

## Theory of radiative $B$ decays

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This talk discusses the theory of inclusive and exclusive radiative  $B$  decays, emphasizing the interplay of perturbative and non-perturbative effects and the importance CP and isospin asymmetries.

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## 1. The Big Picture

Why radiative  $B$  decays?

- Radiative  $B$  decays are... an important probe of New Physics. The process  $b \rightarrow s\gamma$  is a flavor changing neutral current (FCNC). In the Standard Model (SM) there are no FCNCs at tree level. They only arise at a loop level. As a result,  $b \rightarrow s\gamma$  can have contributions from new physics e.g. Supersymmetry. Thus radiative  $B$  decays constrain many models of new physics.
- Radiative  $B$  decays are... theoretically clean. Since the  $b$ -quark mass ( $m_b \sim 5$  GeV) is much larger than  $\Lambda_{\text{QCD}} \sim 0.5$  GeV, observables can be expanded as a power series in  $\Lambda_{\text{QCD}}/m_b \sim 0.1$ . For example, the  $Q_{7\gamma} - Q_{7\gamma}$  contribution to  $\Gamma(\bar{B} \rightarrow X_q\gamma)$  is known to  $\mathcal{O}(\Lambda_{\text{QCD}}^5/m_b^5)$  [1]. This allows to systematically control non-perturbative effects.
- Radiative  $B$  decays are... theoretically interesting. They test basic quantum field theory tools such as the operator product expansion (OPE) and factorization theorems that are important in other areas of physics. They also open a window for non-perturbative physics. For example, the photon spectrum of  $\bar{B} \rightarrow X_s\gamma$  decays measured by CLEO [2], BaBar [3], and Belle [4] collaborations is (at leading twist) the  $B$ -meson parton distribution function.
- Finally, radiative  $B$  decays have... a large impact. Of the CLEO collaboration top cited papers, papers on radiative  $B$  decays are at the first ( $\bar{B} \rightarrow X_s\gamma$  [5]), third ( $\bar{B} \rightarrow K^*\gamma$  [6]) and fourth ( $\bar{B} \rightarrow X_s\gamma$  [2]) place<sup>1</sup>. Similarly a paper on  $\bar{B} \rightarrow X_s\gamma$  [8] is the fourth most cited of the Belle collaboration. For the BaBar collaboration the most highly cited paper on radiative  $B$  decays ( $\bar{B} \rightarrow X_s\ell^+\ell^-$  [9]) is “only” at the 21st place<sup>2</sup>. Furthermore theory papers on radiative  $B$  decays have hundreds of citations.

Radiative  $B$  decays exhibit the interplay of perturbative (short distance) and non-perturbative (long distance) physics, where the latter can be quite complex in structure. The perturbative physics is described by the effective Hamiltonian [11]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\dots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right). \quad (1.1)$$

The most important operators for radiative  $B$  decays are

$$\begin{aligned} Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) F^{\mu\nu} b, \\ Q_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b, \\ Q_1^q &= (\bar{q}b)_{V-A} (\bar{s}q)_{V-A} \quad (q = u, c). \end{aligned} \quad (1.2)$$

The Wilson coefficients  $C_i$  known at next-to-next-to-leading order (NNLO). Non-perturbative effects arise at  $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$ .

<sup>1</sup>A paper concerning the CLEO-II detector [7] is at the second place.

<sup>2</sup>All citation information is based on inspirehep [10] citation count at the time of CKM 2016.

In the following we survey the current theoretical status of inclusive and exclusive radiative  $B$  decays. Among the themes common to both are the interplay of perturbative (short distance) and non-perturbative (long distance) physics and the importance of asymmetries: isospin asymmetries ( $\bar{B} \rightarrow X_s \gamma, \bar{B} \rightarrow K^* \gamma, \bar{B} \rightarrow \rho \gamma$ ), CP asymmetries ( $\bar{B} \rightarrow X_s \gamma, \bar{B} \rightarrow K^* \gamma, \bar{B} \rightarrow \rho \gamma$ ) and even isospin difference of CP asymmetries [12, 13]. While not discussed here, one should mention radiative decays that contain more than one photon:  $B \rightarrow \gamma \gamma$  and  $B \rightarrow X_q \gamma \gamma$ . These were not observed yet.

## 2. Inclusive $\bar{B} \rightarrow X_q \gamma$

The most recent theoretical prediction for inclusive  $\bar{B} \rightarrow X_q \gamma$  decays are  $\text{Br}(\bar{B} \rightarrow X_s \gamma) = (3.36 \pm 0.23) \times 10^{-4}$  and  $\text{Br}(\bar{B} \rightarrow X_d \gamma) = (1.73_{-0.22}^{+0.12}) \times 10^{-5}$  [14]. These should be compared to the experimental values of  $\text{Br}(\bar{B} \rightarrow X_s \gamma) = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$  [15] and  $\text{Br}(\bar{B} \rightarrow X_d \gamma) = (1.41 \pm 0.57) \times 10^{-5}$  [16, 14]. It should be noted that these measurements are obtained by extrapolating the experimental measurement with a photon energy cut in the range of [1.7, 2.0] GeV to 1.6 GeV in order to compare it to theory.

At leading power in  $\Lambda_{\text{QCD}}/m_b$  we have  $\Gamma(\bar{B} \rightarrow X_s \gamma) = \Gamma(b \rightarrow s \gamma)$ , namely the decay is equal to that of a free  $b$ -quark. Since  $\Gamma \propto |\mathcal{H}_{\text{eff}}|^2$ , pairs of operators contribute to the decay rate. The contribution of  $Q_{7\gamma} - Q_{7\gamma}$  and  $Q_{7\gamma} - Q_{8g}$  is known at NNLO, while  $Q_{1,2} - Q_{7\gamma}$  are known at NNLO for two  $m_c$  limits. Interpolation in  $m_c$  leads to  $\pm 3\%$  perturbative uncertainty. Future improvement requires calculation at physical  $m_c$  which is challenging.

Turning to non-perturbative effects, the  $Q_{7\gamma} - Q_{7\gamma}$  contribution obeys a local OPE of the form

$$\Gamma_{77} = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_k c_{k,n} \langle O_{k,n} \rangle, \quad (2.1)$$

where  $c_{k,n}$  are perturbative Wilson coefficients and  $\langle O_{k,n} \rangle$  are non-perturbative matrix elements of HQET operators. The  $n=0$  term gives the free quark contribution discussed above, while the  $n=1$  term vanishes. The coefficients  $c_{k,2}$  are known at  $\mathcal{O}(\alpha_s)$  [17]. The coefficients  $c_{k,\{3,4,5\}}$  are known at  $\mathcal{O}(\alpha_s^0)$  [1] but  $\langle O_{k,\{3,4,5\}} \rangle$  are not well known, see [18]. Still, this is arguably the best known OPE prediction. Very recently, even higher dimensional matrix elements were presented in [19], see the talk ‘‘Higher dimensional HQET parameters’’ in these proceedings. Their contribution to  $\Gamma_{77}$  is not known yet.

Non-perturbative effects from other operators are more complicated. These operators lead to resolved photon contributions, namely processes in which the photon is not directly produced. Thus the operator  $Q_{8g}$  can contribute to the process  $b \rightarrow sg \rightarrow s\bar{q}q\gamma$  and  $Q_1$  to  $b \rightarrow s\bar{c}c \rightarrow sg\gamma$ . The final state contain more partons and might seem to be highly power suppressed, but a more careful analysis shows that these effects give a  $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$  contribution to the rate. For more details see [20] and the CKM 2010 talk [21]. The rate can be written symbolically as  $\Gamma \sim \bar{J} \otimes h$ , where  $\bar{J}$  are perturbative functions and  $h$  are non-perturbative. These non-perturbative functions must be modeled or extracted from (future) data. At  $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$  one finds contributions from  $Q_1 - Q_{7\gamma}$ ,  $Q_{7\gamma} - Q_{8g}$ , and  $Q_{8g} - Q_{8g}$ , leading to a total uncertainty of 5% on  $\Gamma(\bar{B} \rightarrow X_s \gamma)$ . This is also the largest uncertainty on the rate [14].

Can the non-perturbative uncertainty be reduced? For the  $Q_1 - Q_{7\gamma}$  contribution, the improved knowledge of  $\langle O_{k,n} \rangle$  [18] from semileptonic data can better constrain it [22]. A better measurement

of the isospin asymmetry between  $B^+$  and  $\bar{B}^0$  can better constrain the  $Q_{7\gamma} - Q_{8g}$  contribution. Thus data driven methods can better constrain the theoretical uncertainty.

Resolved photon contributions affect the CP asymmetry in  $\bar{B} \rightarrow X_s \gamma$ . It changes it from  $\sim 0.5\%$  from perturbative effects alone to  $[-0.6\%, 2.8\%]$ . For details see [12] and the CKM 2012 talk [23]. They also imply a test of new physics. The isospin difference of CP asymmetries,  $\Delta A_{CP}$ , is proportional to  $\text{Im}(C_{8g}/C_{7\gamma})$ , which can be non-zero in models of new physics.  $\Delta A_{CP}$  was so far measured only by BaBar in 2014:  $\Delta A_{CP} = +(5.0 \pm 3.9 \pm 1.5)\%$  [13]. While consistent with zero, this measurement can be used to constrain  $\text{Im}(C_{8g}/C_{7\gamma})$ . In particular they find  $-1.64 \leq \text{Im}(C_{8g}/C_{7\gamma}) \leq 6.52$  at 90% confidence level [13].

Finally, resolved photon effects for the photon spectrum are not known numerically. They are relevant for extraction of HQET parameters and  $|V_{cb}|$  and  $|V_{ub}|$ . Also, the comparison between theory and experiment relies on extrapolation from measured  $E_\gamma \sim 1.9$  GeV to  $E_\gamma > 1.6$  GeV. The issue of extrapolation should be revisited. Both issues can benefit from detailed  $E_\gamma$  cut effects. For example, instead of optimizing the  $E_\gamma$  cut, it would be useful to have measurements with different values of the photon energy cut. This will allow to test the extrapolation and the resolved photon contributions to the photon spectrum against the data.

### 3. Exclusive $\bar{b} \rightarrow q\gamma$

The theoretical prediction for decays such as  $b \rightarrow s\gamma: B_{(q,s)} \rightarrow (K^*, \phi)\gamma$  and  $b \rightarrow d\gamma: B_{(q,s)} \rightarrow (\rho/\omega, \bar{K}^*)\gamma$  also show an interplay of short distance (SD) effects from  $Q_{7\gamma}$  (while still requiring  $B \rightarrow V$  form factors) and long distance (LD) effects from other operators. In order to reduce hadronic uncertainties one can look at ratios and asymmetries that can be more sensitive to new physics effects.

As an example for ratios consider  $R_{K^*\gamma/\phi\gamma} = \text{Br}(B \rightarrow K^*\gamma)/\text{Br}(B_s \rightarrow \phi\gamma)$ . The SM prediction from [24] is  $R_{K^*\gamma/\phi\gamma}^{\text{SM}} = 0.78 \pm 0.18$ . This should be compared to the LHCb measurement of  $R_{K^*\gamma/\phi\gamma}^{\text{exp}} = 1.23 \pm 0.12$  [25, 26].

Other observables that are sensitive to new physics are isospin asymmetries. The SM prediction for the isospin asymmetry in  $B$  meson decays to  $K^*\gamma$  and  $\rho\gamma$  are  $\bar{a}_I^{\text{SM}}(K^*\gamma) = (4.9 \pm 2.6)\%$  and  $\bar{a}_I^{\text{SM}}(\rho\gamma) = (5.2 \pm 2.8)\%$ . The former agrees with the experimental value  $\bar{a}_I^{\text{exp}}(K^*\gamma) = (5.2 \pm 2.6)\%$  [27], while the latter is in tension with  $\bar{a}_I^{\text{exp}}(\rho\gamma) = (30_{-13}^{+16})\%$  [27].

Belle II can study the time dependent CP asymmetry in  $B$  meson decays to  $f\gamma$ , where  $f$  is a CP eigenstate [28]. The SM prediction changes from  $S_{K^*(K_s\pi^0)\gamma}^{\text{SM, SD}} = -2\frac{m_s}{m_b} \sin 2\phi_1$  and  $S_{\rho^0(\pi^+\pi^-)\gamma}^{\text{SM, SD}} = 0$  from short distance effects alone, to  $S_{K^*(K_s\pi^0)\gamma}^{\text{SM}} = -(2.3 \pm 1.6)\%$  and  $S_{\rho^0(\pi^+\pi^-)\gamma}^{\text{SM}} = (0.2 \pm 1.6)\%$  when including also long distance effects. This is reminiscent of the inclusive CP asymmetry.

In principle the photon can have two different polarizations in  $B \rightarrow V\gamma$  decays. The photon polarization is sensitive to ‘‘right handed’’ new physics. LHCb observed up-down asymmetry proportional to the photon polarization in  $B^\pm \rightarrow K^\pm \pi^\mp \pi^\pm \gamma$  [29]. But unfortunately as they say at the end of the paper ‘‘.. the values for the up-down asymmetry, may be used, if theoretical predictions become available, to determine for the first time a value for the photon polarization, and thus constrain the effects of physics beyond the SM in the  $b \rightarrow s\gamma$  sector’’. See also Kou’s talk at CKM 2014 [30]. Hopefully future measurements of other decay channels could be interpreted in terms of the photon polarization.

## 4. Conclusions

Radiative  $B$  decays have rich structure and show intricate interplay of perturbative and non-perturbative physics. At the same time they are an important probe of new physics. While the theory of radiative  $B$  decays is mature, there is still room for improvements. New results from LHCb and soon Belle II will motivate further theoretical work.

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