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$|V_{us}|$ from τ decays in theory

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The puzzle of the $> 3\sigma$ low (c.f. three-family-unitarity expections) determination of V_{us} from the conventional implementation of flavor-breaking (FB) finite-energy sum rules (FESRs) employing inclusive hadronic τ decay data is revisited, problems with this implementation identified, and an alternative implementation which cures them described. Applying this new implementation using preliminary BaBar results for the exclusive $\tau \to K^- \pi^0 \nu_\tau$ branching fraction, we find $|V_{us}| = 0.2229(22)_{exp}(4)_{th}$, in good agreement with results from other sources. Limitations on near-term possibilities for reducing experimental errors are discussed, and a new approach to improving this situation via dispersive analyses of strange hadronic τ data using lattice input described.

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1. Introduction

With $|V_{ud}| = 0.97417(21)$ [1], three-family-unitary implies $|V_{us}| = 0.2258(9)$. Direct $K_{\ell 3}$ and $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ determinations, using recent 2014 FlaviaNet experimental results [2] and 2016 lattice input [3], yield results $|V_{us}| = 0.2231(9)$ and $0.2253(7)$, respectively, compatible with this expectation. The most recent update of the conventional implementation of the FB FESR hadronic τ decay approach [4], in contrast, yields the 3.6σ low result $|V_{us}| = 0.2176(21)$ [5].

In the Standard Model (SM), denoting the differential distributions for flavor $ij = ud, us$, vector (V) or axial-vector (A) current-mediated decays by $dR_{V/A;ij}/ds$, where $R_{V/A;ij} \equiv \Gamma[\tau^- \to \nu_\tau \, \mathrm{hadrons}_{V/A;ij}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$, one has, with $\rho^{(J)}_{V/A;ij}$ the spectral function of the scalar polarization, $\Pi^{(J)}_{V/A;ij}$, of the corresponding current-current 2-point function [6]

$$\frac{dR_{V/A;ij}}{ds} = \frac{12\pi^2 |V_{ij}|^2 S_{EW}}{m_\tau^2} (1-y_\tau)^2 \, \tilde{\rho}(s) \,, \tag{1.1}$$

with $y_\tau = s/m_\tau^2$, $\tilde{\rho}(s) = (1+2y_\tau)\rho^{(1)}_{V/A;ij}(s) + \rho^{(0)}_{V/A;ij}(s)$, S_{EW} a known short-distance electroweak correction, and V_{ij} the flavor ij CKM matrix element. Rewritten in terms of kinematic-singularity-free combinations, the dominant $\rho^{(0+1)}_{V/A;ij}$ term appears multiplied by the "kinematic weight" $w_\tau(y) = (1-y)^2(1+2y)$. The non-chirally-suppressed π and K pole contributions dominate $\rho^{(0)}_{A;ud,us}(s)$. The remaining, doubly-chirally-suppressed continuum $J=0$ contributions are negligible for $ij = ud$. For $ij = us$, they are small and can be estimated using the related $ij = us$ scalar and pseudoscalar sum rules [7, 8]. The experimental $dR_{V/A;ij}/ds$ distributions then yield $\rho^{(0+1)}_{V/A;ud,us}(s)$.

The inclusive τ $|V_{us}|$ determination employs FESRs for the FB difference $\Delta\Pi \equiv \Pi^{(0+1)}_{V+A;ud} - \Pi^{(0+1)}_{V+A;us}$, and associated spectral function, $\Delta\rho \equiv \rho^{(0+1)}_{V+A;ud} - \rho^{(0+1)}_{V+A;us}$ [4]. Generically,

$$\int_0^{s_0} w(s)\Delta\rho(s)\, ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\Delta\Pi(s)\, ds \,, \tag{1.2}$$

valid for any $s_0 > 0$ and any analytic $w(s)$. For large enough s_0, the OPE is to be employed on the RHS. On the LHS, for general w, subtracting $J=0$ contributions yields the $J = 0+1$ analogue, $dR^{(0+1)}_{V/A;ij}/ds$, of $dR_{V/A;ij}/ds$. Defining the re-weighted integrals

$$R^w_{V+A;ij}(s_0) \equiv \int_0^{s_0} ds \, \frac{w(s)}{w_\tau(s)} \frac{dR^{(0+1)}_{V+A;ij}(s)}{ds} \,, \tag{1.3}$$

Eq. (1.2) can be used to replace the FB difference $\delta R^w_{V+A}(s_0) \equiv \frac{R^w_{V+A;ud}(s_0)}{|V_{ud}|^2} - \frac{R^w_{V+A;us}(s_0)}{|V_{us}|^2}$ with its OPE representation, yielding [4],

$$|V_{us}| = \sqrt{R^w_{V+A;us}(s_0) / \left[\frac{R^w_{V+A;ud}(s_0)}{|V_{ud}|^2} - \delta R^{w,OPE}_{V+A}(s_0) \right]} \,. \tag{1.4}$$

This result should be s_0- and w-independence, providing self-consistency tests.

The $> 3\sigma$ low $|V_{us}|$ results noted above are produced by a conventional implementation of Eq. (1.4) [4] using $s_0 = m_\tau^2$ and $w = w_\tau$ only. This choice allows the spectral integrals to be obtained from the inclusive non-strange and strange branching fractions, but precludes s_0- and w-independence tests. Since w_τ has degree 3, $\delta R^{w_\tau,OPE}_{V+A}(s_0)$ has OPE contributions up to dimension $D=8$. $D=2$ and 4 contributions, involving only α_s and the quark masses and condensates [3, 9, 10, 11], are known. Experimentally unknown $D=6$ condensates are estimated using the vacuum saturation approximation (VSA), and $D=8$ contributions neglected [4, 12]. This treatment of $D=6$ and 8 contributions (especially the use of the VSA) is known to be potentially dangerous [13]. The slow convergence of the $D=2$ OPE series which, to 4-loops, has the form [9]

$$\frac{3}{2\pi^2}\frac{\bar{m}_s}{Q^2}\left[1+\frac{7}{3}\bar{a}+19.93\bar{a}^2+208.75\bar{a}^3\right], \tag{1.5}$$

with $\bar{a}=\alpha_s(Q^2)/\pi$, and $\bar{m}_s=m_s(Q^2)$, $\alpha_s(Q^2)$ the running $\overline{MS}$ strange mass and coupling, is also potentially problematic, given that $\bar{a}(m_\tau^2)\simeq 0.1$.

Figure 1: $|V_{us}|$ from the w_τ and $\hat{w}$ FESRs, with conventional implementation OPE assumptions as input.

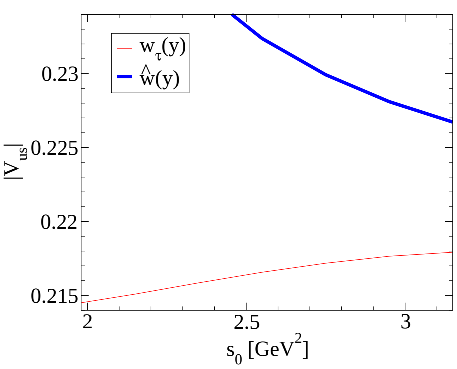
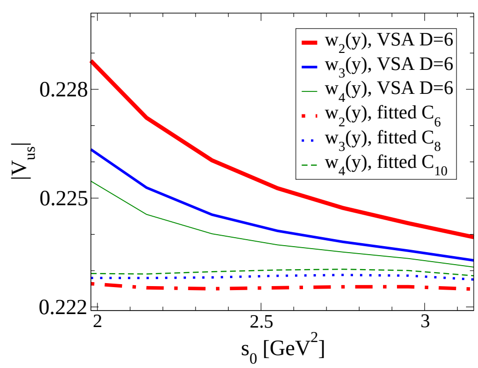


Figure 2: $|V_{us}|$ from the conventional (solid lines) and new implementations (dashed lines) of the w_N FESRs.



Conventional implementation $D>4$ assumptions are testable by comparing $w_\tau(y)=1-3y^2+2y^2$ and $\hat{w}(y)=1-3y+3y^2-y^3$ $(y=s/s_0)$ $|V_{us}|$ results. Integrated $D=6$ and 8 OPE contributions for $\hat{w}$ are -1 and $-1/2$ times, respectively, those for w_τ. Small $D=6$ and negligible $D=8$ contributions for w_τ thus require small $D>4$ contributions for $\hat{w}$. The two FESRS should produce compatible, s_0-stable $|V_{us}|$ results. A breakdown of these $D>4$ assumptions would, in contrast, produce s_0-instabilities of opposite sign for the two FESRs and an output $|V_{us}|$ difference decreasing with increasing s_0. The results of this comparison, shown in Fig. 1, obviously support scenario two.

The $D=2$ convergence issue was investigated by comparing OPE expectations to $n_f=2+1$ RBC/UKQCD lattice results for $\Delta\Pi(Q^2)$ [14]. An excellent match of $D=2+4$ OPE to lattice results was observed in the broad high-Q^2 interval $4\ GeV^2<Q^2<10\ GeV^2$ when 3-loop truncation and a fixed- (rather than local-) scale treatment of logarithmic contributions were employed for the $D=2$ series [15].[1] Conventional $D=2+4$ OPE error estimates were also found to be extremely

[1]The fixed- and local-scale treatments are the analogues of the "fixed-order" (FOPT) and "contour-improved" (CIPT) FESR $D=2$ series prescriptions.

conservative [15]. Much larger deviations of the $D = 2+4$ OPE sum from the lattice data were also seen below $Q^2 \sim 4\ GeV^2$ than conventional implementation $D > 4$ assumptions would imply [15].

2. A new FB FESR implementation

The above observations suggest an alternate FB FESR implementation in which the 3-loop-truncated FOPT version of $D = 2$ OPE contributions favored by lattice data is used and the effective $D > 4$ OPE condensates, C_D, are fit to data [15]. FESRs based on the weights

$$w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N\,, \tag{2.1}$$

are convenient as they involve only a single unknown $D = 2N+2 > 4$ OPE contribution. The $1/s_0^N$ scaling of this contribution allows both $|V_{us}|$ and C_{2N+2} to be obtained from the w_N FESR fit.

We determine the weighted non-strange and strange spectral integrals as follows. K and π pole contributions are evaluated using $K_{\mu 2}$, $\pi_{\mu 2}$ and SM expectations, and continuum *ud* contributions using the ALEPH *ud* V+A distribution [16]. Continuum *us* V+A contributions are obtained by summing over exclusive modes, with Belle [17] and BaBar [18, 19] results used for the $\bar{K}^0\pi^-$ and $K^-\pi^0$ distributions, BaBar [20] and Belle [21] results for the $K^-\pi^+\pi^-$ and $\bar{K}^0\pi^-\pi^0$ distributions, and 1999 ALEPH results [22] for the combined distribution of exclusive *us* modes not re-studied at the B-factories. We consider two different possibilities for the $K^-\pi^0\nu_\tau$ branching fraction which normalizes the exclusive $K^-\pi^0$ distribution: $0.00433(15)$ from the 2014 HFAG summer fit [23] (dominated by BaBar), and $0.00500(14)$ from a preliminary BaBar thesis update [19]. Central results below correspond to the latter choice, which is favored by BaBar.

Figure 2 shows results for $|V_{us}|$ obtained from the $w_{2,3,4}$ FESRs. The solid lines result from conventional implementation OPE assumptions/input, the dashed lines from analyses using instead as input the central effective $D > 4$ condensate values from the new-implementation w_N FESR fits. The switch to fitted $C_{D>4}$ input is seen to completely cure the s_0- and w-instablilities of the conventional implementation approach. With the different w_N FESRs yielding $|V_{us}|$ in good agreement, we base our final result on a combined 3-weight fit. Normalizing the exclusive $K^-\pi^0$ distribution with the favored preliminary BaBar branching fraction, we find [15]

$$|V_{us}| = 0.2229(22)_{exp}(4)_{th}\,. \tag{2.2}$$

The theory error is dominated by the uncertainty in $\langle m_s\bar{s}s\rangle$, the experimental error by the errors and covariances of the strange exclusive distributions [15]. The result agrees well with that from $K_{\ell 3}$, and, within errors, with 3-family unitarity expectations.[2] Roughly half of this improved agreement results from the data-based treatment of higher D OPE contributions, and half from the use of the new preliminary BaBar $K^-\pi^0\nu_\tau$ branching fraction. The curing of the s_0- and w-instability problem, however, results entirely from the data-based $D > 4$ OPE treatment.

Significant reductions in the $|V_{us}|$ error are possible through improvements to the low-multiplicity strange exclusive branching fractions [15]. The $\sim 25\%$ uncertainties in the weighted spectral integrals of the combined, higher-multiplicity 1999 ALEPH "residual mode" distribution, however,

[2] Normalizing the $K^-\pi^0$ distribution using the HFAG 2014 branching fraction, yields $|V_{us}| = 0.2204(23)_{exp}(4)_{th}$, 0.0024 higher than the conventional implementation result obtained from the same experimental input.

Table 1: Relative w_N-weighted *us* spectral integral contributions in the s_0 fit window of the alternate FB FESR implementation. s_0 is in GeV^2. $K\pi$ column entries are the sum of the $K^-\pi^0$ and $\bar{K}^0\pi^-$ contributions, $K\pi\pi$ column entries the sum of the $K^-\pi^+\pi^-$ and $\bar{K}^0\pi^-\pi^0$ contributions, and *Residual* column entries the contributions of the residual mode part of the 1999 ALEPH distribution.

Weight	s_0	K	$K\pi$	$K\pi\pi$	Residual
w_2	2.15	0.496	0.426	0.062	0.017
	3.15	0.360	0.414	0.162	0.065
w_3	2.15	0.461	0.446	0.073	0.019
	3.15	0.331	0.415	0.182	0.074
w_4	2.15	0.441	0.456	0.082	0.021
	3.15	0.314	0.411	0.194	0.081

represent an important limiting factor. A competitive $|V_{us}|$ determination requires sub-0.5% precision, hence weighted inclusive *us* spectral integrals with sub-% precision. The relative contributions of the lower-multiplicity exclusive modes and residual mode sum to the inclusive w_2-, w_3- and w_4-weighted *us* spectral integrals are shown in Table 1, at the lowest and highest s_0 in the analysis fit window. The $\sim 25\%$ residual mode error corresponds to $\sim 2\%$ inclusive *us* spectral integral errors at the lower end of this window. A factor of > 2 improvement in the residual mode sum distribution errors would thus be needed to make the FB FESR approach fully competitive.

It is possible to circumvent this limitation by switching to a dispersive analysis using inclusive *us* data and weights designed to allow lattice data, rather than the OPE, to be used as theory input [15, 24]. Explicitly, one starts from $|V_{us}|^2\,\tilde{\rho}(s)$, obtained from the experimental $dR_{us;V+A}/ds$ distribution via Eq. (1.1). $\tilde{\rho}(s)$ is the spectral function of the kinematic-singularity-free *us* V+A polarization combination, $\tilde{\Pi}_{us;V+A}(Q^2)$, with $Q^2 = -s$ and

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left(1 - 2\frac{Q^2}{m_\tau^2}\right)\Pi^{(J=1)}_{us;V+A}(Q^2) + \Pi^{(J=0)}_{us;V+A}(Q^2)\,. \tag{2.3}$$

Choosing weights, $\bar{W}_N(s) = 1/\left[\prod_{k=1}^N (s+Q_k^2)\right]$, with poles at the N distinct Euclidean locations $Q^2 = Q_1^2, \cdots Q_N^2$, $Q_k^2 > 0$, one has, for $N \geq 3$, the convergent, unsubtracted dispersion relation

$$\int_0^\infty ds\,\bar{W}_N(s)\,\tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^N \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\prod_{j\neq k}\left(Q_j^2 - Q_k^2\right)}\,. \tag{2.4}$$

The $\tilde{\Pi}_{us;V+A}(Q_k^2)$ on the RHS of this relation can be determined with good accuracy on the lattice if all Q_k^2 are kept to a few to several tenths of a GeV^2 [24]. The $s \leq m_\tau^2$ contribution to the LHS is determinable from experimental $dR_{us;V+A}/ds$ data, up to the unknown factor $|V_{us}|^2$. Keeping all Q_k^2 below $\sim 1\ GeV^2$, and choosing the number of poles, N, large enough allows one to also suppress spectral integral contributions from the region $s > m_\tau^2$ (where data do not exist and pQCD is used for $\tilde{\rho}(s)$), and that part of the kinematically allowed region $s < m_\tau^2$ where *us* data errors are large. Increasing N, improves this suppression, at the cost of an increase in the errors on the lattice side (the level of cancellation in the sum of residues grows with increasing N). The error on $|V_{us}|$ is to be minimized by optimizing the choice of N and the Q_k^2, subject to these two competing constraints. Space limitations preclude a discussion of the preliminary results from this approach presented at the conference. A paper containing the final results is in preparation [24].

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