

The Loop-Tree Duality: Progress Report

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We review the recent developments of the Loop-Tree Duality method, focussing our discussion on the first numerical implementation and its use in the direct numerical computation of multi-leg Feynman integrals. Non-trivial examples are presented.

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1. Introduction

The Loop-Tree Duality (LTD) method [1–14] turns N -leg loop quantities (integrals and amplitudes) into a sum of connected tree-level-like diagrams with a remaining integration measure that is similar to the $(N + 1)$ -body phase-space [1]. Therefore, loop and tree-level corrections of the same order, may in principle be treated under a common integral sign with the use of a proper numerical integrator (usually a Monte Carlo routine) [11, 12]. The LTD method fits into a broader effort to produce fully automated next-to-leading order (NLO) computations. Many steps toward that direction have been taken in the last years [15–38]. Substantial progress has also been made at higher orders [39–41].

Here we focus on the use of the LTD framework in computing one-loop Feynman diagrams. The numerical implementation of the LTD had been initially tested on integrals with up to six external legs [14]. Here we report on the performance of the method for diagrams with up to eight external legs and we present non-trivial examples of a scalar and tensor octagon with different internal mass configurations. The motivation for the work presented here originated from our intention to use the method for the computation of the N -photon amplitude ($2\gamma \rightarrow (N - 2)\gamma$) [21, 23, 42–45].

2. Numerical Implementation of the LTD

In dimensional regularisation, a one-loop scalar diagram can be represented by

$$L^{(1)}(p_1, p_2, \dots, p_N) = -i \int \frac{d^d \ell}{(2\pi)^d} \prod_{i=1}^N G_F(q_i), \quad (2.1)$$

where $\ell = (\ell_0, \boldsymbol{\ell})$ is the loop momentum, $G_F(q_i) = 1/(q_i^2 - m_i^2 + i0)$ are Feynman propagators and q_i are the momenta of the internal lines which depend on ℓ . By applying the LTD, we essentially integrate over the energy component ℓ_0 using the residue theorem. The loop diagram turns then into a sum of integrals over the three-momentum $\boldsymbol{\ell}$ each of which is called a “dual contribution”. The dual contributions emerge from the original integral after cutting one of the internal lines:

$$L^{(1)}(p_1, p_2, \dots, p_N) = - \sum_{i=1}^N \int \frac{d^3 \boldsymbol{\ell}}{(2\pi)^3} \tilde{\delta}(q_i) \frac{1}{q_{i+1}^2 - m_{i+1}^2 - i0} \eta p_{i+1}$$

where $\tilde{\delta}(q_i) = 2\pi i \delta_+(q_i^2 - m_i^2)$ with the “+” subscript stating that we are taking the positive-energy solution. To integrate the dual contributions over $\boldsymbol{\ell}$ requires most of the times a contour deformation due to the presence of the so-called *ellipsoid* and *hyperboloid* singularities [9] that in general are present at the integrand level.

The LTD method has been implemented in a C++ code [9] and for the numerical integration the Cuba library [46] was used. One needs only to provide the external four-momenta and the internal propagator masses. There is freedom from the side of the user to change various parameters, e.g. the parameters of the contour deformation, choose an integration routine between Cuhre [47, 48] and VEGAS [49] and specify the desired number of evaluations or the required accuracy. At run time, the code initially reads in and assigns masses and external momenta. Then it proceeds with an analysis of the ellipsoid and hyperboloid singularity structure to set up the details of the contour deformation and finally performs the numerical integration using either Cuhre or VEGAS. It has been tested for a large number of scalar and tensor diagrams with different number of external legs using as third-party reference values results from LoopTools 2.10 [50] and SecDec 3.0 [51]. The running time for a precision of 4-digits, on a typical Desktop machine (Intel i7 @ 3.4 GHz processor, 4-cores 8-threads), varied from below a second to around 30 seconds.

In Table 1, we present results for a scalar and tensor octagon. The former has all internal masses different whereas the latter has all internal masses equal. The external momenta configu-

| Diagram | Real Part | Imaginary Part |
|----------------|--------------------------------|----------------------------------|
| Scalar octagon | $6.8263(4) \times 10^{-10}$ | $+i 9.17379(37) \times 10^{-10}$ |
| Tensor octagon | $-3.77449(34) \times 10^{-10}$ | $+i 2.82760(3) \times 10^{-9}$ |

Table 1: Tensor octagon with all internal masses equal.

ration used for both the scalar and tensor octagons is shown in Eq. (2.2)

$$\begin{aligned}
 p_1 &= (-2.500000, & 0, & 0, & -2.500000) \\
 p_2 &= (-2.500000, & 0, & 0, & 2.500000) \\
 p_3 &= (-0.427656, & 0.041109, & -0.180818, & 0.385362) \\
 p_4 &= (-0.907144, & 0.289299, & 0.859318, & 2.805929) \\
 p_5 &= (-0.414246, & 0.329547, & 0.249476, & -0.027570) \\
 p_6 &= (-1.907351, & -0.950926, & -1.460214, & 0.775566) \\
 p_7 &= (-0.271157, & 0.155665, & 0.039639, & -0.218456) \\
 p_8 &= -p_1 - p_2 - p_3 - p_4 - p_5 - p_6 - p_7
 \end{aligned} \tag{2.2}$$

whereas the numerators and the masses for the two cases are given below:

Scalar octagon numerator: 1

masses:

$$\begin{aligned}
 m_1 &= 4.506760, m_2 = 2.814908, m_3 = 1.427626, m_4 = 7.621541 \\
 m_5 &= 5.269166, m_6 = 3.521039, m_7 = 5.888145, m_8 = 4.422515
 \end{aligned}$$

Tensor octagon numerator: $\ell.p_2 \times \ell.p_4$

$$\text{masses: } m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = 4.506760.$$

3. Conclusions

The LTD method exhibits many interesting theoretical properties when processes with many external legs and different mass scales are under consideration. Our numerical implementation of

the LTD demonstrates many of the method's appealing characteristics. The code has an excellent performance for integrals with many external legs since it shows only a moderate rise in the running time as the number of legs increases.

Our next step will be to apply our LTD numerical implementation on the computation of N -photon amplitudes. It would also be interesting although more technically involved, to apply the LTD in processes with N -gluon one-loop amplitudes demanding two of gluons to be off-shell.

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