PROCEEDINGS OF SCIENCE

Numerical simulations of causal relativistic viscous hydrodynamics for high-energy heavy-ion collisions

Kazuhisa Okamoto*
Department of Physics, Nagoya University, Nagoya 464-8602, Japan
E-mail: okamoto@hken.phys.nagoya-u.ac.jp

Chiho Nonaka
Department of Physics, Nagoya University, Nagoya 464-8602, Japan,
Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya 464-8602, Japan
Department of Physics, Duke University, Durham, North Carolina 27708, USA
E-mail: nonaka@hken.phys.nagoya-u.ac.jp

Relativistic hydrodynamic simulations play a key role in exploring the QGP bulk property and the QCD phase transition in analyses of high-energy heavy-ion collisions at RHIC and LHC. For the quantitative understanding of the QGP property, hydrodynamic calculation with high-precision is important. Using a Riemann solver based on the two-shock approximation, we construct a new 3+1 dimensional relativistic viscous hydrodynamics code. The new hydrodynamics algorithm has less numerical viscosity, which is important to discuss the physical viscosities at RHIC and LHC. For one of applications of the new hydrodynamics code, we argue the possible development of Kelvin-Helmholtz instability in high-energy heavy-ion collisions.

The 3rd International Symposium on "Quest for the Origin of Particles and the Universe"
5-7 January 2017
Nagoya University, Japan

*Speaker.

PoS(KMI2017)046

1. Introduction

The experimental results at RHIC and LHC suggest that strongly interacting quark-gluon plasma (QGP) is created. The relativistic hydrodynamic simulation is one of promising phenomenological model for description of the space-time evolution of hot and dense matter after collisions. Recently analysis of higher flow harmonics using relativistic viscous hydrodynamic simulations with the event-by-event initial fluctuations has shown a significant development of understanding of the QGP property [1]. In particular, the shear viscosity of the QGP has been actively evaluated from the comparison between hydrodynamic calculations and experimental data. In addition, new observables which are sensitive to the QGP property are also reported. The study of the experimental data gives a possibility of quantitative determination of values of shear and bulk viscosities and their temperature dependence.

Recently, we constructed a new relativistic viscous hydrodynamics code in Cartesian coordinates toward a quantitative understanding of the QGP property [2]. We employed the Riemann solver with the two-shock approximation which is stable even with small numerical viscosity. Then we extended the algorithm to that in Milne coordinates to describe the strong and rapid longitudinal expansion of the hot matter at RHIC and LHC [3]. Furthermore we constructed a new relativistic hydrodynamics code with viscosity effects in Milne coordinates based on the algorithm of Ref. [2]. Using our hydrodynamics code, we shall argue the Kelvin-Helmholtz (KH) instability in high-energy heavy-ion collisions. The KH instability in high-energy heavy-ion collisions was argued using ideal fluid calculations in Ref. [4]. However, since the hydrodynamic instability is sensitive to the viscosity effects, the analysis with our relativistic viscous hydrodynamics code may reveal the detailed mechanism of the KH instability in high-energy heavy-ion collisions.

2. Relativistic viscous hydrodynamic equation

The relativistic hydrodynamics is based on the conservation equations,

$$N^{\mu}_{;\mu} = 0, \quad T^{\mu\nu}_{;\mu} = 0, \tag{2.1}$$

where N^{μ} is the net charge current and $T^{\mu\nu}$ is the energy-momentum tensor. In the Landau frame, the net charge current and energy-momentum tensor are decomposed as $N^{\mu} = nu^{\mu} + n^{\mu}$ and $T^{\mu\nu} = eu^{\mu}u^{\nu} - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$, where n^{μ}, Π, and $\pi^{\mu\nu}$ are the charge diffusion current, bulk pressure, and shear tensor respectively. In the relativistic Navier-Stokes theory, the viscous tensors are given by,

$$n^{\mu}_{\rm NS} = \sigma T \nabla_{\mu}\left(\frac{\mu}{T}\right), \tag{2.2}$$

$$\pi^{\mu\nu}_{\rm NS} = \eta\left(\nabla^{\mu}u^{\nu} + \nabla^{\nu}u^{\mu} - \frac{2}{3}\Delta^{\mu\nu}\theta\right), \tag{2.3}$$

$$\Pi_{NS} = -\zeta\theta. \tag{2.4}$$

We define the convective time derivative D and the spatial gradient operator as $DA^{\mu_1\cdots\mu_n} \equiv u^{\beta}A^{\mu_1\cdots\mu_n}_{;\beta}$, $\nabla_{\alpha}A^{\mu_1\cdots\mu_n} \equiv \Delta^{\beta}_{\alpha}A^{\mu_1\cdots\mu_n}_{;\beta}$. Naive extension of relativistic Navier-Stokes theory suffers from the acausality and instability. The second-order hydrodynamics resolves this problem [5]. In the second-order hydrodynamics, the viscous tensors are treated as dynamical variables. The constitutive equations for the evolution of viscous tensors are given by

$$\Delta^{\mu}{}_{\alpha} D n^{\alpha} = -\frac{1}{\tau_n}(n^{\mu} - n^{\mu}_{\rm NS}) - I^{\mu}_n, \tag{2.5}$$

$$\Delta^{\mu}{}_{\alpha}\Delta^{\nu}{}_{\beta} D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}(\pi^{\mu\nu} - \pi^{\mu\nu}_{\rm NS}) - I^{\mu\nu}_{\pi} \tag{2.6}$$

$$D\Pi = -\frac{1}{\tau_\Pi}(\Pi - \Pi_{\rm NS}) - I_\Pi, \tag{2.7}$$

where $I^{\mu}_n, I^{\mu\nu}_{\pi}$, and I_Π are second-order terms, τ_n, τ_π, and τ_Π are relaxation times.

3. Numerical algorithm

We construct a new algorithm to solve the second-order hydrodynamic equations Eqs. (2.1), (2.5)-(2.7) in the Milne coordinates. Using the Strang splitting method, we split the conservation equations into two parts; an ideal part and a viscous part. The ideal part is solved by the Riemann solver [3] based on the two-shock approximation. We also split the constitutive equations of viscous tensors into convection equations, relaxation equations, and equations with source terms. The convection equations are solved by the high-resolution upwind scheme with the third-order piecewise parabolic method (PPM). The relaxation equations are solved by the finite difference method if the relaxation time is larger than the time step size. If the relaxation time is smaller than the time step size, the finite difference method for the relaxation equation become unstable. To avoid this problem, we use the piecewise exact solution (PES) method when the relaxation time is smaller than the time step size. The space derivatives in source terms and Navier-Stokes value of viscous tensors are evaluated by the second-order MC limiter.

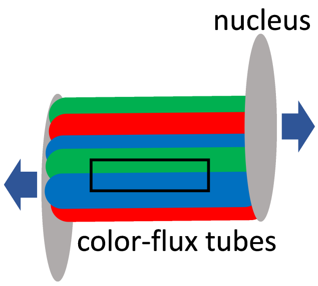
We check validity of our algorithm of solving the relativistic viscous hydrodynamic equation for the viscous Bjorken flow and the viscous Gubser flow [6]. We can obtain good agreement between numerical results and analytic solutions in the test problems.

4. Kelvin-Helmholtz instability in heavy-ion collisions

We argue the development of the KH instability in high-energy heavy-ion collisions using the new relativistic viscous hydrodynamics code. Detailed arguments can be seen in Ref. [6]. The KH instability can happen on the boundary between two horizontal streams which have different fluid velocities. The color-flux tube structure is considered as the initial condition of high-energy heavy-ion collisions (left panel of Fig. 1). We suppose different color-flux tubes start to expand from different points. Then the shear flow exists between different color-flux tubes. Assuming two flux-tubes are located in $x > 0$ and $x < 0$, we consider following initial condition on the (x, η) plane.

$$e(\tau_0, x, \eta) = \frac{e_{\rm U}(\tau_0,\eta) + e_{\rm D}(\tau_0,\eta)}{2} + \frac{e_{\rm U}(\tau_0,\eta) - e_{\rm D}(\tau_0,\eta)}{2}\tanh\left(\frac{x - x_b}{\Delta}\right), \tag{4.1}$$

$$v^{\eta}(\tau_0, x, \eta) = \frac{v^{\eta}_{\rm U}(\tau_0,\eta) + v^{\eta}_{\rm D}(\tau_0,\eta)}{2} + \frac{v^{\eta}_{\rm U}(\tau_0,\eta) - v^{\eta}_{\rm D}(\tau_0,\eta)}{2}\tanh\left(\frac{x - x_b}{\Delta}\right), \tag{4.2}$$



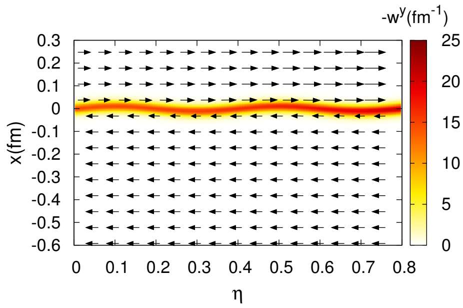


Figure 1: Left:The color-flux tube structure in the initial state of high-energy heavy-ion collisions. Right: The initial condition of Eqs. (4.1) and (4.2). The color profile shows the vorticity $-w^y = -(\partial u^x/\partial\eta - \tau^2\partial u^\eta/\partial x)/\tau$. The arrows stand for the velocity field in $(\tau v^\eta, v^x)$.

where e_U and e_D are the energy density in $x > 0$ and $x < 0$, v^η_U and v^η_D are the rapidity component of velocity in $x > 0$ and $x < 0$, x_b is the boundary between two flux tubes. We assume the boundary has fluctuations $x_b = 0.01\sin(2\pi\eta/\lambda)$. The state in $x > 0$ ($x < 0$) is derived by translating the Bjorken's scaling solution $e_B = e_0(\tau_0/\tau)^{4/3}$ and $v^\eta_B = 0$ to $-\Delta z$ $(+\Delta z)$. As a result, the energy density and velocity are given by $e_U(\tau,\eta) = e_B(t,z+\Delta z)$, $v^\eta_U = v^\eta_B(t,z+\Delta z)$, $e_U(\tau,\eta) = e_B(t,z-\Delta z)$, $v^\eta_U = v^\eta_B(t,z-\Delta z)$, respectively. The right panel of Fig. 1 shows the initial conditions Eqs. (4.1) and (4.2). We start the numerical calculation from the initial conditions Eqs. (4.1) and (4.2) with $\tau_0 = 1$ fm, $\lambda = 0.4$, $\Delta = 0.02$ fm, and the initial energy density (temperature) $e_0 = 741$ GeV/fm^3($T_0 = 800$ MeV). We use the ideal gas equation of state $e = 3p$. We set the grid $(\Delta x,\Delta\eta) = (0.005\text{ fm}, 0.00625)$ and the time-step size $\Delta\tau = 0.2\Delta x$.

Figure 2 shows the results of ideal fluid calculations at $\tau = 4$ and 7 fm. The vortex formation due to the KH instability is observed. The vortices expand and are weakened at later times because of the Bjorken flow. Figure 3 shows the results of viscous fluid calculations at $\tau = 4$ and 7 fm. We set the shear viscosity $\eta/s = 0.01$. In contrast to the ideal fluid calculation, we can not observe the clear vortex formation. In viscous fluids, a small size vortex compared with the Kolmogorov length scale is smeared by the viscosity and can not exist. The fluctuation with the wave length $\lambda = 0.4$ at $\tau_0 = 1$ fm may be smaller than the Kolmogorov length scale. On the other hand, since the growing speed of fluctuations with larger wave length is slower, the fluctuations with large wave length ($\lambda > 0.5$) are smeared by the Bjorken expansion before the vortex formation is achieved. Therefore, fluctuations with any scale wave length can not cause the KH instability at mid-rapidity ($|\eta|$<0.8). However, since the initial conditions Eqs. (4.1) and (4.2) have larger shear flow at larger rapidity, there is a possibility the KH instability occurs at forward rapidity. Meanwhile, if the initial longitudinal flow is smaller than the Bjorken flow, a fluctuation with long wave length survives and can cause the KH instability.

5. Summary

We construct a new relativistic viscous hydrodynamics code. In the test calculations, the new code can reproduce the analytic solutions of the viscous Bjorken flow and the viscous Gubser flow with good accuracy. We argue the KH instability in high-energy heavy-ion collisions using the new

PoS(KMI2017)046

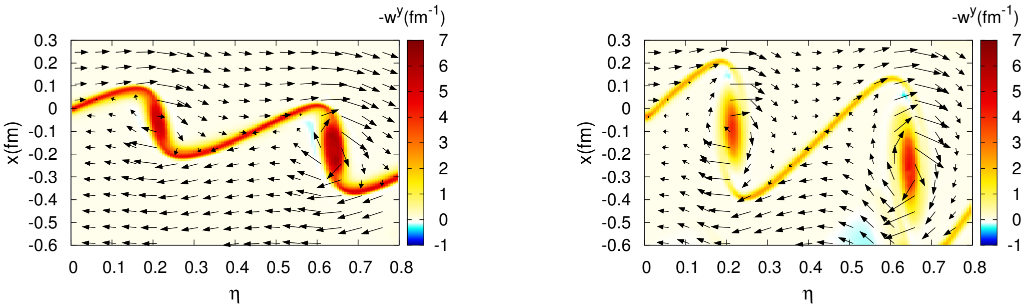


Figure 2: The evolution of KH instability in Bjorken expansion. The results of the ideal fluid calculation at $\tau = 4$ fm (left) and 7 fm (right) are shown.

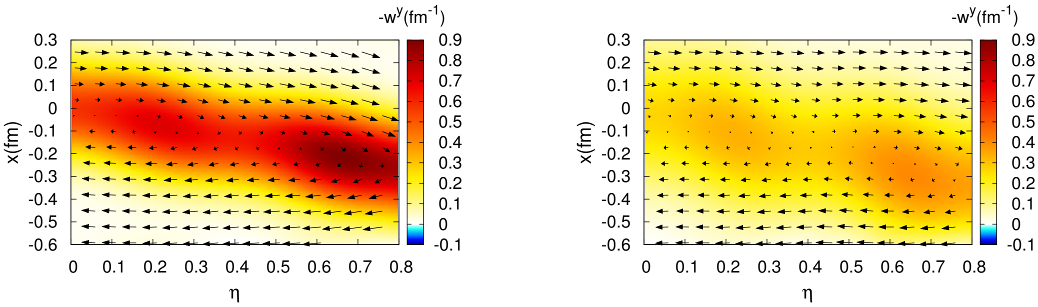


Figure 3: The evolution of KH instability in Bjorken expansion. The results of the viscous fluid calculation with $\eta/s = 0.01$ at $\tau = 4$ fm (left) and 7 fm (right) are shown.

hydrodynamics code. At mid-rapidity, the KH instability is suppressed due to the viscosity and Bjorken expansion effects.

Acknowledgments

We would like to thank Berndt Mueller and Rainer Fries for valuable discussions. C.N. is grateful for the hospitality of the members of Cyclotron Institute and Department of Physics and Astronomy in Texas A&M. The work of C.N. is supported by the JSPS Grant-in-Aid for Scientific Research (S) No. 26220707 and US Department of Energy grant DE-FG02-05ER41367.

References

[1] C. Gale, S. Jeon, B. Schenke, P. Tribedy, and R. Venugopalan, Phys. Rev. Lett. **110**, 012302 (2013)

[2] Y. Akamatsu, S. Inutsuka, C. Nonaka, and M. Takamoto, J. Comput. Phys. **256**, 34 (2014)

[3] K, Okamoto, Y. Akamatsu, and C. Nonaka, Eur. Phys. J. C (2016) **76**: 579

[4] L. P. Csernai, D. D. Strottman, and Cs. Anderlik, Phys. Rev. C **85** 054901 (2012)

[5] W. Israel and J. M. Stewart, Ann. Phys. (N.Y.) **118** (1979), 341

[6] K, Okamoto and C. Nonaka, arXiv:1703.01473 [nucl-th]