

## Lepton asymmetry in $S_3$ extended Standard Model

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Standard model (SM) of electroweak interaction seems to be complete and consistent with almost all the data obtained so far, nevertheless, some deviations in the B sector are observed apart from the neutrino oscillation. It is believed that the SM is not a complete theory as we cannot explain the matter-antimatter asymmetry in our Universe in addition to the fact that the visible Universe contains just 5% of the total energy budget. We consider leptogenesis in a minimal  $S_3$  extended standard model with a Higgs doublet and 3 right handed singlet Majorana neutrinos. We studied the neutrino phenomenology from the flavor structure of the  $S_3$  invariant mass matrix in compatible with the  $1\sigma$  experimental oscillation parameters. We have chosen the out of equilibrium decays of the lightest right handed Majorana neutrino to be in the temperature range less than  $10^8$  GeV, where one flavor approximation isn't valid as all the charged lepton yukawa couplings are in equilibrium. Hence we can distinguish between the  $\tau$  and other lepton flavors. Thereafter, we generate the lepton asymmetry by adding flavor effects coming individually from all the leptons sector. This flavor approximation can generate an appreciable lepton asymmetry which can convert to the baryon asymmetry through sphaleron process and can account for the experimental observation. Imposition of  $S_3$  flavor structure of the yukawa couplings restrict the asymmetry in the  $\tau$  sector to vanish whereas the asymmetries in other leptons sector remain finite.

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## 1. Introduction

With the advent of new technology we are able to understand Physics at smaller distances. The series of spectacular experiments over the past few decades in this area established the framework of the standard model (SM). The final missing link in the SM, the Higgs boson, has already been discovered at the Large Hadron Collider (LHC). Interestingly, another set of experiments found that the mysterious neutrinos do oscillate from one flavor to another and this observation has hinted us the possibility of Physics beyond that of the SM. There are other observations, namely, the Baryon asymmetry of the Universe, the existence of Dark Matter etc. give us enough reason to think that the successful SM is not the whole story but rather may be a part of larger theory which is not yet understood. Therefore, the study of neutrino physics has taken center stage in recent times in particle physics.

The mechanism for the generation of neutrino mass is a current topic of deliberation. From the literature one can infer that the type-I seesaw mechanism looks to be an elegant way to explain the neutrino mass and the lepton asymmetry. Moreover, we have learnt that the lepton asymmetry generated during the early universe should have been converted to baryon asymmetry by the sphaleron process. In this article, we explore the possibility of using the type-I seesaw mechanism with additional right handed (RH) singlet fermion to address the problem at hand. In the thermal leptogenesis the lightest RH neutrino when the temperature is below  $10^{12}$  GeV, the  $\tau$  lepton Yukawa coupling comes to equilibrium and hence within the temperature range of  $10^9 < T < 10^{12}$  GeV, the two flavor approximation is acceptable. Here we have explained the flavor effects below the temperature  $10^9$  GeV, where all the charged lepton yukawas are in equilibrium. The SM is extended with three RH Majorana neutrino and having two Higgs doublets. We studied the neutrino phenomenology in the model satisfying  $1\sigma$  neutrino oscillation parameters and mixing in the Higgs sector. In the rest of the article, we will first introduce the model and using the experimental values of various parameters obtain the allowed parameter space both for Dirac and Majorana sector and the flavored leptogenesis.

## 2. The model framework

The Standard Model with gauge group  $\mathcal{G}_{SM} \equiv SU(2)_L \times U(1)_Y$  is extended with an extra non-abelian discrete flavor symmetry  $S_3$ . In addition to the SM particles, we have added 3 right handed singlet neutrinos and one extra higgs doublet to explain the neutrino phenomenology and leptogenesis. All the particles are considered as the reducible representations of  $S_3$ . The triplet representation of  $S_3$  can be reduced to a doublet and a singlet (i.e.,  $3 = 2 \oplus 1$ ) [1-4]. According to the group representation of  $S_3$ , the inner products of two doublets  $P = (x_1, x_2)$  and  $Q = (y_1, y_2)$  will have a form  $(x_1y_2 + x_2y_1, x_1y_1 - x_2y_2)$ . The trivial singlet will have a form of  $(x_1y_1 + x_2y_2)$ , the non trivial singlet will have a form  $(x_1y_2 - x_2y_1)$  and the tensor product of two doublets will give one doublet, one trivial singlet and one non-trivial singlet [1].

The group charge assignment can be assigned as follows:

Particles	SM-Group	$S_3$
$(L_1, L_2), (Q_1, Q_2)$	2	2
$L_s, Q_s$	2	1
$(E_{1R}, E_{2R}), (Q_{1R}, Q_{2R})$	1	2
$E_{3R}, Q_{3R}$	1	1
$\nu_{1R}, \nu_{2R}$	1	2
$\nu_{sR}$	1	1
$H_1, H_2$	2	2

where  $L, Q$  are left handed leptons and quark doublets respectively,  $E_R$  and  $N_R$  are the right handed charged leptons and right handed majorana neutrinos.

The  $SM \times S_3$  invariant Lagrangian for yukawa interaction in charged and neutral lepton sector is given by

$$\begin{aligned}
\mathcal{L}_{Mass} = & y_1 [\bar{L}_e \tilde{H}_2 N_{1R} + \bar{L}_\mu \tilde{H}_1 N_{1R} + \bar{L}_e \tilde{H}_1 N_{2R} - \bar{L}_\mu \tilde{H}_2 N_{2R}] \\
& + y_3 [\bar{L}_\tau \tilde{H}_1 N_{1R} + \bar{L}_\tau \tilde{H}_2 N_{2R}] + y_2 [\bar{L}_e \tilde{H}_1 N_{3R} + \bar{L}_\mu \tilde{H}_2 N_{3R}] \\
& + y_{11} [\bar{L}_e H_2 E_{1R} + \bar{L}_\mu H_1 E_{1R} + \bar{L}_e H_1 E_{2R} - \bar{L}_\mu H_2 E_{2R}] \\
& + y_{13} [\bar{L}_\tau H_1 E_{1R} + \bar{L}_\tau H_2 E_{2R}] + y_{12} [\bar{L}_e H_1 E_{3R} + \bar{L}_\mu H_2 E_{3R}] \\
& + \frac{1}{2} \bar{N}_R^c M_R N_R + \text{h.c.}
\end{aligned}$$

## 2.1 Neutrino sector

The full mass matrix for neutral leptons is given by in the basis  $\tilde{N} = (\nu_L^c, N_R)^T$  as

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}. \quad (2.1)$$

One can write the flavor structure of Dirac mass matrix from the  $S_3$  invariant lagrangian mentioned above.

$$M_D = \begin{pmatrix} y_1 v_2 & y_1 v_1 & y_2 v_1 \\ y_1 v_1 & -y_1 v_2 & y_2 v_2 \\ y_3 v_1 & y_3 v_2 & 0 \end{pmatrix} \quad (2.2)$$

Alike the Dirac mass matrix for neutrinos, one can write the charged lepton mass matrix of the form

$$M_l = \begin{pmatrix} y_{l1} v_2 & y_{l1} v_1 & y_{l2} v_1 \\ y_{l1} v_1 & -y_{l1} v_2 & y_{l2} v_2 \\ y_{l3} v_1 & y_{l3} v_2 & 0 \end{pmatrix} \quad (2.3)$$

The light neutrino mass formula is governed by type-I seesaw mechanism as,

$$m_\nu = M_D M_R^{-1} (M_D)^T \quad (2.4)$$

Hence, by using type I seesaw mechanism, we can obtain the simple form of small neutrino mass matrix,

$$M_\nu = \begin{pmatrix} A & B & C \\ B & D & E \\ C & E & F \end{pmatrix}.$$

It may be noted here that

$$U_\nu M_\nu U_\nu^T = \text{Diag}(m_1, m_2, m_3). \quad (2.5)$$

The charged lepton mass matrix is diagonalized numerically for a set of values of each entries that gives the mass of charged leptons as 0.511 eV, 105 MeV and 1777 MeV respectively. The matrix that diagonalizes the charged lepton mass matrix ( $U_{el}^\dagger$ ) is obtained. The PMNS matrix is parameterized as  $U_{PMNS} = U_{el}^\dagger U_\nu$ . For a typical set of values of the entries in the mass matrix, ( $A=0.012968$ ,  $B=0.0093801$ ,  $C=0.017575$ ,  $D=0.0194216$ ,  $E=0.153206$ ,  $F=0.027024$ ), we get  $\Delta m_{sol}^2 \simeq 7.7 \times 10^{-5} eV^2$ ,  $\Delta m_{atm}^2 \simeq 2.4 \times 10^{-3} eV^2$ ,  $\theta_{13} \simeq 0.25$ ,  $\theta_{12} \simeq 0.54$ ,  $\theta_{23} \simeq 0.67$ .

$$U_{el} = \begin{pmatrix} -0.70712 & -0.708066 & -0.00633713 \\ 0.0760625 & -0.0849724 & 0.993476 \\ -0.702991 & 0.7030 & 0.113867 \end{pmatrix} \quad U_\nu = \begin{pmatrix} -0.802682 & 0.0825209 & -0.590681 \\ -0.361107 & -0.855456 & 0.37122 \\ -0.47466 & 0.511258 & 0.716461 \end{pmatrix}$$

The Dirac rephasing invariant is given by

$$J_{cp} = \text{Im}(U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*) = s_{23} c_{23} s_{12} c_{12} s_{13} c_{13} e^{i\delta} \sim 0.031. \quad (2.6)$$

## 2.2 Higgs Sector

The higgs sector in this model contains two  $SU(2)_L$  doublets  $H_1$  and  $H_2$  which transform as doublet under  $S_3$ . Furthermore, it may be noted that the Higgs sector consist of 4 neutral higgs fields; 2 pseudoscalar and 2 scalar fields. After the symmetry breaking, the mixing between the two higgs doublets will lead to the SM higgs where the pseudo scalar and the charged scalar fields will be the goldstone bosons to give mass to the gauge bosons. This  $S_3$  invariant higgs sector is quite similar with the two higgs doublet model [5-9]. The structure of potential is a bit complicated as compared to the SM higgs potential. The Potential in this model is given by

$$\begin{aligned} V(H_1, H_2) = & -\mu[(H_1^\dagger H_1) + (H_2^\dagger H_2)] - \mu_{12}(H_1^\dagger H_2 + h.c) + \lambda_1[(H_1^\dagger H_2 - H_2^\dagger H_1)^2] \\ & \lambda_2[(H_1^\dagger H_1)^2 + (H_2^\dagger H_2)^2 + (H_1^\dagger H_2)(H_2^\dagger H_1) + (H_2^\dagger H_1)(H_1^\dagger H_2)] \\ & + \lambda_3[(H_1^\dagger H_2 + H_2^\dagger H_1)^2 + (H_1^\dagger H_1 - H_2^\dagger H_2)^2], \end{aligned}$$

where

$$H_1 = \begin{pmatrix} h_1^\pm \\ v_1 + h_1 + iA_1 \end{pmatrix} \quad H_2 = \begin{pmatrix} h_2^\pm \\ v_2 + h_2 + iA_2 \end{pmatrix}$$

The minimization conditions for the above potential when both the higgs field get non zero vev is given by  $\frac{\partial V}{\partial v_1} = 0$  and  $\frac{\partial V}{\partial v_2} = 0$ , Which leads to the conditions

$$\begin{aligned} 4v_1^3 \lambda_2 + 4v_1 v_2^2 \lambda_2 + 4v_1^3 \lambda_3 + 4v_1 v_2^2 \lambda_3 + 2v_1 \mu + 2v_2 \mu_{12} &= 0 \\ 4v_1^2 v_2 \lambda_2 + 4v_2^3 \lambda_2 + 4v_1^2 v_2 \lambda_3 + 4v_2^3 \lambda_3 + 2v_2 \mu + 2v_1 \mu_{12} &= 0. \end{aligned}$$

And from the minimum condition  $\frac{\partial^2 V}{\partial v_1^2} > 0$  and  $\frac{\partial^2 V}{\partial v_2^2} > 0$ , we can get

$$\begin{aligned} 4\lambda_2(3v_1^2 + v_2^2) + 4\lambda_3(3v_1^2 + v_2^2) + 2\mu &> 0 \\ 4\lambda_2(3v_2^2 + v_1^2) + 4\lambda_3(3v_2^2 + v_1^2) + 2\mu &> 0. \end{aligned}$$

The vacuum stability conditions are given by  $\lambda_1, \lambda_2, \lambda_3 > 0$  and  $\lambda_1 > -2\sqrt{\lambda_2\lambda_3}$ .

The explicit symmetry breaking term in the expression of potential leads to a symmetry of  $H_1 \longleftrightarrow H_2$ . The VEV, invariant under this symmetry are most economic as the freedom can be completely absorbed into the yukawa coupling, so that the most general form of mass matrix is constructed above. Hence the two higgs doublets can be rotated from flavor basis to mass basis through a orthogonal transformation between the fields. In the neutral Higgs sector we can get the rotation of fields as:  $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = R \begin{pmatrix} h_L \\ h_H \end{pmatrix}$ , where R is an orthogonal matrix that rotates the flavor state of the neutral higgs field to mass eigen state. Hence the square mass matrix for the neutral higgs from the mixing is given by

$$M_h^2 = \begin{pmatrix} M_{h1}^2 & M_{h1h2} \\ M_{h1h2} & M_{h2}^2 \end{pmatrix} \quad (2.7)$$

In doing so the mixing angle can be written as

$$\begin{aligned} \tan 2\gamma &= \frac{2M_{h1h2}}{M_{h2}^2 - M_{h1}^2} \\ &= 2 \frac{(v_1 v_2 (\lambda_2 + \lambda_3) + \mu_{12})}{(v_1^2 - v_2^2)(\lambda_2 + \lambda_3)} \end{aligned}$$

The mass spectrum of the charged and neutral higgs after diagonalization of the mixed mass matrices are given by

$$M_{h_L}^2 \approx 2v^2(\lambda_2 + \lambda_3) + \mu^2 - 2\sqrt{(v_1^4 + v_2^4)(\lambda_2 + \lambda_3) + 8v_1 v_2 \lambda_2 \mu_{12} + \mu_{12}^2}$$

$$M_{h_H}^2 \approx 4v^2(\lambda_2 + \lambda_3) + \mu^2 + 2\sqrt{(v_1^4 + v_2^4)(\lambda_2 + \lambda_3) + 8v_1 v_2 \lambda_2 \mu_{12} + \mu_{12}^2}$$

$$M_{A_H}^2 \approx 2(\lambda_2 - \lambda_1)v^2 + \mu$$

$M_{ch}^2 \approx 2\lambda_2 v^2 + \mu$ , where  $h_L$  is the standard model like higgs which acquire the electroweak vacuum expectation value of  $v = \sqrt{v_1^2 + v_2^2} \simeq 174$  GeV. The charged and CP odd scalar  $h_L^\pm$ ,  $A_L$  are the goldstone bosons to be absorbed by the gauge bosons to acquire mass in the electroweak symmetry breaking.

### 3. Leptogenesis

The CP asymmetry will be generated by the out of equilibrium decay of the lightest right handed neutrino in one loop level in addition to the interference of tree level contribution, which is given by [10-12]

$$\epsilon_{N_\alpha} = \frac{\Gamma_{(N \rightarrow l H_L)} - \Gamma_{(N \rightarrow l^c H_L^c)}}{\Gamma_{(N \rightarrow l H_L)} + \Gamma_{(N \rightarrow l^c H_L^c)}} + \frac{(\Gamma_{(N \rightarrow l H_H)} - \Gamma_{(N \rightarrow l^c H_H^c)})}{\Gamma_{(N \rightarrow l H_H)} + \Gamma_{(N \rightarrow l^c H_H^c)}}.$$

### 3.1 Flavored Leptogenesis in heierarchical RH neutrino masses ( $N_1 < N_2 < N_3$ )

If we consider the normal hierarchy in RH neutrino sector, then  $N_2$  and  $N_3$  will decouple from the thermal plasma earlier than the lightest neutrino  $N_1$ . Hence the asymmetry created by  $N_2$  and  $N_3$  can be washed out by the inverse decay of  $N_1$  and the total asymmetry will be generated by the out of equilibrium decay of  $N_1$ . If the flavors are considered, the CP asymmetry will be stored in each individual flavor. For three flavor regime, below the temperature range  $M \sim T < 10^8$  GeV, as all the charged lepton yukawa couplings remain in equilibrium, the asymmetry generated by the decay of individual lepton flavor can be washed out only by the inverse decay of the same flavor. Hence even if the total CP asymmetry of all the flavors vanishes, the asymmetry will be stored in each individual lepton sectors.

The CP asymmetry will be generated by the out of equilibrium decay of the lightest right handed neutrino in one loop level in addition to the interference of tree level contribution, which is given by [10-14]

$$\epsilon_{N_\alpha} = \sum_{j,i \neq j} \frac{1}{8\pi} \left[ \frac{\text{Im}(YY^\dagger)_{ij} Y_{\alpha i} Y_{\alpha j}^*}{(YY^\dagger)_{ii}} \right] \left( g \left( \frac{M_j^2}{M_i^2} \right) + f \left( \frac{M_j^2}{M_i^2} \right) \right),$$

where  $\alpha$  is the flavor index.

Imposition of  $S_3$  symmetry gives a particular flavor structure to the Yukawa coupling matrix, given by

$$Y = \begin{pmatrix} y_1 \sin \gamma & y_1 \cos \gamma & y_2 \cos \gamma \\ y_1 \cos \gamma & -y_1 \sin \gamma & y_2 \sin \gamma \\ y_3 \cos \gamma & y_3 \sin \gamma & 0 \end{pmatrix}$$

CP asymmetry in the different lepton flavors are given by

$$\begin{aligned} \epsilon_{1\tau} &= \left( \frac{-3}{16\pi} \right) \frac{\text{Im} \left( y_2^2 y_3^2 \cos^2 \gamma \sin^2 \gamma \right) \left( \frac{M_1}{M_2} \right)}{y_1^2 + y_2^2 \cos^2 \gamma} \\ \epsilon_{1\mu} &= \left( \frac{-3}{16\pi} \right) \frac{\text{Im} \left( (-y_2^2 y_1^2 \cos^2 \gamma \sin^2 \gamma) \left( \frac{M_1}{M_2} \right) - (2y_3 y_1^*) \sin^2 \gamma \cos^2 \gamma y_1 y_2^* \left( \frac{M_1}{M_3} \right) \right)}{y_1^2 + y_2^2 \cos^2 \gamma} \\ \epsilon_{1e} &= \left( \frac{-3}{16\pi} \right) \frac{\text{Im} \left( (y_2^2 y_1^2 \cos^2 \gamma \sin^2 \gamma) \left( \frac{M_1}{M_2} \right) + (2y_3 y_1^*) \sin^2 \gamma \cos^2 \gamma y_1 y_2^* \left( \frac{M_1}{M_3} \right) \right)}{y_1^2 + y_2^2 \cos^2 \gamma} \end{aligned} \quad (3.2)$$

The magnitude of CP asymmetry in electron and muon sectors are same where in the tau sector there is no complex phase that gives rise to CP asymmetry. The flavored CP asymmetry [13] depends on the phases present in the  $y_2$  and  $y_3$ , where the expression is given by

$$\epsilon_\alpha \sim \left( \frac{-3}{16\pi} \right) \frac{\text{Im}(y_3 y_2 y_1^2 \sin^2 \gamma e^{i(\phi_3 - \phi_2)})}{y_1^2 + y_2^2 \cos^2 \gamma} \left( \frac{M_1}{M_3} \right) \quad (3.3)$$

Hence from the lagrangian we can arbitrarily chose the phases in the yukawa couplings to absorb the phase transformation of the fields. which can be written as  $\phi_3 - \phi_2 = p_L - p_R$ , where  $p_L$  and  $p_R$  are the phases in the left and right handed fields. Hence the CP asymmetry depends on both the low and high energy phases of neutrinos. The washout parameters are given by

$$\begin{aligned}\tilde{m}_l &= \frac{(Y_{l1}^* Y_{l1}) v^2}{M_1} \\ \tilde{m}_\tau &= \frac{y_3^2 v^2 \cos^2 \gamma}{M_1} \\ \tilde{m}_\mu &= \frac{y_1^2 v^2 \cos^2 \gamma}{M_1} \\ \tilde{m}_e &= \frac{y_1^2 v^2 \sin^2 \gamma}{M_1}\end{aligned}$$

In the strong washout region, if we consider the right handed neutrino mass to be  $M_1 \sim T = 10^7$  GeV,

$$\begin{aligned}\tilde{m}_l &\gg m^* \\ \Rightarrow \tilde{m}_e &\gg 10^{-3} eV \\ \Rightarrow \frac{\tilde{m}_e}{m^*} &= \frac{y_1^2 \sin^2 \gamma \times 3.02 \times 10^4}{10^{-12} \times M_1} \gg 1 \\ y_1^2 &\gg 0.6 \times 10^{-9}\end{aligned}$$

and washout condition for tau sector is given by

$$\begin{aligned}\tilde{m}_\tau &\gg 10^{-3} eV \\ \Rightarrow \frac{\tilde{m}_\tau}{m^*} &= \frac{y_3^2 \cos^2 \gamma \times 3.02 \times 10^4}{10^{-12} \times M_1} \gg 1 \\ y_3^2 &\gg 0.33 \times 10^{-9}\end{aligned}$$

The total lepton asymmetry is given by:  $Y_L = \frac{\eta \varepsilon}{g^*}$ . Where  $\eta$  is the efficiency factor and given by

$$\eta(\tilde{m}_l) = \left[ \left( \frac{\tilde{m}_l}{8.25 \times 10^{-3} eV} \right)^{-1} + \left( \frac{0.2 \times 10^{-3} eV}{\tilde{m}_l} \right)^{-1.16} \right]^{-1}$$

and  $g^*$  is the relativistic degrees of freedom, which is 106.75. Hence the baryon asymmetry can be written as

$$\begin{aligned}Y_B &= a Y_L \\ &= \frac{a}{g^*} \left[ \varepsilon_e \eta \left( \frac{151}{179} \tilde{m}_e \right) + \varepsilon_\mu \eta \left( \frac{344}{537} \tilde{m}_\mu \right) + \varepsilon_\tau \eta \left( \frac{344}{537} \tilde{m}_\tau \right) \right],\end{aligned}$$

where

$$a = \frac{8N_F + 4N_H}{14N_F + 9N_H} \simeq 0.53.$$

The baryon to photo ratio is  $(6.19 \pm 0.14) \cdot 10^{-10}$  from WMAP experiment and from Planck experiment it is  $(6.02 - 6.18) \cdot 10^{-10}$  [15]. Hence from the above expression the fraction  $a = 0.53$  of the lepton asymmetry converts to the baryon asymmetry through electroweak sphaleron process and the CP asymmetry should be in the order of  $10^{-6} - 10^{-7}$  [16] to give the observed baryon asymmetry of order  $10^{-11}$ .

#### 4. Discussion

We studied the neutrino and higgs phenomenology in a  $S_3$  extended standard model. We discussed the mixing in the higgs sector which is quite similar with the 2 HDM and the explicit symmetry breaking parameter controls the mass of one of the higgs field. For a mass of TeV order for Higgs handles the tree level FCNCs and due to the large mass splitting between the SM higgs and other higgs, the mixing angle becomes very small. In Previous literature, it is studied that there is no direct connection between the CP violating phase present in the low energy neutrino mixing and the CP violation that gives rise to the lepton asymmetry in the higher temperature regime. In the temperature range  $< 10^8$  GeV, as all the charged lepton yukawa coupling are in equilibrium, the asymmetry will be stored in each flavor sector even if the total CP asymmetry vanishes. In this model the extension of  $S_3$  symmetry restricts the CP asymmetry generated in the tau lepton sector to vanish, where as the asymmetry in the electron and muon sector remains non zero. In the strong washout region we have obtained bounds to the dirac neutrino yukawa couplings.

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