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Analytic results for two-loop Yang-Mills

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Recent Developments in computing very specific helicity amplitudes in two loop QCD are presented. The techniques focus upon the singular structure of the amplitude rather than on a diagramatic and integration approach.

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1. Introduction

There has been excellent progress in computing the matrix elements for $2 \longrightarrow 2$ NNLO processes which together with work on factorisation has led to robust predictions for many processes. For higher point matrix elements there has been very limited progress except in highly symmetric, particularly supersymmetric, theories and indeed the only for four points are the two-loop QCD amplitude known for all helicities [1, 2, 3]. Beyond four point the five-point all-plus amplitude which recently constructed using generalised unitarity techniques [4] followed by integration [5]. In this talk, it was shown how, for this very specific helicity configuration, the singular structure of the amplitude can be used to determine the two loop-amplitude. Specifically we have re-computed the five-point case [6] and obtained results for the six and seven-point all-plus amplitudes [7, 8].

The all-plus helicity amplitude at leading colour may be written

$$\mathscr{A}_n(1^+,2^+,...,n^+)|_{\text{leading color}} = g^{n-2}\sum_{L\geq 1}\left(g^2 N_c c_\Gamma\right)^L \times \sum_{\sigma\in S_n/Z_n} \text{tr}(T^{a_{\sigma(1)}}T^{a_{\sigma(2)}}T^{a_{\sigma(3)}}\cdots T^{a_{\sigma(n)}}) \times A_n^{(L)}(\sigma(1)^+,\sigma(2)^+,...,\sigma(n)^+) \tag{1.1}$$

and it is $A_n^{(2)}(1^+,2^+,\cdots,n^+)$ which is the subject of this talk. This helicity amplitude vanishes at tree level and consequently has a purely rational one-loop expression to order ε given by [9]

$$A_n^{(1)}(1^+,2^+,\cdots,n^+) = -\frac{i}{3}\sum_{1\leq k_1<k_2<k_3<k_4\leq n}\frac{\langle k_1\,k_2\rangle\,[k_2\,k_3]\,\langle k_3\,k_4\rangle\,[k_4\,k_1]}{\langle 1\,2\rangle\,\langle 2\,3\rangle\cdots\langle n\,1\rangle} + O(\varepsilon)\,. \tag{1.2}$$

while the all-ε forms of the one-loop amplitudes are given in terms of higher dimensional scalar integrals [10]. and for $n\leq 6$ are [10]

$$A_4^{(1)}(1^+,2^+,3^+,4^+) = \frac{2i\varepsilon(1-\varepsilon)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle}\times s_{12}s_{23}I_4^{D=8-2\varepsilon}\,,$$

$$\begin{aligned}A_5^{(1)}(1^+,2^+,3^+,4^+,5^+) &= \frac{i\varepsilon(1-\varepsilon)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}\\ &\times\Big[s_{23}s_{34}I_4^{(1),D=8-2\varepsilon}+s_{34}s_{45}I_4^{(2),D=8-2\varepsilon}+s_{45}s_{51}I_4^{(3),D=8-2\varepsilon}\\ &+s_{51}s_{12}I_4^{(4),D=8-2\varepsilon}+s_{12}s_{23}I_4^{(5),D=8-2\varepsilon}+(4-2\varepsilon)\varepsilon(1,2,3,4)I_5^{D=10-2\varepsilon}\Big]\,,\end{aligned}$$

$$\begin{aligned}A_6^{(1)}(1^+,2^+,3^+,\,4^+\,,5^+,6^+) &= \frac{i\varepsilon(1-\varepsilon)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 6\,1\rangle}\frac{1}{2}\Big[\\ &-\sum_{1\leq i_1<i_2\leq 6}\text{tr}[\not{k}_{i_1}\not{K}_{i_1+1,i_2-1}\not{k}_{i_2}\not{K}_{i_2+1,i_1-1}]I_4^{(i_1,i_2),D=8-2\varepsilon}+(4-2\varepsilon)\,\text{tr}[123456]\,I_6^{D=10-2\varepsilon}\\ &+(4-2\varepsilon)\sum_{i=1}^{6}\varepsilon(i+1,i+2,i+3,i+4)I_5^{(i),D=10-2\varepsilon}\Big]\,,\end{aligned} \tag{1.3}$$

where $I_m^{(i),D}$ denotes the D dimensional scalar integral obtained by removing the loop propagator between legs $i-1$ and i from the $(m+1)$-point scalar integral etc. [11], $\not{K}_{a,b}\equiv\sum_{i=a}^{b}\not{k}_i$ and $\varepsilon(a,b,c,d) = [a\,b]\,\langle b\,c\rangle\,[c\,d]\,\langle d\,a\rangle - \langle a\,b\rangle\,[b\,c]\,\langle c\,d\rangle\,[d\,a]$.

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2. Techniques

We will attempt to compute the amplitude purely from the singularities much in the tradition of S-matrix theory [12]. Specifically we will consider

- IR and UV singular structure under regularisation
- Unitarity
- Factorisation

Regularisation structure. The IR and UV behaviours of the two-loop amplitude in dimensional regularisation are known [13] and for this amplitude motivates a partition:

$$A_n^{(2)}(1^+,2^+,...,n^+) = A_n^{(1)}(1^+,2^+,...,n^+)I_n^{(2)} + F_n^{(2)} + \mathcal{O}(\varepsilon)\,. \tag{2.1}$$

where

$$I_n^{(2)} = \left[-\sum_{i=1}^{n} \frac{1}{\varepsilon^2} \left(\frac{\mu^2}{-s_{i,i+1}} \right)^{\varepsilon} \right] \tag{2.2}$$

In this equation $A_n^{(1)}$ is the all-ε form of the one-loop amplitude. There are no ε^{-1} terms in this expression (outside of I_n) although the amplitude has both a UV divergence and a collinear IR divergence [14]. However since the tree amplitude vanish both are proportional to n and cancel leaving only the infinities within $I_n^{(2)}$ which are the soft IR singular terms. The finite remainder function $F_n^{(2)}$ can be split into polylogarithmic and rational pieces,

$$F_n^{(2)} = P_n^{(2)} + R_n^{(2)}\,. \tag{2.3}$$

Unitarity. D-dimensional unitarity techniques can be used to generate the integrands [4] for the five-point amplitude which can then be integrated to give the result [5]. However the organisation of the amplitude in the previous section allows us to obtain the finite polylogarithms using four-dimensional unitarity [15, 16] where the cuts are evaluated in four dimension with the corresponding simplifications. With this simplification the all-plus one-loop amplitude effectively becomes an additional on-shell vertex and the two-loop cuts effectively become one-loop cuts with a single insertion of this vertex. The non-vanishing four dimensional cuts are shown in fig. 1.

The cuts allow us to determine the coefficients of box and triangle functions to the amplitude. These contain both IR terms and finite polylogarithms. The IR terms combine overall [17],

$$\sum \mathcal{C}_i I_{4,i}^{\rm 2m}\bigg|_{IR} + \sum \mathcal{C}_i I_{3,i}^{\rm 2m} + \sum \mathcal{C}_i I_{3,i}^{\rm 1m} = A_n^{(1),\varepsilon^0}(1^+,2^+,\cdots,n^+) \times I_n \tag{2.4}$$

where $A_n^{(1),\varepsilon^0}(1^+,2^+,\cdots,n^+)$ is the order ε^0 truncation of the one-loop amplitude. A key step is to promote the coefficient of these terms to the all-ε form of the one-loop amplitude. This ensures that the two-loop amplitude has the correct singular structure.

The remaining parts of the box integral functions become the polylogarithms. The full expression for $P_n^{(2)}$ is [17] is

$$P_n^{(2)} = -\frac{i}{3\langle 12\rangle\langle 23\rangle\langle 34\rangle\cdots\langle n1\rangle} \sum_{i=1}^{n}\sum_{r=1}^{n-4} c_{r,i} F_{n:r,i}^{2m} \tag{2.5}$$

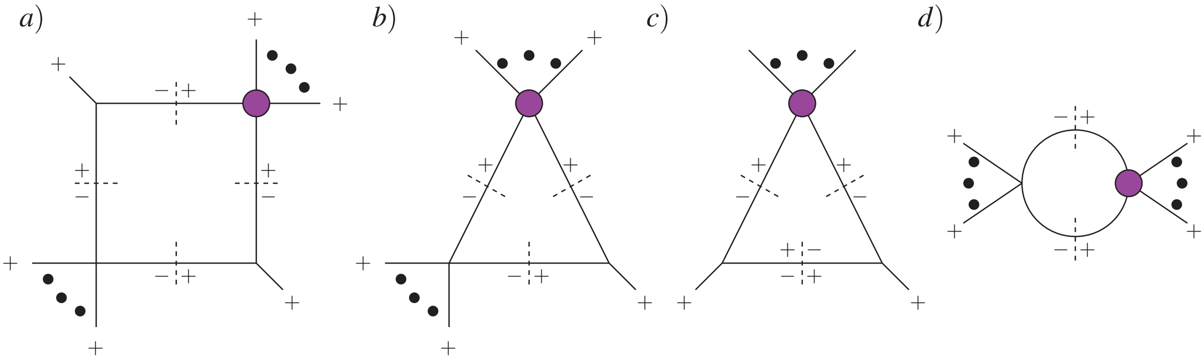

Figure 1: Four dimensional cuts of the two-loop all-plus amplitude involving an all-plus one-loop vertex (indicated by $\bullet$)

where

$$c_{r,i} = \left(\sum_{a<b<c<d\in K_4} \mathrm{tr}_-[abcd] - \sum_{a<b<c\in K_4} \mathrm{tr}_-[abcK_4] + \sum_{a<b\in K_4} \frac{\langle i-1|K_4 ab K_4|i+r\rangle}{\langle i-1\, i+r\rangle} \right) , \tag{2.6}$$

$$F^{2m}_{n:r,i} = F^{2m}[t^{[r+1]}_{i-1}, t^{[r+1]}_{i}, t^{[r]}_{i}, t^{[n-r-2]}_{i+r+1}] , \tag{2.7}$$

$t^{[r]}_i = (k_i + k_{i+1} + \cdots + k_{i+r-1})^2$ and

$$\begin{aligned} F^{2m}[S,T,K_2^2,K_4^2] = \quad & \mathrm{Li}_2[1-\tfrac{K_2^2}{S}] + \mathrm{Li}_2[1-\tfrac{K_2^2}{T}] + \mathrm{Li}_2[1-\tfrac{K_4^2}{S}] \\ & + \mathrm{Li}_2[1-\tfrac{K_4^2}{T}] - \mathrm{Li}_2[1-\tfrac{K_2^2K_4^2}{ST}] + \mathrm{Log}^2(S/T)/2 \, . \end{aligned} \tag{2.8}$$

Factorisation. The remaining part of the amplitude is the rational $R_n^{(2)}$. As a rational function we may wish to obtain this via recursion provided we can control its singularities. We wish to use complex recursion to determine $R(z)$. Britto-Cachazo-Feng-Witten recursion [18] exploited the analytic properties of n-point tree amplitude under a complex shift of its external momenta to compute the amplitude. The momentum shift introduces a complex parameter, z, whilst preserving overall momentum conservation and keeping all external momenta null. Possible shifts include the original BCFW shift which acts on two momenta, say p_a and p_b, by

$$\bar\lambda_a \to \bar\lambda_{\hat a} = \bar\lambda_a - z\bar\lambda_b, \lambda_b \to \lambda_{\hat b} = \lambda_b + z\lambda_a \, , \tag{2.9}$$

and the Risager shift [19] which acts on three momenta, say p_a, p_b and p_c, by shifting λ_a

$$\lambda_a \to \lambda_{\hat a} = \lambda_a \; + z\,[bc]\lambda_\eta \, , \tag{2.10}$$

and cyclically λ_b and λ_c. In the last case λ_η must satisfy $\langle a\,\eta\rangle \neq 0$ etc., but is otherwise unconstrained. After applying the shift, the rational quantity of interest is a complex function parametrized by z i.e. $R(z)$. If $R(z)$ vanishes at large $|z|$, the Cauchy's theorem applied to $R(z)/z$ over a contour at infinity implies

$$R = R(0) = -\sum_{z_j\neq 0} \mathrm{Res}\Big[\frac{R(z)}{z}\Big]\Big|_{z_j} \, . \tag{2.11}$$

Tree amplitudes have simple poles when a shifted propagator vanishes and the corresponding residues are readily obtained from general factorisation theorems leading to the BCFW recursion formulae for tree amplitudes [18]. For the rational part of the two-loop all-plus amplitude the BCFW shift generates a shifted quantity that does not vanish at infinity and so cannot be used to reconstruct the amplitude (the one-loop all-plus amplitudes also behave in this way). However, using the Risager shift (2.10) does yield a shifted quantity with the desired asymptotic behaviour. Also loop amplitudes in non-supersymmetric theories may have double poles in complex momenta. Mathematically this is not a problem since if we consider a function with a double pole at $z = z_j$ and Laurent expansion,

$$R(z) = \frac{c_{-2}}{(z-z_j)^2} + \frac{c_{-1}}{(z-z_j)} + \mathscr{O}((z-z_j)^0) \,, \tag{2.12}$$

then the required residue is

$$\mathrm{Res}\Big[\frac{R(z)}{z}\Big]\Big|_{z_j} = -\frac{c_{-2}}{z_j^2} + \frac{c_{-1}}{z_j} \tag{2.13}$$

and we can use Cauchy's theorem *provided* we know the value of both the leading and sub-leading poles. The leading pole can be obtained from factorisation theorems, but, at this point, there are no general theorems determining the sub-leading pole and we need to determine the sub-leading pole for each specific case.

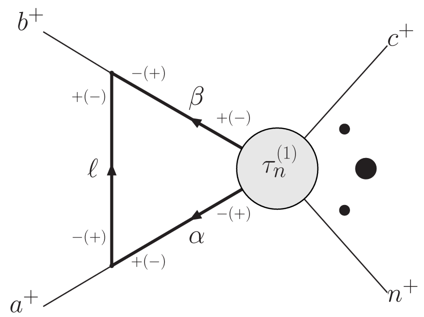

Figure 2: Diagram containing the leading and sub-leading poles as $s_{ab} \to 0$. The axial gauge construction permits the off-shell continuation of the internal legs.

We determine the sub-leading pole by determining the pole in the diagram shown in fig 2 using an axial gauge formalism. We have used this approach previously to compute one-loop amplitudes [20, 21, 22]. and labeled this process *augmented recursion*. The principal helicity assignment in fig 2 gives the integral

$$\frac{i}{(2\pi)^D} \int \frac{d^D\ell}{\ell^2\alpha^2\beta^2} \frac{[a|\ell|q\rangle [b|\ell|q\rangle}{\langle a\, q\rangle \langle b\, q\rangle} \frac{\langle \beta\, q\rangle^2}{\langle \alpha\, q\rangle^2} \tau_n^{(1)}(\alpha^-, \beta^+, c^+, ..., n^+) \,. \tag{2.14}$$

To determine 2.14 in general we would need to consider $\tau_n^{(1)}$ to be the doubly off-shell one-loop current. However, as we are only interested in the residue on the $s_{ab} \to 0$ pole, we do not need the full current. This process is detailed in ref. [8]. The resultant sub-leading pole is quite complex but can be substituted into 2.11 to yield the unshifted $R(0)$. The initial expression, after combining all factorisation, can be simplified into quite compact forms. We obtain a form for $R_6^{(2)}$ that is explicitly independent of q, has manifest cyclic symmetry and no spurious poles.

$$R_6^{(2)} = \frac{i}{9} \sum_{cyclicperms} \frac{G_6^1 + G_6^2 + G_6^3 + G_6^4 + G_6^5}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle}. \tag{2.15}$$

where

$$\begin{aligned} G_6^1 &= \frac{s_{cd}s_{df}\langle f|aK_{abc}|e\rangle}{\langle f e\rangle t_{abc}} + \frac{s_{ac}s_{dc}\langle a|fK_{def}|b\rangle}{\langle ab\rangle t_{def}} , \\ G_6^2 &= \frac{[ab]\,[fe]}{\langle ab\rangle \langle fe\rangle} \langle ae\rangle^2 \langle fb\rangle^2 + \frac{1}{2}\frac{[af]\,[cd]}{\langle af\rangle \langle cd\rangle} \langle ac\rangle^2 \langle df\rangle^2 , \\ G_6^3 &= \frac{s_{df}\,\langle af\rangle \langle cd\rangle [ca]\,[df]}{t_{abc}} \\ ,\ G_6^4 &= \frac{\langle a|be|f\rangle t_{def}}{\langle af\rangle} \end{aligned} \tag{2.16}$$

and

$$\begin{aligned} G_6^5 = {} & 2s_{ac}^2 + s_{eb}^2 + s_{ab}\left(-3s_{ac} - 2s_{ad} + 6s_{ae} + 4s_{bc} + s_{bd} + 2s_{be} + 4s_{bf} + 7s_{cd} - s_{ce} - s_{de} + 3s_{df}\right) \\ & + s_{ac}\left(2s_{ad} + 3s_{ae} - 2s_{bd} - s_{be} + s_{cf} - \tfrac{5}{2}s_{df}\right) + \tfrac{3}{2}s_{ad}s_{be} \\ & -8\,\langle bc\rangle [cd]\,\langle de\rangle [eb] + 5\,\langle fa\rangle [ac]\,\langle cd\rangle [df] , \end{aligned} \tag{2.17}$$

This was confirmed in an subsequent independent calculation [23].

3. The Seven-Point Rational Piece

We have also computed the seven point $R_7^{(2)}$ ref. [8]. The seven-point rational piece can be calculated in an identical fashion. The seven-point current $\tau_7^{(1)}(\alpha^-, \beta^+, c^+, d^+, e^+, f^+, g^+)$ is built from the corresponding seven-point single minus amplitude [24] just as the six-point current was built from the six-point amplitude. $R_6^{(2)}$ as determined above is also required for recursion. Defining

$$\begin{aligned} G_7^1 = {} & \frac{\langle ga\rangle}{t_{abc}t_{efg}}\Bigg(\frac{\langle cd\rangle [eg]\,[d|K_{abc}|e\rangle [a|K_{abc}|e\rangle [c|K_{abc}|f\rangle}{\langle ef\rangle} - \frac{\langle de\rangle [ca]\,[d|K_{efg}|c\rangle [g|K_{efg}|c\rangle [e|K_{efg}|b\rangle}{\langle bc\rangle} \\ & + \frac{\langle ef\rangle \langle cd\rangle [ca]\,[fg]\,[e|K_{efg}|a\rangle [d|K_{efg}|b\rangle}{\langle ab\rangle} - \frac{\langle bc\rangle \langle de\rangle [eg]\,[ab]\,[c|K_{abc}|g\rangle [d|K_{abc}|f\rangle}{\langle fg\rangle} \Bigg) , \\ G_7^2 = {} & \frac{1}{t_{abc}t_{efg}} s_{cd}s_{de}\,\langle ga\rangle\,[g|K_{efg}K_{abc}|a] , \\ G_7^3 = {} & \frac{1}{t_{cde}}\Bigg(s_{ce}\left(\frac{s_{ef}\langle c|K_{ab}K_{fga}|d\rangle}{\langle cd\rangle} - \frac{s_{bc}\langle e|K_{fg}K_{gab}|d\rangle}{\langle de\rangle} \right) + \frac{\langle ef\rangle \langle bc\rangle [fb]\,[c|K_{cde}|g\rangle [e|K_{cde}|a\rangle}{\langle ga\rangle} \\ & + \frac{\langle bc\rangle [c|K_{cde}|b\rangle [e|K_{cde}|a\rangle [b|K_{fg}|e\rangle}{\langle ab\rangle} + \frac{\langle ef\rangle [e|K_{cde}|f\rangle [c|K_{cde}|g\rangle [f|K_{ab}|c\rangle}{\langle fg\rangle} \Bigg) , \end{aligned}$$

$$
\begin{aligned}
G_7^4 &= \frac{[g\,a]}{\langle g\,a\rangle}\langle g\,e\rangle\langle a\,e\rangle\left(\frac{[d\,e]}{\langle d\,e\rangle}\langle d\,g\rangle\langle d\,a\rangle+\frac{[e\,f]}{\langle e\,f\rangle}\langle f\,g\rangle\langle f\,a\rangle\right)\,,\\
G_7^5 &= \frac{1}{t_{cde}}\big([c\,e]\,(\langle e\,f\rangle\,[d\,f]\,\langle c|K_{ab}K_{fga}|d\rangle+\langle b\,c\rangle\,[d\,b]\,\langle e|K_{fg}K_{gab}|d\rangle)\\
&\qquad\qquad +\langle b\,c\rangle\,\langle e\,f\rangle\,(2\,\langle g\,a\rangle\,[c\,e]\,[f\,g]\,[a\,b]+[b\,f]\,[e|K_{ab}K_{fg}|c])\big)\,,\\
G_7^6 &= \frac{1}{\langle g\,a\rangle}(\langle g|fK_{bc}|a\rangle t_{efg}-\langle a|bK_{ef}|g\rangle t_{abc})\\
G_7^7 &= s_{bf}^2-2s_{ga}^2-3s_{db}s_{df}+4s_{da}s_{dg}-6s_{ac}s_{eg}+7(s_{eb}s_{fc}+s_{ea}s_{gc})+s_{ab}s_{fg}+3s_{fa}s_{gb}\\
&\quad +s_{ce}(s_{cf}+s_{eb}-4(s_{ab}+s_{fg}+s_{ga})+5[d|K_{ga}|d\rangle)\\
&\quad +4[e|bcf|e\rangle-2[f|gab|f\rangle+3[g|baf|g\rangle+2[g|cea|g\rangle, \qquad (3.1)
\end{aligned}
$$

the full function in this case is

$$
R_7^{(2)}=\frac{i}{9}\sum_{\text{cyclic perms}}\frac{G_7^1+G_7^2+G_7^3+G_7^4+G_7^5+G_7^6+G_7^7}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 6\,7\rangle\,\langle 7\,1\rangle}\,. \qquad (3.2)
$$

This expression has the full cyclic and flip symmetries required and has all the correct factorisations and collinear limits. It been generated under the assumption that the shifted rational function vanishes at infinity: if this had been unjustified we would not have generated a function with the appropriate symmetries. This completes the seven-point calculation: the first seven point helicity amplitude obtained in QCD.

4. Summary and Prospects

[27] We have been able to use the singularity structure of amplitudes to obtain higher-point two-loop QCD amplitudes. The methods have avoided complicated two-loop integrations. Correspomding results have been obtained for two-loop gravity where the UV term may be simple obtained [25]. So far, we have only been able to generate the simplest helicity amplitude by these methods but have obtained these in compact analytic forms which complement recent progress in numerical techniques [26, 27] The effort to extend these methods to further helicity configurations is on-going.

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