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TMDs at small x

Jian Zhou[*]
School of Physics and Key Laboratory of Particle Physics and Particle Irradiation (MOE), Shandong University, Jinan, Shandong 250100, China
E-mail: jzhou@sdu.edu.cn

Our recent work on the topical issue, the interplay of transverse momentum dependent(TMD) physics and saturation physics is summarized in this contribution. The main focus is on polarization dependent phenomenology and TMD evolution at small x.

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[*]Speaker.

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1. Introduction

Among other Quantum Chromodynamics(QCD) based frameworks, TMD factorization [1] and Color Glass Condensate(CGC) effective theory [2] are two of the most powerful theoretical tools to explore the internal structure of nucleon and of nuclei. The information on nucleon internal structure is encoded in the universal nonperturbative parts, i.e. TMD parton distributions and the Wilson lines which enter into the cross section formulas computed in TMD factorization and CGC framework, respectively. During the past few decades, two frameworks are more or less parallelly developed, and apply in the different kinematical regions. The validation of TMD factorization requires the existence of an additional hard scale Q^2 that is much larger than parton intrinsic transverse momentum $k_\perp^2$ in a hard scattering process, while CGC can be employed in the region where hard scale Q^2 is much smaller than the center mass energy S. Apparently, there could exist an overlap kinematical region $k_\perp^2 \ll Q^2 \ll S$ where both approaches can apply.

In recent years, a lot of efforts have been made to work out a unified method to describe physics in such overlap region by combining TMD factorization and CGC effective theory. As a formulation in the leading power approximation, it is not legitimate to use TMD factorization as a starting point to compute physical observables at small x since multiple rescattering is significantly enhanced by very high gluon number density. Instead, one should carry out calculations in the CGC formalism first, and then Talyor expand impact factor in terms of the power $\frac{k_\perp^2}{Q^2}$ to isolate the leading power contribution. The first nonvanishing term of the Taylor expansion was shown to be identical to the hard coefficients computed in TMD approach, while the derivative of Wilson lines with respect to transverse coordinates can be related to gluon TMD matrix elements [3]. This procedure later was generalized to the polarization dependent cases [4] and finite N_c case [4, 5]. In this way, the application of TMD factorization in small x region is justified at tree level. Beyond tree level, the formalism dealing with the joint TMD evolution and small x evolution developed in Refs. [6, 7, 8] will be reviewed in Sec.IV.

With the establishment of TMD factorization at small x, one can explore novel polarization dependent effects at small x in terms of TMD parton distributions. The information on gluon TMDs inside an unpolarized or transversely polarized nucleon/nuclei(with S_T being nucleon transverse spin vector) is formally encoded in the following matrix element,

$$\Gamma^{\mu\nu[U,U']} = \frac{1}{xP^+}\int \frac{dy^- d^2 y_T}{(2\pi)^3} e^{ik\cdot y} \langle P, S_T | 2\mathrm{Tr}\left[F_T^{+\nu}(0) U F_T^{+\mu}(y) U'\right] |P, S_T\rangle \big|_{y^+=0}, \tag{1.1}$$

where U and U' are process dependent gauge links in the fundamental representation. At leading power, this correlator can be parameterized by six independent tensor structures [9],

$$\begin{aligned}\Gamma^{\mu\nu} = {} & \delta_T^{\mu\nu} f_1^g - \left(\frac{2k_T^\mu k_T^\nu}{k_T^2} + \delta_T^{\mu\nu}\right) h_1^{\perp g} - \delta_T^{\mu\nu} \frac{\varepsilon_{T\alpha\beta} k_T^\alpha S_T^\beta}{M} f_{1T}^{\perp g} \\ & - i\varepsilon_T^{\mu\nu} \frac{k_T \cdot S_T}{M} g_{1T}^g - \frac{\tilde{k}_T^{\{\mu} S_T^{\nu\}} + \tilde{S}_T^{\{\mu} k_T^{\nu\}}}{2M} h_{1T}^g + \frac{\tilde{k}_T^{\{\mu} k_T^{\nu\}}}{k_T^2} \frac{k_T \cdot S_T}{M} h_{1T}^{\perp g},\end{aligned} \tag{1.2}$$

where TMDs are functions of x and $k_T^2 \equiv -k_\perp^2$. The short hand notation $\tilde{k}_T^\nu = \varepsilon_T^{\mu\nu} k_{T\mu}$ is used.

The first two gluon TMDs, f_1^g and $h_1^{\perp g}$, are the unpolarized and linearly polarized gluon distribution, respectively. Among the four transverse spin dependent gluon TMDs, the three T-odd gluon TMDs, $f_{1T}^{\perp g}$, $h_{1T}^{\perp g}$ and h_{1T}^{g}, are relevant for the transverse single spin asymmetry studies. All of these TMDs contain process dependent gauge links. The two most important cases are the the Weizsäcker-Williams (WW) distribution involving only single future or past pointing staple-like gauge links, denoted by $\Gamma^{[+,+\dagger]}$ or $\Gamma^{[-,-\dagger]}$, and the dipole type distribution involving a closed loop gauge link $\Gamma^{[+,-\dagger]}$ or $\Gamma^{[-,+\dagger]}$, respectively.

2. Small x gluon TMDs inside an unpolarized target

Parton TMDs at large $k_T \gg \Lambda_{QCD}$ are perturbatively calculable. Generally speaking, a T-even TMD with different gauge link structure possess the same perturbative tail at large $k_\perp$. The contributions from gauge link only become relevant at low transverse momentum $k_\perp \sim \Lambda_{QCD}$ in the dilute region. At small x, a new semi-hard scale, so-called the saturation momentum is dynamically generated due to the high number density of gluons. This provides us an unique chance to study how transverse momentum spectrum is affected by different gauge links in a kinematical window, namely the dense medium region $\Lambda_{QCD} \ll k_\perp \sim Q_s$, where gauge link contributions become significant while TMDs still can be computed using the perturbative method.

The most notable example is two widely used unpolarized gluon TMDs: the dipole type gluon distribution $G_{DP}(x,k_\perp)$ and the WW type gluon distribution $G_{WW}(x,k_\perp)$. Both of the distributions can be computed in the MV model,

$$xG_{WW}(x,k_\perp) = \frac{N_c^2-1}{N_c}\frac{\pi R_0^2}{4\pi^4\alpha_s}\int d^2r_\perp\, e^{-ik_\perp\cdot r_\perp}\frac{1}{r_\perp^2}\left(1-e^{-\frac{r_\perp^2 Q_{sg}^2}{4}}\right), \tag{2.1}$$

$$xG_{DP}(x,k_\perp) = N_c k_\perp^2\frac{\pi R_0^2}{2\pi^2\alpha_s}\int\frac{d^2r_\perp}{(2\pi)^2}e^{ik_\perp\cdot r_\perp}e^{-\frac{1}{4}r_\perp^2 Q_s^2} \tag{2.2}$$

where $Q_{sg}=\frac{C_A}{C_F}Q_s$ is the gluon saturation momentum. For $k_\perp \gg Q_{sg}$, as expected the perturbative tail is recovered from the both distributions: $xG_{WW}(x,k_\perp)=xG_{DP}(x,k_\perp)\propto\frac{1}{k_\perp^2}$. In contrast, for $\Lambda_{QCD}\ll k_\perp \ll Q_s$, they behave quiet differently: $xG_{WW}(x,k_\perp)\propto \ln\frac{Q_{sg}^2}{k_\perp^2}$,$xG_{DP}(x,k_\perp)\propto\frac{k_\perp^2}{Q_s^2}e^{-k_\perp^2/Q_s^2}$.

Following the standard procedure, one also can compute both the dipole type and the WW type linearly polarized gluon distributions in the MV model [10],

$$xh_{1,WW}^{\perp g}(x,k_\perp) = \frac{N_c^2-1}{8\pi^3}\pi R_0^2\int d|r_\perp|\frac{K_2(|k_\perp||r_\perp|)}{\frac{1}{4\mu_A}|r_\perp|Q_{sg}^2}\left(1-e^{-\frac{r_\perp^2 Q_{sg}^2}{4}}\right). \tag{2.3}$$

$$xh_{1,DP}^{\perp g}(x,k_\perp) = xG_{DP}(x,k_\perp) \tag{2.4}$$

where K_2 is the second order Bessel function. From the above expressions, one finds that at large transverse momentum $k_\perp \gg Q_{sg}$, the polarized WW type gluon distribution saturates the positivity limit, $xh_{1,WW}^{\perp g}(x,k_\perp)=xG_{WW}(x,k_\perp)\propto\frac{1}{k_\perp^2}$, while at low transverse momentum $\Lambda_{QCD}\ll k_\perp\ll Q_s$, the linear polarization of gluons is suppressed in the WW case: $h_{1,WW}^{\perp g}/G_{WW}\ll 1$.

The dipole type linearly polarized gluon TMD computed in the MV model always saturates the positivity bound at any $k_\perp$. However, after taking into account TMD evolution, the effect of linear gluon polarization is strongly suppressed as shown in Fig.1 [11]. Despite the strong Sudakov suppression, the $\cos 2\phi$ asymmetry generated by $h_{1,DP}^{\perp g}$ is still sizeable at RHIC energy, and may allow to test k_T-resummation formalism at small x and the theoretical expectation that the CGC state is in fact polarized. More phenomenological studies on linear gluon polarization can be found in Refs. [12, 13, 14, 15].

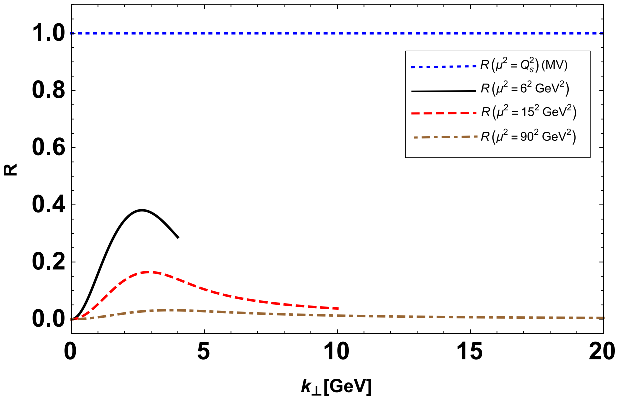

Figure 1: The linearly polarization of gluons at different scales for the dipole case.

3. Small x gluon TMDs inside a transversely polarized target

All of three T-odd gluon TMDs, $f_{1T}^{\perp g}$, $h_{1T}^{\perp g}$ and h_{1T}^{g} responsible for transverse spin asymmetries can be computed in collinear factorization at large $k_\perp$. Due to the twist-3 nature of transverse spin asymmetry phenomenon, they receive leading contribution at twist-3 level. The hard coefficients entering in these expressions are usually different for different gauge links appearing in the gluon matrix element given in Eq. (1.1). For the dipole case, in the small x limit, one has [16],

$$f_{1T,DP}^{\perp g} \approx h_{1T,DP}^{g} \approx h_{1T,DP}^{\perp g} \approx \frac{M}{k_T^4}\frac{1}{x}\int_x^1 dz \left\{ C_2 \sum_{q-\bar{q}} T_{F,q}(z,z) + C_1 T_G^{(-)}(z,z) \right\} \tag{3.1}$$

where $T_{F,q}$, $T_G^{(-)}$ are the Qiu-Sterman quark gluon correlation function and the C-odd tri-gluon correlation respectively. As for the WW case, all leading $1/x$ contribution cancel out between different twist-3 pieces. This result motivates us to come up with a small x formalism treatment for these dipole type T-odd gluon TMDs. In the small x limit one can relate the TMD matrix element by partial integration to a close loop Wilson line $U^{[\Box]}(0_T, y_T)$. The imaginary part of the Wilson loop gives rise to the T-odd part of gluon TMD matrix which can be cast into the form,

$$\Gamma_{\text{T-odd}}^{\mu\nu}(x,k_T;S_T) = \frac{k_T^\mu k_T^\nu}{g^2 V x P^+} \int \frac{d^2 y_T}{(2\pi)^3} e^{ik_T \cdot y_T} \langle P, S_T | \mathrm{Tr}\left[U^{[\Box]}(0_T, y_T) - U^{[\Box]\dagger}(0_T, y_T) \right] | P, S_T \rangle, \tag{3.2}$$

When parameterizing the above tensor structure, we get only one spin dependent term [17],

$$\Gamma_{\text{T-odd}}^{\mu\nu}(x,k_T;S_T) = \frac{k_T^\mu k_T^\nu N_c}{2\pi^2 \alpha_s x} \frac{\epsilon_T^{\alpha\beta} S_{T\alpha} k_{T\beta}}{M} O_{1T}^{\perp}(x, k_T^2), \tag{3.3}$$

where $O_{1T}^{\perp}(x,k_T^2)$ is identified as a spin dependent odderon in [17]. By equalling two parametrization, one derives [16],

$$x f_{1T}^{\perp g} = x h_{1T}^{g} = x h_{1T}^{\perp g} = \frac{-k_T^2 N_c}{4\pi^2 \alpha_s} O_{1T}^{\perp}(x,k_T^2)\,, \tag{3.4}$$

We have now obtained a consistent picture at small x involving only one independent TMD, determined by the spin-dependent odderon. This universal distribution should govern the single transverse spin asymmetries in $p^{\uparrow}p$ and $p^{\uparrow}A$ scattering at RHIC in the small-x regime. This description differs from SSA involving the spin-*in*dependent odderon [18]. Note that the above analysis later was extended to the spin one nucleon case [19].

The expectation value of the spin-dependent odderon has been evaluated using the McLerran-Venugopalan (MV) model[17] and the diquark model [20]. The key observation from model calculations is that the spin dependent odderon is dynamically generated through the asymmetrical color source distribution in the transverse plane of transversely polarized nucleon. More theoretical and phenomenological studies on SSAs at small x can be found in Refs. [21, 22, 23].

4. The evolution of small x gluon TMDs

Whenever there are widely different scales involved in a hard scattering process, large logarithm terms which need to be resummed to all orders in a systemical way, will show up in high order calculations. It has been confirmed by an explicit one loop cross section calculation [6] that the large logarithms $\ln\frac{S}{Q^2}$ and $\ln\frac{Q^2}{k_\perp^2}$ simultaneously arise in the overlap region $k_\perp^2 \ll Q^2 \ll S$ where both CGC and TMD approach can apply. Provided that the large logarithm terms appear in the same pattern at even higher order(beyond one loop), they can be resummed by means of the Collins-Soper evolution equation and the BK equation, respectively. The relative size of two type large logarithms is solely determined by the kinematics of a physical process.

An alternative way of doing resummation is to study the evolution of gluon TMD matrix elements. To this end, we computed NLO correction to gluon TMD in a simple quark model using the Ji-Ma-Yuan scheme [7]. As expected, the resulted gluon TMD contains both the Collins-Soper type large logarithm and the logarithm $\ln\frac{1}{x}$, and satisfies the Collins-Soper equation and the BFKL equation at the same time.

The calculation also can be formulated in a more formal way. We computed gluon TMDs in CGC framework with the Collins-2011 scheme, and express them in terms of the Wilson lines. Both large logarithm terms arise in corresponding hard coefficients [8]. We first remove $\ln\frac{1}{x}$ logarithm by replacing the bare Wilson lines with the renormalized ones whose rapidity dependence is controlled by the BK equation. The large $k_\perp$ logarithm is further resummed into the Sudakov factor using the Collins-Soper equation. This procedure essentially follows the spirit of the effective theory by noticing that small x physics and TMD physics actually live at different scales.

5. Summary

In summary, a unified method combing CGC framework and TMD factorization has been established. While employing powerful saturation physics formalism and TMD approach to polarization effects at small x has already produced fruitful results, we believe that there is much more left to explore in the future study from both theoretical and experimental sides.

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