PROCEEDINGS OF SCIENCE

The glueball and meson spectrum, the meson weak decay constants and the strong effective coupling with the analytic (infrared) confinement

Gurjav Ganbold*
Bogoliubov Laboratory of Theoretical Physics, JINR, Joliot-Curie 6, 141980 Dubna, Russia;
Institute of Physics and Technology, Enkh Taivan ave. 54b, 13330 Ulaanbaatar, Mongolia
E-mail: ganbold@theor.jinr.ru

The phenomena of effective strong coupling and hadron mass generating, the properties of two-particle bound states have been studied in the framework of a QCD-inspired relativistic model of quark-gluon interaction with infrared confined propagators. The spectra of quark-antiquark and two-gluon stable states are defined by master equations similar to the ladder Bethe-Salpeter equation. We derived a meson mass equation and revealed a specific new behaviour of the mass-dependent strong coupling $\hat{\alpha}_s(M)$ defined in the time-like region. A new infrared freezing point $\hat{\alpha}_s(0) = 1.03198$ at origin has been found and it did not depend on the confinement scale $\Lambda > 0$. Independent and new estimates on the scalar glueball mass (we found it around 1739 MeV), 'radius' and gluon condensate value have been performed. The spectrum of conventional mesons have been calculated by introducing a minimal set of parameters: the masses of constituent quarks and Λ. The obtained values are in good agreement with the latest experimental data with relative errors less than 1.8 percent. Accurate estimates of the leptonic decay constants of pseudoscalar and vector mesons have been performed.

XVII International Conference on Hadron Spectroscopy and Structure - Hadron2017
25-29 September, 2017
University of Salamanca, Salamanca, Spain

*Speaker.

PoS(Hadron2017)036

1. Introduction

QCD predicts a dependence of the physical coupling g under changes of energy scale Q (or, distance $\sim 1/Q$). This dependence $\alpha_s(Q) \doteq g^2/(4\pi)$ is described theoretically by the renormalization group equations and determined experimentally at relatively high energies $Q > 1$ GeV [1]. Meanwhile, understanding of a number of phenomena such as quark confinement, hadronization etc., requires a correct description of hadron dynamics in the infrared (IR) region below $Q \leq 1$ GeV. Particularly, many quantities in particle physics are affected by the IR behavior of α_s. However, the long-distance behavior of α_s has not been well defined yet, it needs to be more specified. The correct description of the effective strong coupling in the IR regime remains one of the actual problems in particle physics.

2. Model

Let's consider a QCD-inspired relativistic field model with Lagrangian [2]:

$$\mathscr{L} = -\frac{1}{4}\left(\partial^\mu \mathscr{A}^A_\nu - \partial^\nu \mathscr{A}^A_\mu - g f^{ABC}\mathscr{A}^B_\mu \mathscr{A}^C_\nu\right)^2 + \left(\bar{q}^a_f\left[\gamma_\alpha \partial^\alpha - m_f\right]^{ab} q^b_f\right) + g\left(\bar{q}^a_f\left[\Gamma^\alpha_C \mathscr{A}^C_\alpha\right]^{ab} q^b_f\right), \tag{2.1}$$

where $\mathscr{A}^C_\alpha$ is the gluon and q^a_f is a quark field of flavor f with mass $m_f = \{m_{ud}, m_s, m_c, m_b\}$ and $\Gamma^\alpha_C = i\gamma_\alpha t^C$. For the spectra of quark-antiquark and di-gluon bound states we solve Bethe-Salpeter type equations obtained in [3, 4]. By omitting intermediate stages of calculation (see for details [5]) we rewrite the master equation determining the meson mass as follows:

$$1 = \alpha_s \cdot \lambda_{JJ}(M_J^2, m_1, m_2) = \alpha_s \frac{16\pi C_J}{9}\int \frac{d^4k}{(2\pi)^4} \iint dxdy\, e^{-ik(x-y)}\, U_{\mathscr{N}}(x)\sqrt{D(x)D(y)}\,U_{\mathscr{N}}(y)$$
$$\cdot \mathrm{Tr}\left[O_J \tilde{S}_{m_1}(\hat{k} + \xi_1 \hat{p})\, O_J \tilde{S}_{m_2}(\hat{k} - \xi_2 \hat{p})\right]\Big|_{-p^2 = M_J^2}, \tag{2.2}$$

where $C_J = \{1, 1, 1/2, -1/2\}$, $\xi_i = m_i/(m_1 + m_2)$, $O_J = \{I, i\gamma_5, i\gamma_\mu, \gamma_5\gamma_\mu\}$ and the gluon ($D(x)$) and quark propagator ($\tilde{S}_{m_1}(\hat{p})$) are represented in Euclidean space.

Note, the polarization kernel $\lambda_{JJ'}(-p^2)$ has to be diagonalized on a complete system of orthonormal functions $\{U_{\mathscr{N}}\}$, where $\mathscr{N} = \{n, l, \mu, ...\}$ is a set of quantum numbers. The solution of Eq.(2.2) is nothing else but the solution of the corresponding ladder Bethe-Salpeter equation.

Ultraviolet divergences in the model have been removed by renormalization of wave function and charge, but infrared singularities remain in Eq.(2.2) because of integration over variable k. To avoid the appearance of the singularities in the mass formula, we follow theoretical predictions in favor of an IR-finite behavior of the gluon propagator [6, 7] and introduce a scheme of infrared cutoffs on the limits of scale integrations for the propagators as follows:

$$D(x) = \frac{1}{4\pi^2 x^2} \to \int\limits_{\Lambda/2}^{\infty} ds\, e^{-sx^2}, \quad \tilde{S}_{m_f}(\hat{p}) = \frac{1}{-i\hat{p} + m_f} \to (i\hat{p} + m_f)\int\limits_0^{1/\Lambda} dt\, e^{-t(p^2 + m_f^2)}, \tag{2.3}$$

where Λ is the mass scale of the IR confinement domain. These propagators are entire analytic functions in the Euclidean space. Note, another type of IR confinement applied to whole 'quark-antiquark' loop was applied in [8, 9]. The analytic confinement disappears as $\Lambda \to 0$.

3. Effective Strong Coupling in the IR Region

The meson mass M_J is defined by Eq.(2.2) at given α_s, quantum numbers $\mathcal{N}$ and constituent quark masses $\{m_1, m_2\}$. And vice versa, α_s can be estimated for given masses. The QCD coupling may feature an IR-finite behavior (e.g., in [10, 11]). To study this, we choose M_J as an appropriate energy-scale for α_s and consider a special case $m_1 = m_2 \sim M/2$ in the ground-state. A new effective (mass-dependent) strong coupling $\alpha_s(M)$ in time-like domain may be defined by:

$$\alpha_s(M) = 1/\lambda(M, M/2, M/2) . \tag{3.1}$$

The polarization kernel λ_{JJ} in Eq. (2.2) is natively obtained real and symmetric that allows us to find a simple variational solution to this problem. A new variational upper bound $\alpha(M)$ to $\alpha_s(M)$ is shown in Fig.1. The slope of $\alpha(M/\Lambda)$ depends on $\Lambda > 0$, but the newly revealed origin $\alpha(0) = 1.032$ (or, $\alpha(0)/\pi = 0.328$) remains unchanged. This IR-fixed value $\alpha(0)$ is in a reasonable agreement with often quoted estimates [12]:

$$\alpha_s^0/\pi \simeq 0.19 \div 0.25 \quad [13], \qquad \alpha_s^0/\pi \simeq 0.265 \quad [14], \qquad \alpha_s^0/\pi \simeq 0.26 \quad [15].$$

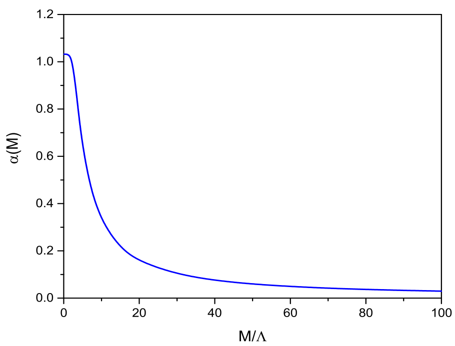

Figure 1: Dependence of the effective strong coupling α_s on relative mass scale M/Λ.

4. Lowest Glueball State

The existence of glueballs, the bound states of gluons, is predicted by QCD because of the self-interaction of gluons. Most known experimental signatures for glueballs are an enhanced production in gluon-rich channels of radiative decays and some decay branching fractions incompatible with $(q\bar{q})$ states. There are predictions expecting glueball-like states in the mass range $M_G \sim 1.5 \div 5.0$ GeV with spin $J = 0, 1, 2, 3$ [16]. Below we consider a two-gluon scalar bound state with $J^{PC} = 0^{++}$. By omitting details of intermediate calculations (similar to those represented in the previous section) we define the scalar glueball mass $M_{0^{++}}$ from equation:

$$1 - \frac{8\,\alpha}{3\pi} \int dz\, e^{izp}\, \Pi_G(z) = 0, \qquad p^2 = -M_{0^{++}}^2 , \tag{4.1}$$

where $\Pi_G(z)$ is the self-energy (polarization) function of the scalar glueball. Particularly, for $\Lambda = 236$ MeV and $\alpha(M_G)$ defined from Eq.(3.1) we obtain new estimates:

$$M_{0^{++}} = 1739 \text{ MeV}, \qquad \alpha(M_{0^{++}}) = 0.451 . \tag{4.2}$$

The new value of $M_{0^{++}}$ is in reasonable agreement with other predictions [2, 16, 17, 18, 19].

We also estimate the scalar glueball 'radius' (r_G) which leads to a new value:

$$r_{0^{++}} \cdot M_{0^{++}} = 4.41 \tag{4.3}$$

that is in reasonable agreement with data in [16].

5. Meson Spectrum

The dependence of meson mass on α_s and other model parameters is defined by Eq.(2.2). Particularly, for $\Lambda = 236\,MeV$ and $\alpha(M)$ defined in Eq.(3.1) we derive variationally meson mass formula Eq.(2.2) by fitting the conventional meson masses with adjustable constituent quark masses $\{m_{ud}, m_s, m_c, m_b\}$. We have fixed a new final set of model parameters (in units of MeV) as follows:

$$\Lambda = 236\,, \quad m_{ud} = 227.6\,, \quad m_s = 420.1\,, \quad m_c = 1521.6\,, \quad m_b = 4757.2\,. \tag{5.1}$$

We represent our new estimates on the pseudoscalar (P) and vector (V) meson masses in Tab.1.

Table 1: Estimated masses of conventional mesons compared to the recent experimental data [16].

$J^{PC} = 0^{-+}$	M_P (in MeV)	Data	$J^{PC} = 1^{--}$	M_V (in MeV)	Data
D	1893.6	1869.62	ρ	774.3	775.26
D_s	2003.7	1968.50	K^*	892.9	891.66
η_c	3032.5	2983.70	D^*	2003.8	2010.29
B	5215.2	5259.26	D_s^*	2084.1	2112.3
B_s	5323.6	5366.77	J/Ψ	3077.6	3096.92
B_c	6297.0	6274.5	B^*	5261.5	5325.2
η_b	9512.5	9398.0	Υ	9526.4	9460.30

Our estimates fit the latest experimental data with relative errors less than 1.8 per cent.

6. Leptonic Decay Constants

The leptonic decay constants f_J are important quantities in meson physics. The precise knowledge of their values provides more improvement in our understanding of various processes convolving meson decays.

To describe effectively the 'sawtooth'-type unsmooth dependence of f_J on meson masses (see Tab.3), we introduce additional parameters R_J characterizing the meson 'size' in units of mass scale as follows: $\tilde{U}_R(k) = \int_0^1 ds\, h(s) \exp\left[-sk^2/R_J^2\right]$, where $h(s)$ is a smooth function. Then, by using our model parameters in Eq.(5.1) we find the optimal meson 'sizes' and estimate new results on the leptonic decay constants of conventional mesons shown in Tab. 2.

Table 2: Estimated 'size' parameters R_J (in GeV) and leptonic decay constants f_J of conventional mesons (in MeV) compared to experimental data in [20, 21, 22, 23].

0^{-+}	R_P	f_P	Data	Ref.	1^{--}	R_V	f_V	Data	Ref.
D	0.93	207	206.7 ± 8.9	[20]	ρ	0.33	221	221 ± 1	[20]
D_s	1.08	257	257.5 ± 6.1	[20]	K^*	0.38	217	217 ± 7	[20]
η_c	1.83	238	238 ± 8	[22]	D^*	0.78	245	245 ± 20	[23]
B	1.73	193	192.8 ± 9.9	[21]	D_s^*	0.90	271	272 ± 26	[23]
B_s	2.18	239	238.8 ± 9.5	[21]	J/Ψ	2.40	416	415 ± 7	[20]
B_c	3.34	488	489 ± 5	[22]	B^*	3.34	196	196 ± 44	[23]
η_b	3.80	800	801 ± 9	[22]	Υ	2.80	715	715 ± 5	[20]

References

[1] S. Bethke, Eur. Phys. J. C **64**, 689 (2009).

[2] G. Ganbold, Phys. Rev. D **79**, 034034 (2009).

[3] G. Ganbold, Phys. Rev. D **81**, 094008 (2010).

[4] G. Ganbold, Phys. Part. Nucl. **45**, 10 (2014).

[5] G. Ganbold, Phys. Part. Nucl. **43**, 79 (2012).

[6] C.S. Fischer , R. Alkofer and H. Reinhardt, Phys. Rev. D **65**, 094008 (2002).

[7] C. Lerche and L. von Smekal, Phys. Rev. D **65**, 125006 (2002).

[8] G. Ganbold *et al.*, J. Phys.: Nucl. Part. Phys. G **42**, 075002 (2015).

[9] G. Ganbold, Eur. Phys. J.: Web of Conference **138**, 04004 (2017).

[10] A.C. Aguilar, A. Mihara and A.A. Natale, Int. J. Mod. Phys. A **19**, 249 (2004).

[11] S.J. Brodsky and G.F. de Teramond, Phys. Lett. B **582**, 211 (2004).

[12] M. Baldicchi *et al.*, Phys. Rev. D **77**, 034013 (2008).

[13] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[14] T. Zhang and R. Koniuk, Phys. Lett. B **261**, 311 (1991).

[15] F. Halzen, G.I. Krein and A.A. Natale, Phys. Rev. D **47**, 295 (1993).

[16] C. Patrignani *et al.*, Chin. Phys. C **40**, 100001 (2016).

[17] C.J. Morningstar and M. Peardon, Phys. Rev. D **60**, 034509 (1999).

[18] Y. Chen *et al.*, Phys. Rev. D **73**, 014516 (2006).

[19] E. Gregory *et al.*, J. High Energ. Phys. **10**, 170 (2012).

[20] K.A. Olive *et al.*, (Particle Data Group Collaboration) Chin. Phys. C **38**, 090001 (2014).

[21] J. Laiho, E. Lunghi and R.S. Van de Water, Phys. Rev. D **81**, 034503 (2010).

[22] T.-W. Chiu *et al.*, (TWQCD Collaboration) Phys. Lett. B **651**, 171 (2007).

[23] D. Becirevic *et al.*, Phys. Rev. D **60**, 074501 (1999).