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chiral model for the $D^+ \to K^+K^-K^+$ decay amplitude

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The *isobar model*, widely used for describing heavy-meson decays into three pseudoscalars, assumes that these processes are dominated by spectator-resonance intermediate states, but also includes non-resonant contributions. It yields guess functions which include free phases, to be obtained by fitting data displayed in Dalitz plots. However, the question arises as to whether the parameters obtained in the case of a successful fit have meanings which go beyond the reproduction of a particular set of experimental data. In particular, can they be used to shed light into yet unknown two-body processes, especially those concerning kaons? Can they yield reliable information about scattering amplitudes? As we argue in the sequence, the answers to these questions do not favour the isobar model. This happens because, on general grounds, *there is no simple connection between a heavy-meson decay amplitude T and two-body scattering amplitudes A, involving the same particles*. Reasons read:

a. dynamics - The dynamical contents of T and A are rather different, since the former includes weak vertices, which cannot be present in the latter. Therefore, although scattering amplitudes A can be substructures of T, there is no reason whatsoever for assuming that these A's are either identical or proportional to T. This is supported by case studies, such as that regarding the process $D^+ \to K^-\pi^+\pi^+$, which explained the difference between observed S-wave decay and scattering phases by describing the weak sector of the problem in terms of meson loops[1].

b. isospin: - Scattering amplitudes A depend both on the *angular momentum J* and on the *isospin I* of the channel considered, whereas just a J dependence can be extracted from an experimental decay amplitude T. Thus, the latter is always, at best, a linear combination of different isospin states, of the form $T^{[J]} = H_1 A_1^{[J,I_1]} + H_2 A_2^{[J,I_2]} + \cdots$, where the weights H_k are energy dependent functions determined by the weak vertex. It is impossible to derive directly $A^{[J,I]}$ from $T^{[J]}$ simply because the former contains more structure than the latter.

c. non-resonant term - The non-resonant term may involve proper three-body interactions and also tends to blur scattering information contained in the sub amplitudes $T^{[J]}$.

d. unitarity - Good fits to Dalitz plots data usually require several resonances with the *same quantum numbers*. The isobar model describes each of them by means of a line shape (Breit-Wigner), but sums of line shapes violate unitarity, even in the case of scattering amplitudes[2].

e. coupled channels - The simple guess functions provided by the isobar model do not incorporate properly the couplings of intermediate states. For instance, $K\bar{K}$ intermediate states do contribute to elastic $\pi\pi$ scattering, as discussed by Hyams et al.[3] and guess functions better suited for accommodating data should have structures similar to eq.(11) of that paper.

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1. lagrangian approach

Although the point of departure of the isobar model is sound, all the problems mentioned in the abstract tend to corrode the physical meaning of parameters obtained from fits based on it. As a consequence, one cannot rely on them for obtaining scattering information from decay data.

The most conservative way of ensuring that the meaning of parameters does not change from process to process is to rely on lagrangians. At present, intermediate energy reactions cannot be described by the fundamental QCD lagrangian, written in terms of gluons and quarks, and one is forced to emlploy effective theories. Here, we depart from chiral perturbation theory (ChPT), which is ideally suited for describing low-energy interactions of pseudoscalar mesons. In this effective theory[4] [5], amplitudes are systematically expanded in terms of polynomials, involving both kinematical variables and quark masses, which describe multi-meson contact interactions. As resonances are important, we use an extension of chiral perturbation theory, known as (ChPTR), in which they are explicitly included [6]. In this framework, leading order (LO) contributions correspond to multi-meson contact interactions, whereas resonance exchanges are next-to-leading order (NLO). In principle, a heavy-meson decay into three pseudoscalars should be described by a properly unitarized three-body amplitude. However, deriving it is a task too involved and, following the usual practice, we work in the so called $(2+1)$ approximation, in which two-body unitarized amplitudes, coupled to spectator particles, are used to extend the range of validity of the theory[7]. If needed, the explicit inclusion of heavy mesons[8] is also possible.

In a lagrangian approach, the only free parameters are masses and coupling constants, whereas widths are not fundamental but, rather, constructed quantities. In case the free parameters of a lagrangian model can be extracted from fits to data, the derivation of a whole family of scattering amplitudes becomes straightforward.

2. model for $D^+ \to K^+K^-K^+$

In order to illustrate the lagrangian approach, we consider the specific reaction $D^+ \to K^+K^-K^+$, which is doubly-Cabibbo-suppressed. The main features of the discussion are, nevertheless, quite general. Our model is based on the assumption that the quarks c and $\bar{s}$ in the D^+ annihilate into a W^+, which subsequently hadronizes in the vacuum, as described in fig.1.

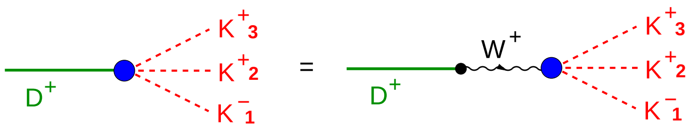

Figure 1: The decay $D^+ \to K^- K^+ K^+$ (left) is assumed to proceed thought the intermediate steps $D^+ \to W^+$ and $W^+ \to K^- K^+ K^+$ (right); throughout the paper, the label 1 refers to the K^-.

This *decay* amplitude is given by

$$T = -\left[\frac{G_F}{\sqrt{2}} \sin^2 \theta_C\right] \langle K^-(p_1) K^+(p_2) K^+(p_3)|A^\mu|0\rangle \langle 0|A_\mu|D^+(P)\rangle \,, \tag{2.1}$$

where G_F is the Fermi decay constant, θ_C is the Cabibbo angle, the A^μ are axial currents and $P = p_1 + p_2 + p_3$. Denoting the D^+ decay constant by F_D, we write $\langle 0|A_\mu|D^+(P)\rangle = -i\sqrt{2}F_D P_\mu$ and find a *decay* amplitude proportional to the divergence of the remaining axial current, given by

$$T = i\left[\frac{G_F}{\sqrt{2}}\sin^2\theta_C\right]\sqrt{2}F_D\;[P_\mu\,\langle A^\mu\rangle]\;, \tag{2.2}$$

with $\langle A^\mu\rangle = \langle K^-(p_1)K^+(p_2)K^+(p_3)|A^\mu|0\rangle$.

The dynamics of the vertex $\langle A^\mu\rangle$ is displayed in fig.2 and diagrams are evaluated using the techniques described in Refs.[5, 6]. In chiral perturbation theory, the primary couplings of the W^+ to the $K^-K^+K^+$ system always involve a direct interaction, accompanied by a kaon-pole term, denoted by (A) and (B) in the figure. Only their joint contribution is compatible with the implementation of chiral symmetry. Diagrams (1A+1B) are LO and describe a non-resonant term, a proper three body interaction, which goes beyond the $(2+1)$ approximation, whereas Figs. (2A+2B) allow for the possibility that two of the mesons rescatter, after being produced in the primary weak vertex.

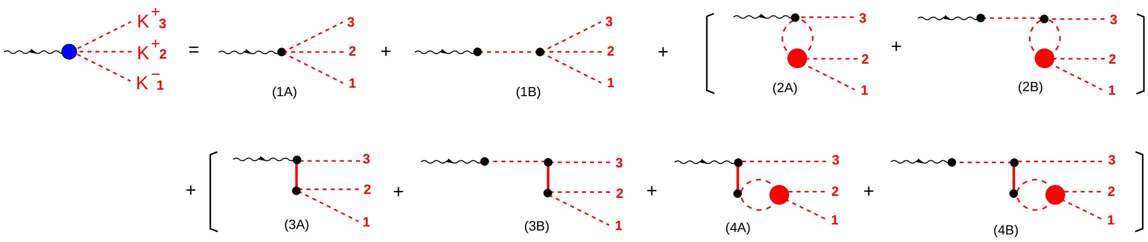

Figure 2: Dynamical structure of the blue blob in fig.1; the wavy line is the W^+, dashed lines are mesons, continuous lines are resonances and the full red blob represent meson-meson scattering amplitudes, described in fig.3; all diagrams within square brackets should be symmetrized, by making $2 \leftrightarrow 3$.

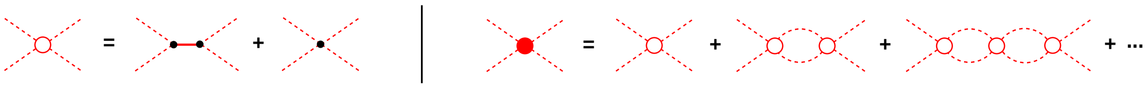

Figure 3: Left: tree-level structure for the two-body interaction kernel $\mathscr{K}_{ab\to cd}$: a NLO s-channel resonance, added to a LO contact term. Right: unitarized sacattering amplitude.

The structure of the meson-meson *scattering* amplitude A is described in fig.3, as a LO four point interaction, typical of chiral symmetry, supplemented by NLO resonance exchanges in the s-channel. The latter are real functions and contain bare poles. However, the ressummation of the geometric series, indicated on right figure, unitarizes the scattering amplitude and endows the s-channel resonance with a width. Accordingly, diagrams (3A+3B) in fig.2 are NLO and describe the *production* of bare resonances at the weak vertex, whereas final state rescattering processes (4A+4B) endow them with widths. Therefore, the dynamical content of families (3A+3B+4A+4B) is somehow similar to that of the isobar model.

At this point, it is worth noting that figs.2 and 3 do clearly support statement **a. dynamics**, in the abstract. The former describes the *decay* amplitude T and includes the weak vertex, whereas the latter represents A, the *scattering* amplitude. Although A is a substructure of T, the relationship between both amplitudes is by no means straightforward.

In these diagrams, all *vertices* represent interactions derived from chiral lagrangians, that are real functions. The imaginary parts of the amplitudes come from two-meson propagators which, in general, involve both real and imaginary components. The former contain divergent contributions and their regularization brings unknown parameters into the problem. In order to avoid this considerable nuisance, we work in the K-matrix approximation, whereby just the imaginary parts of the two-meson propagators are kept.

We consider just S- and P- waves in the two-body K^+K^- subsystem, corresponding to $J = 1,0$ and isospin $I = 1,0$ channels. The associated resonances are the ρ, the ϕ, the a_0 and the two scalar-isoscalar states allowed by $SU(3)$ ChPTR. They are denoted by S_1 and S_o and correspond, respectively, to a singlet and to a member of an octet, with the same quantum numbers of the vacuum. The physical $f_0(980)$ and $f_0(1370)$ can, then, be linear combinations of S_1 and S_o. The intermediate two-meson propagators shown in Figs.2 and 3 can involve $\pi\pi$, KK, $\eta\eta$ and $\pi\eta$ intermediate systems and hence there is a large number of **coupled channels** to be considered.

The *decay* amplitude for the process $D^+ \to K^-K^+K^+$, given by eq.(2.2), has the general structure

$$T = T_{NR} + \left[T^{(1,1)} + T^{(1,0)}\right] + \left[T^{(0,1)} + T^{(0,0)}\right] \,, \tag{2.3}$$

where T_{NR} is the non-resonant contribution from diagrams (1A+1B) of fig.2 and the $T^{(J,I)}$ are the resonant contributions from diagrams (3A+3B+4A+4B), in the various spin and isospin channels. This generic expression also applies to the isobar model and it is important to stress that there, even in favourable situations, just the linear combinations with well defined angular momentum, within square brackets, can be extracted from fits, whereas isospin information remains concealed.

Owing to the use of chiral symmetry, all amplitudes are proportional to M_K^2, included in a common factor $C = \{[(G_F/\sqrt{2})\,\sin^2\theta_C]\;(2F_D/F)\,[M_K^2/(M_D^2 - M_K^2)]\}$, where F is the $SU(3)$ pseudoscalar decay constant. Below we display, as examples, just T_{NR} and $T^{(0,1)}$, the amplitude in the a_0 channel, but similar results hold for the other components. Using the kinematical variables $m_{ij}^2 = (p_i - p_j)^2$, the non-resonant contribution is the real polynomial

$$T_{NR} = C\,\left\{\left[(m_{12}^2 - M_K^2) + (m_{13}^2 - M_K^2)\right]\right\} \,, \tag{2.4}$$

and corresponds to a proper three-body interaction. The amplitude $T^{(0,1)}$ reads

$$T^{(0,1)} = \frac{\Gamma_{a_0}}{D_{a_0}} \equiv -\frac{1}{2}\,\frac{\left[M_{21}^{(0,1)}\,\Gamma_{(0)\,\pi 8}^{(0,1)} + \left(1 - M_{11}^{(0,1)}\right)\Gamma_{(0)\,KK}^{(0,1)}\right]}{\left[\left(1 - M_{11}^{(0,1)}\right)\left(1 - M_{22}^{(0,1)}\right) - M_{12}^{(0,1)} M_{21}^{(0,1)}\right]} + (2 \leftrightarrow 3)\,, \tag{2.5}$$

where the functions $\Gamma_{(0)\,ab}^{(0,1)}$ describe the tree-level *weak decay process* $W^+ \to a\,b\,K^+$ and the matrices $M_{jk}^{(0,1)}$ implement the *coupling of different channels*, by means of strong final state *scattering* interactions. These functions are given by

$$\Gamma_{(0)\,\pi 8}^{(0,1)} = C\left[\frac{2\sqrt{2}\left[-c_d\,P\cdot p_3 + c_m M_D^2\right]\left[c_d\,\left(m_{12}^2 - M_\pi^2 - M_8^2\right) + 2\,c_m M_\pi^2\right]}{\sqrt{3}F^2\;\;[m_{12}^2 - m_{a_0}^2]} - \frac{\sqrt{3}\,\left[M_D^2/3 - P\cdot p_3\right]}{\sqrt{2}}\right],$$

$$\Gamma_{(0)\,KK}^{(0,1)} = C\left[\frac{2\left[-c_d\,P\cdot p_3 + c_m M_D^2\right]\left[c_d\,\left(m_{12}^2 - 2M_K^2\right) + 2\,c_m M_K^2\right]}{F^2\;\;[m_{12}^2 - m_{a_0}^2]} - \frac{\left[M_D^2 - P\cdot p_3\right]}{2}\right], \tag{2.6}$$

with the coupling constants c_d and c_m defined in Ref.[5]. The matrices M_{jk} read

$$M_{11}^{(0,1)} = -\mathscr{K}_{\pi 8|\pi 8}^{(0,1)}\,[\bar{\Omega}_{\pi 8}^{S}]\,, \qquad M_{12}^{(0,1)} = -\mathscr{K}_{\pi 8|KK}^{(0,1)}\,[\bar{\Omega}_{KK}^{S}/2]\,,$$

$$M_{21}^{(0,1)} = -\mathscr{K}_{KK|\pi 8}^{(0,1)}\,[\bar{\Omega}_{\pi 8}^{S}]\,, \qquad M_{22}^{(0,1)} = -\mathscr{K}_{KK|KK}^{(0,1)}\,[\bar{\Omega}_{KK}^{S}/2]\,, \tag{2.7}$$

where the imaginary functions $\bar{\Omega}_{ab}^{S}$ represent two-meson propagators, given by

$$\bar{\Omega}_{ab}^{S} = -\frac{i}{8\pi}\,\frac{Q_{ab}}{\sqrt{s}}\,\theta(s-(M_a+M_b)^2)\,, \qquad Q_{ab} = \frac{1}{2}\sqrt{s-2\,(M_a^2+M_b^2)+(M_a^2-M_b^2)^2/s}\,. \tag{2.8}$$

and the $\mathscr{K}_{(cd|ab)}^{(0,1)}$ are interaction kernels, which read

$$\mathscr{K}_{(\pi 8|\pi 8)}^{(0,1)} = -\left[\frac{4}{3F^4}\right]\frac{\left[c_d\,(s-M_\pi^2-M_8^2)+c_m\,2M_\pi^2\right]^2}{s-m_{a_0}^2} + \left[\frac{2M_\pi^2}{3F^2}\right]\,,$$

$$\mathscr{K}_{(\pi 8|KK)}^{(0,1)} = -\left[\frac{2\sqrt{2}}{\sqrt{3}F^4}\right]\frac{\left[c_d\,(s-M_\pi^2-M_8^2)+c_m\,2M_\pi^2\right]\,\left[c_d\,s-(c_d-c_m)\,2M_K^2\right]}{s-m_{a_0}^2} + \left[\frac{(3s-4M_K^2)}{\sqrt{6}F^2}\right]\,,$$

$$\mathscr{K}_{(KK|KK)}^{(0,1)} = -\left[\frac{2}{F^4}\right]\frac{\left[c_d\,s-(c_d-c_m)\,2M_K^2\right]^2}{s-m_{a_0}^2} + \left[\frac{s}{2F^2}\right]\,. \tag{2.9}$$

These expressions can already be used as guess functions. Nevertheless, it is useful to rewrite the denominator of eq.(2.5), just to show that it resembles the shape lines used in the isobar model. Neglecting the polynomial contributions in eqs.(2.9), the results displayed above yield the typical form

$$D_{a_0}(s) = \left[(s-m_{a_0}^2)+i\,m_{a_0}\,\Gamma_{a_0}(s)\right]/(s-m_{a_0}^2) \tag{2.10}$$

$$m_{a_0}\Gamma_{a_0}(s) = \frac{1}{8\pi F^4\sqrt{s}}\Big\{(4/3)\,\left[c_d\,(s-M_\pi^2-M_8^2)+2\,c_m M_\pi^2\right]^2 Q_{\pi 8} + \left[c_d\,(s-2M_K^2)+2\,c_m M_K^2\right]^2 Q_{KK}\Big\}\,. \tag{2.11}$$

3. scattering amplitudes

The same structures allow the construction of all *scattering* amplitudes $A_{cd|ab}^{(J,I)}$. For instance, that describing elastic $K\bar{K}$ scattering in the a_0 channel reads

$$A_{KK|KK}^{(0,1)} = \frac{N_{KK|KK}^{(0,1)}}{D_{a_0}} = \frac{\left[M_{21}^{(0,1)}\,\mathscr{K}_{\pi 8|KK}^{(0,1)} + \left(1-M_{11}^{(0,1)}\right)\mathscr{K}_{KK|KK}^{(0,1)}\right]}{\left[\left(1-M_{11}^{(0,1)}\right)\left(1-M_{22}^{(0,1)}\right)-M_{12}^{(0,1)}M_{21}^{(0,1)}\right]}\,. \tag{3.1}$$

In this way, the use a a lagrangian model allows free parameters, masses and coupling constants, extracted from heavy meson *decay* data to be used in the prediction of *scattering* amplitudes.

The *decay* and *scattering* amplitudes, eqs.(2.5) and (3.1), share the same denominator and therefore can be related directly

$$T^{(0,1)} = H^{(0,1)}_{KK|KK} \times A^{(0,1)}_{KK|KK}\,,$$
$$H^{(1,0)}_{KK|KK} = -\frac{1}{2}\,\frac{\left[M^{(0,1)}_{21}\,\Gamma^{(0,1)}_{(0)\,\pi 8} + \left(1 - M^{(0,1)}_{11}\right)\Gamma^{(0,1)}_{(0)\,KK}\right]}{\left[M^{(0,1)}_{21}\,\mathscr{K}^{(0,1)}_{\pi 8|KK} + \left(1 - M^{(0,1)}_{11}\right)\mathscr{K}^{(0,1)}_{KK|KK}\right]}\,. \tag{3.2}$$

As just a J dependence can be extracted from a Dalitz plot, an empirical *decay* amplitude T can, at most, yield $T^{[J]} = H_1 A_1^{[J,1]} + H_0 A_0^{[J,0]}$, a linear combination of *scattering* amplitudes with different isospins ans weight functions H_k.

4. summary

Isobar models, successful as they are in providing fits for heavy meson decays, rely on parameters which are not physically transparent. As an alternative, we presented a Multi-Meson-Model (Triple-M), applied to the $D^+ \to K^+K^-K^+$ amplitude. It is based on lagrangians from chiral perturbation theory with resonances (ChPTR), and main features read:
1. it incorporates resonances and extends the isobar model;
2. it includes a non-resonant contribution, a consequence of chiral symmetry, which is a real three-body interaction, fully given by theory;
3. all imaginary terms in the amplitude are completely determined by unitarity and no free complex parameters are employed;
4. all free parameters represent either meson masses or coupling constants and, therefore, have a rather transparent physical meaning;
5. parameters obtained from fits yield predictions for meson-meson *scattering* amplitudes.

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