV}}}$ MeV	M_{PDG} MeV
1^{--}	$1s$	9489	9460.30 ± 0.26
	$2s$	10023	10023.26 ± 0.31
	$1d$	10147	10163.7 ± 1.4
	$3s$	10354	10355.2 ± 0.5
	$2d$	10435	
	$4s$	10621	10579.4 ± 1.2
	$3d$	10681	
	$5s$	10854	$10889^{+3.2}_{-2.6}$
	$4d$	10903	
$(0,1,2)^{++}$	$1p$	9903	$9899.87 \pm 0.28 \pm 0.31$
	$2p$	10254	$10260.24 \pm 0.24 \pm 0.50$
	$3p$	10531	10512.1 ± 2.3

Table 1: Calculated J^{PC} bottomonium masses from $V(r)_{\Lambda=320\mathrm{MeV}}$ and $m_b = 4450$ MeV. Masses for experimental resonances, M_{PDG}, have been taken from [10]. For $1p$ and $2p$ states the experimental centroids are quoted. For $3p$ states the only known experimental mass is listed.

In Table 1 we list the calculated masses for bottomonium for $\Lambda = 320$ MeV as compared to data (for the same comparison in charmonium see [9]). To denote the sates we use the spectroscopic notation nl, in terms of the radial, n, and orbital angular momentum, l, quantum numbers of the quark-antiquark system. As we are dealing with a spin independent potential we compare as usual the calculated $s-$ wave state masses with spin-triplet data, the $p-$ wave state masses with the centroids obtained from data and the $d-$ wave states with the few existing experimental candidates.

As can be checked a very good spectral description is attained (notice that some of the differences between the calculated values and data come from the fact that no mixing between the s and d states has been taken into account).

4. Summary

A heavy quark-antiquark potential from a modified Schwinger-Dyson strong coupling has been built. More precisely the non perturbative momentum dependence associated to the dynamic generation of a gluon effective mass in the Schwinger-Dyson approach to QCD has been implemented. The potential has been successfully applied to the calculation of the quenched bottomonium (and charmonium) spectrum. It is worth to emphasize that this potential is very much like a Cornell potential between 0.1 and 4 fm. This provides some justification from QCD to the phenomenological success of the Cornell potential

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