

Spinor helicity methods in DIS at small x : 3-parton production

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We use spinor helicity methods to calculate the azimuthal angular correlations between three partons produced in Deep Inelastic Scattering (DIS) at small Bjorken x . We show that gluon saturation effects in the proton or nucleus target lead to a broadening and disappearance of the two away side peaks. We show how our results may be used to include the effects of fully coherent cold matter energy loss in di-jet production in DIS at small x .

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1. Introduction

It is well-known that parton (quark and gluon) distribution functions in QCD grow very rapidly at small x [1]. This is understood to be due to the large phase space (in x) available for radiation of more partons, primarily gluons which naturally leads to the question, can this growth go on for ever? If so, it would lead to a violation of Froissart bound on growth of physical cross sections with energy. Therefore one expects that some QCD dynamics should come into play when parton number densities become large, i.e. at small x .

Gluon saturation was proposed long time ago as a dynamical QCD mechanism by which this fast growth of gluon distribution function can be tamed. The main idea is simple; partons are the quasi-free degrees of freedom in perturbative QCD and the underlying interaction is parton on parton scattering. However since parton densities are large at small x , one can scatter from many partons together. This is perhaps easiest to visualize in DIS on a nucleus target in the rest frame of the nucleus; the incoming photon splits into a quark anti-quark pair long before it reaches the nucleus and has a large coherence length. The created quark anti-quark pair, called a dipole, interacts coherently with all the nucleons along its direction of motion. This is the multiple scattering picture of DIS at small x . One can formulate gluon saturation in an effective action approach in QCD [2, 3].

The dipole-nucleus scattering cross section depends on the energy of the interaction, or equivalently on the Bjorken x of the target. This energy dependence is governed by JIMWLK evolution equation [4] is a non-linear equation re-sums large logs of energy or $1/x$ and leads to (perturbative) unitarization of physical cross sections at high energy. It reduces to the BFKL [5] equation for two Reggeized gluons and to BJKP equation for multi-Reggeized gluons in the low density region [7]. The JIMWLK equation reduces to a particularly simple and closed equation, known as the BK equation [6] in the large N_c limit.

There has been a large volume of work dedicated to understanding gluon saturation dynamics, as manifested in physical cross sections measured in high energy collisions involving at least one hadron or nucleus, such DIS on protons and nuclei, high energy proton-proton, proton-nucleus and nucleus-nucleus collisions. There are two main signatures of saturation in inclusive particle production; transverse momentum broadening due to multiple scatterings and suppression of the p_t spectra in proton-nucleus collisions when normalized to proton-proton collisions (R_{pA}) due to small x evolution [8]. Both of these effects have already been seen in experiments at RHIC and the LHC. However, various models based on nuclear modifications of parton distribution functions combined with a rapidity shift of partons, while not based on first principles QCD, can also fit the data.

It is therefore important to go beyond fully inclusive production and consider less inclusive observables which should have more discrimination power to distinguish between different approaches and to clarify the role of gluon saturation. One such example is production of three hadrons/jets in DIS. Studying the azimuthal angular dependence of physical cross section on Bjorken x and p_t is invaluable since it is known [9, 10] that the magnitude and width of azimuthal angular correlations are highly sensitive to gluon saturation and the so called saturation scale $Q_s(x, A, b_t)$.

2. 3-parton production in DIS

Here we consider production of three partons (a quark, an anti-quark and a gluon) in scattering of a virtual photon on a proton or nucleus target, at small x ,

$$\gamma^* p(A) \rightarrow q \bar{q} g \quad (2.1)$$

The produced partons will multiply scatter from the target before eventually hadronizing or becoming jets. The scattering of each parton from the target is described as propagation of the parton in the color field of the nucleus, taken to be a classical background field. The effective interaction between a parton (quark) and the target is described by

$$\tau_F(p, q) \equiv 2\pi \delta(p^+ - q^+) \not{n} \int d^2 x_t e^{-i(q_t - p_t) \cdot x_t} \{ \theta(p^+) [V(x_t) - 1] - \theta(-p^+) [V^\dagger(x_t) - 1] \} \quad (2.2)$$

where p^+ is the light cone energy of the quark entering the nucleus, and q^+ is the light cone energy of the scattered quark. During this eikonal scattering the transverse position of the quark remains the same. $V(x_t)$ is a Wilson line in fundamental representation given by

$$V(x_t) \equiv \hat{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ A_a^-(x^+, x_t) t_a \right\} \quad (2.3)$$

describing propagation of a quark in the background color field A . There is an analogous expression for a gluon propagating in a background color field. The amplitude can be written as

$$\begin{aligned} i\mathcal{A}_1 &= (ie)(ig) \int \frac{d^4 k_1}{(2\pi)^4} \bar{u}(p) \gamma^\mu t^a S_F(p+k, k_1) \gamma^\nu S_F(k_1-l, -q) [S_F^{(0)}(-q)]^{-1} v(q) \varepsilon_\nu(l) \varepsilon_\mu^*(k), \\ i\mathcal{A}_2 &= (ie)(ig) \int \frac{d^4 k_1}{(2\pi)^4} \bar{u}(p) [S_F^{(0)}(p)]^{-1} S_F(p, k_1) \gamma^\nu S_F(k_1-l, -q-k) \gamma^\mu t^a v(q) \varepsilon_\nu(l) \varepsilon_\mu^*(k) \\ i\mathcal{A}_3 &= (ie)(ig) \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \bar{u}(p) [S_F^{(0)}(p)]^{-1} S_F(p, k_1-k_2) \gamma^\lambda t^c S_F^{(0)}(k_1) \gamma^\nu \\ &\quad S_F(k_1-l, -q) [S_F^{(0)}(-q)]^{-1} v(q) [G_\lambda^\delta]^{ca}(k_2, k) [G_\delta^{(0)\mu}(k)]^{-1} \varepsilon_\nu(l) \varepsilon_\mu^*(k), \\ i\mathcal{A}_4 &= (ie)(ig) \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \bar{u}(p) [S_F^{(0)}(p)]^{-1} S_F(p, l-k_1) \gamma^\nu S_F^{(0)}(-k_1) \gamma^\lambda t^c \\ &\quad S_F(k_2-k_1, -q) [S_F^{(0)}(-q)]^{-1} [G_\lambda^\delta]^{ca}(k_2, k) [G_\delta^{(0)\mu}(k)]^{-1} \varepsilon_\nu(l) \varepsilon_\mu^*(k), \end{aligned} \quad (2.4)$$

where $\varepsilon_\nu(l)$, $\varepsilon_\mu^*(k)$ denote polarization vectors of the incoming virtual photon and the outgoing gluon respectively. To simplify the Dirac algebra involved in calculating the cross section, we use spinor helicity methods which simplify the Dirac algebra tremendously. Therefore we consider a photon of a particular polarization which splits into quark and anti-quark with a fixed helicity, either of which subsequently radiates a gluon, also with fixed helicity. The first diagram can be written as

$$\begin{aligned} i\mathcal{A}_1^a &= ieg \int d^4 x d^4 y \frac{d^4 k_1}{(2\pi)^4} e^{-i(p+k-k_1)x} e^{-i(k_1-l+q)y} \delta(x^+) \delta(y^+) \\ &\quad t^a [\theta(p^+ + k^+) V(x_t) - \theta(-p^+ - k^+) V^\dagger(x_t)] [\theta(-q^+) V(y_t) - \theta(q^+) V^\dagger(y_t)] \\ &\quad \frac{N}{(p+k)^2 k_1^2 (k_1-l)^2} \end{aligned} \quad (2.5)$$

and N is the numerator containing the Dirac matrices,

$$N \equiv \bar{U}(p) \not{\epsilon}^*(k) (\not{p} + \not{k}) \not{k}_1 \not{\epsilon}(l) (\not{k}_1 - \not{l}) \not{V}(q) \quad (2.6)$$

For example, consider the case when a longitudinal photon splits into a quark with positive helicity, an anti-quark with negative helicity. The positive helicity quark then radiates a positive helicity gluon giving this particular helicity amplitude as

$$N^{L;+++} = -\frac{\sqrt{2}}{[nk]} \frac{Q}{l^+} [np] \langle kp \rangle [np] \langle n\bar{k}_1 \rangle [n\bar{k}_1] \langle nq \rangle \left(\langle n\bar{k}_1 \rangle [n\bar{k}_1] - l^+ \langle n\bar{n} \rangle [n\bar{n}] \right) \quad (2.7)$$

Each of factors like $\langle nq \rangle, \dots$ is a simple kinematics factor which is evaluated once and for all, for example, $\langle nq \rangle \sim q^+$. All the various amplitudes for a given helicity configuration can be written in similar form. The rest of calculation involves performing some routine integrals. One can then write the physical cross section for production of 3 partons in DIS in a concise form which involves some kinematic factors and traces of products of Wilson lines in fundamental and adjoint representations, known as dipoles, quadrupoles, \dots , which can be evaluated either using numerical methods or by analytical methods if one makes a Gaussian type averaging approximation. To proceed further, one will need to evaluate the production in the kinematics appropriate to a given experiment, depending on the x and transverse momentum p_t and azimuthal angle coverage of the particular experiment. We refer the reader to [11] for full details and numerical results.

Our results here can be used to include and estimate the effects of fully coherent cold matter energy loss in di-jet production in DIS on a nucleus target at small x . To do so one would need to integrate out the transverse momentum one of the three partons in the final state and take the soft energy limit (this limit dominates radiative energy loss). This is straightforward to do once one defines a medium induced energy loss by performing the same calculation for a proton target and subtracting it from the case of a nucleus target. Work in this direction is in its preliminary stages and will be reported else where.

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