

# Probing TMDs in heavy quarkonium production in $pp$ collision

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**Asmita Mukherjee<sup>\*†</sup> and Sangem Rajesh**

*Department of Physics, Indian Institute of Technology Bombay  
Powai, Mumbai 400076, India*

We present a recent calculation of  $J/\psi$  and  $\Upsilon$  production in unpolarized  $pp$  collision and show that this can be used to probe the unpolarized gluon as well as the linearly polarized gluon transverse momentum dependent parton distributions (TMDs). We use the color evaporation model for the heavy quarkonium production and use a generalized factorized form of the cross section. We compare the results with experimental data.

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<sup>\*</sup>Speaker.

<sup>†</sup>Email :asmita@phy.iitb.ac.in

## 1. Introduction

Single spin asymmetries (SSAs) when either the target or one of the colliding protons is polarized have been experimentally observed since a long time [1, 2, 3, 4, 5]. Two main approaches to explain it theoretically are (1) collinear framework [6] involving higher twist quark or gluon correlators and (2) transverse momentum dependent distribution (TMD) and fragmentation functions [7]. While the former approach is the first one and is free from several complications associated with the TMD framework, the TMD based approach is quite useful for phenomenological studies. Here one uses a generalized factorized framework in terms of the TMDs. Gauge invariance needs the inclusion of gauge links or Wilson lines in the operator definition of TMDs. As these gauge links depend on the process in which the TMDs are probed, this introduces process dependence. Thus there are issues related to universality and applicability of factorization for different processes in the TMD formalism. For simpler processes, like the semi-inclusive deep inelastic scattering (SIDIS) or Drell-Yan (DY) the cross section can be written using a generalized factorization. The TMDs are functions of the longitudinal momentum fraction  $x$  and transverse momentum  $k_{\perp}$  of the partons (quark, antiquark or gluon). These TMDs generate some asymmetries in the azimuthal angle of the observed particle in the final state that can give information on the spin and orbital angular momentum of the quarks and gluons. However, it is also important to understand the unpolarized TMDs, not only because they give the momentum distribution of the partons but also because they appear in the denominator of the spin asymmetries, so one needs a good understanding of them in order to understand the asymmetries [8].  $pp$  collisions are direct tools to probe the gluon TMDs, which play an important role in the cross section and asymmetries in the collider kinematics. It is known that there is a non-zero probability of finding linearly polarized gluons in an unpolarized proton, provided they have non-zero transverse momenta [9]. The corresponding TMD is denoted by  $h_1^{\perp g}$  and it is a time-reversal even object. At leading twist, the gluon correlator of an unpolarized proton is parametrized in terms of the unpolarized TMD  $f_1$  and the linearly polarized gluon TMD  $h_1^{\perp g}$ . Although  $h_1^{\perp g}$  has not been extracted from data yet, there are already quite a few theoretical studies for possible extraction from different experiments [10, 11, 12, 13, 14, 15, 16, 17]. Here we present a recent study of the possibility to probe it in heavy quarkonium production in  $pp$  collision [18].

## 2. CHARMONIUM ( $J/\psi$ ) AND BOTTOMONIUM ( $\Upsilon$ ) PRODUCTION CROSS SECTION

There are mainly three models for heavy quarkonium production. In all these models, the cross section is factorized into a hard part where the quarks and gluons form the heavy quark and antiquark pair, and a soft or non-perturbative part where the heavy quark pair forms a bound state with definite quantum numbers. In the color singlet model (CSM) the heavy quark pair is formed in a color singlet state. In the non-relativistic QCD (NRQCD) based approach the heavy quark pair can be produced in both color octet and color singlet state, and the long distance factor or the non-perturbative matrix element for the formation of the quarkonium can be expanded in powers of  $v$  where  $v$  is the relative velocity of the heavy quark in the quarkonium rest frame. In the color evaporation model (CEM) [19], that we use in this calculation, the heavy quark pair radiates

soft gluons to form a quarkonium state of definite quantum numbers. Color of the  $Q\bar{Q}$  pair does not affect the color of the bound state. The long distance factors in this model are considered to be independent of the process and obtained by fitting data. The cross section for charmonium production in CEM is given by [20] :

$$\sigma = \frac{\rho}{9} \int_{2m_Q}^{2m_{Q\bar{q}}} dM \frac{d\sigma_{Q\bar{Q}}}{dM}, \quad (2.1)$$

where  $m_Q$  is the mass of the heavy quark and  $m_{Q\bar{q}}$  is the mass of lightest heavy meson.  $M$  is the invariant mass of the  $Q\bar{Q}$  pair.  $\rho$  is long distance factor and we took 0.47 and 0.62 for production of  $J/\psi$  and  $\Upsilon$  respectively.

We consider unpolarized proton-proton collision

$$h(P_A) + h(P_B) \rightarrow Q\bar{Q}(q) + X, \quad (2.2)$$

where the four momenta of the particles are given within round brackets. The leading order (LO) subprocesses are  $gg \rightarrow Q\bar{Q}$  and  $q\bar{q} \rightarrow Q\bar{Q}$ . The differential cross section assuming generalized factorization is written as :

$$\frac{d^4\sigma}{dydM^2d^2\mathbf{q}_T} = \frac{\rho}{18} \int dx_a dx_b d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} \delta^4(p_a + p_b - q) \Phi_g^{\mu\nu}(x_a, \mathbf{k}_{\perp a}) \Phi_{g\mu\nu}(x_b, \mathbf{k}_{\perp b}) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}. \quad (2.3)$$

Here  $q_T$  is the transverse momentum of the quarkonium and in the center-of-mass frame of the incident hadrons, where each of the hadrons move along the  $z$  axis.  $k_{\perp a}$  and  $k_{\perp b}$  are the transverse momenta of the incoming gluons;  $\Phi_g^{\mu\nu}$  is the gluon correlator which are parametrized in term soft TMDs. Contribution from the  $q\bar{q}$  channel is found to be very small in the kinematics of the colliders considered. So we consider only the  $gg$  channel. At leading twist, the parametrization of the gluon correlator is given by,

$$\Phi_g^{\mu\nu}(x, \mathbf{k}_{\perp}) = \frac{n_\rho n_\sigma}{(k \cdot n)^2} \int \frac{d(\lambda \cdot P) d^2\lambda_T}{(2\pi)^3} e^{ik \cdot \lambda} \langle P | \text{Tr}[F^{\mu\rho}(0) F^{\nu\sigma}(\lambda)] | P \rangle |_{LF} \quad (2.4)$$

$$= -\frac{1}{2x} \left\{ g_T^{\mu\nu} f_1^g(x, \mathbf{k}_{\perp}^2) - \left( \frac{k_{\perp}^\mu k_{\perp}^\nu}{M_h^2} + g_T^{\mu\nu} \frac{\mathbf{k}_{\perp}^2}{2M_h^2} \right) h_1^{\perp g}(x, \mathbf{k}_{\perp}^2) \right\}. \quad (2.5)$$

$k_{\perp}^2 = -\mathbf{k}_{\perp}^2$ ,  $g_T^{\mu\nu} = g^{\mu\nu} - P^\mu n^\nu / P \cdot n - n^\mu P^\nu / P \cdot n$  and  $M_h$  is the mass of proton. The unpolarized and the linearly polarized gluon distribution functions are denoted by  $f_1^g(x, \mathbf{k}_{\perp}^2)$  and  $h_1^{\perp g}(x, \mathbf{k}_{\perp}^2)$ , respectively. In terms of the TMDs, the differential cross section takes the form :

$$\frac{d^4\sigma}{dydM^2d^2\mathbf{q}_T} = \frac{\rho}{18} \int \frac{dx_a dx_b}{2x_a x_b} d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} \delta^4(p_a + p_b - q) \left\{ f_1^g(x_a, \mathbf{k}_{\perp a}^2) f_1^g(x_b, \mathbf{k}_{\perp b}^2) + wh_1^{\perp g}(x_a, \mathbf{k}_{\perp a}^2) h_1^{\perp g}(x_b, \mathbf{k}_{\perp b}^2) \right\} \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2) \quad (2.6)$$

where  $w$  is weight factor:

$$w = \frac{1}{2M_h^4} \left[ (\mathbf{k}_{\perp a} \cdot \mathbf{k}_{\perp b})^2 - \frac{1}{2} \mathbf{k}_{\perp a}^2 \mathbf{k}_{\perp b}^2 \right]. \quad (2.7)$$

As stated above, we neglect the contribution from the  $q\bar{q}$  channel. Cross section for the gluon initiated subprocess is calculated perturbatively. Using the momentum conserving delta function, we obtain

$$x_{a,b} = \frac{M}{\sqrt{s}} e^{\pm y}, \quad (2.8)$$

where  $y$  is the rapidity and  $\sqrt{s}$  is the center-of-mass energy of the experiment.

### 3. MODEL FOR TMDs AND TMD EVOLUTION

We assume a Gaussian form for the transverse momentum dependence of the TMDs [21] :

$$f_1^g(x, \mathbf{k}_{\perp}^2) = f_1^g(x, Q^2) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-\mathbf{k}_{\perp}^2 / \langle k_{\perp}^2 \rangle}. \quad (3.1)$$

$f_1^g(x, Q^2)$  is the unpolarized gluon distributions (pdfs), the scale is given by  $Q^2 = M^2$ . For the numerical calculation, we have chosen MSTW2008 distribution [27]. The factorized form of  $h_1^{\perp g}$  [17] is given by

$$h_1^{\perp g}(x, \mathbf{k}_{\perp}^2) = \frac{M_h^2 f_1^g(x, Q^2)}{\pi \langle k_{\perp}^2 \rangle^2} \frac{2(1-r)}{r} e^{-\mathbf{k}_{\perp}^2 / r \langle k_{\perp}^2 \rangle}, \quad (3.2)$$

where  $r$  is the parameter which has the range  $0 < r < 1$ . We have chosen two values for  $r$ ,  $r = 1/3$  and  $r = 2/3$ . We use two values for squared intrinsic average transverse momentum of gluons and quarks:  $\langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$  and  $1 \text{ GeV}^2$  [17]. In model I, we have integrated over the full range of  $k_{\perp}$  whereas in model II, we have used an upper bound,  $k_{\max} = \sqrt{\langle k_{\perp a}^2 \rangle}$ . Evolution of the unpolarized pdfs with the scale is given by the DGLAP evolution equation. On the other hand, TMD evolution is performed in  $b$  space [22]. In terms of the TMDs in  $b$  space, the differential cross section is given by :

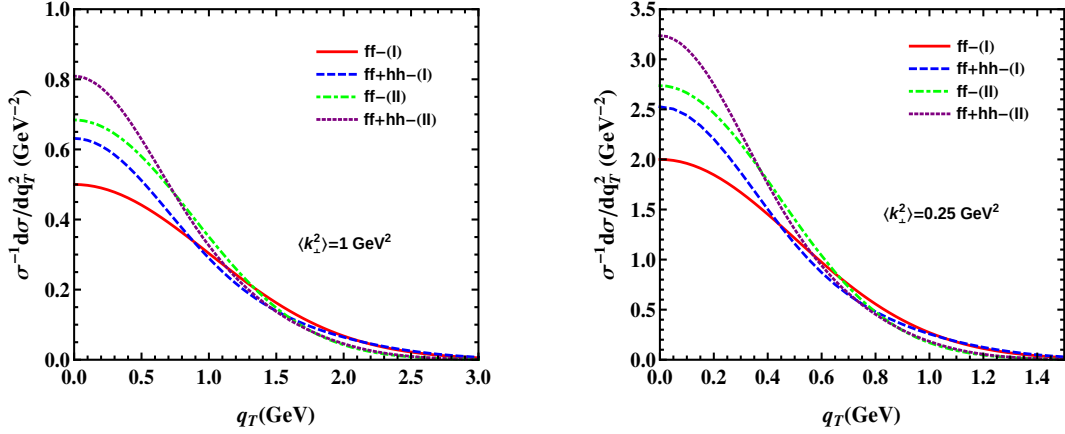
$$\frac{d^4 \sigma}{dy dM^2 d^2 \mathbf{q}_T} = \frac{\rho}{18s} \frac{1}{2\pi} \int_0^{\infty} b_{\perp} db_{\perp} J_0(q_T b_{\perp}) \left\{ f_1^g(x_a, b_{\perp}^2) f_1^g(x_b, b_{\perp}^2) \right. \quad (3.3)$$

$$\left. + h_1^{\perp g}(x_a, b_{\perp}^2) h_1^{\perp g}(x_b, b_{\perp}^2) \right\} \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2), \quad (3.4)$$

where  $J_0$  is the Bessel function. The scale dependence of the TMDs is not explicitly shown above. They depend on the renormalization scale  $\mu$  and the auxiliary parameter  $\zeta$ . Using Collin-Soper and renormalization group equations, we can write [22]:

$$f(x, b_{\perp}, Q_f, \zeta) = f(x, b_{\perp}, Q_i, \zeta) R_{pert}(Q_f, Q_i, b_*) R_{NP}(Q_f, Q_i, b_{\perp}), \quad (3.5)$$

where  $R_{pert}$  and  $R_{NP}$  denote the perturbative and non-perturbative parts of the evolution kernel, respectively.  $c/b_*$  is the initial scale where  $c = 2e^{-\gamma_E}$  with the Euler's constant  $\gamma_E \approx 0.577$ . We



**Figure 1:** Normalized differential cross section of  $J/\psi$  and  $\Upsilon$  production in  $pp \rightarrow Q\bar{Q} + X$  at LHCb ( $\sqrt{s} = 7$  TeV), RHIC ( $\sqrt{s} = 500$  GeV) and AFTER ( $\sqrt{s} = 115$  GeV) energies using DGLAP evolution approach for  $r = \frac{2}{3}$ . The solid (ff-(I)) and dot dashed (ff-(II)) lines are obtained by considering unpolarized gluons and quarks in Model-I and Model-II respectively. The dashed (ff+hh-(I)) and tiny dashed (ff+hh-(II)) lines are obtained by taking into account unpolarized gluons and quarks plus linearly polarized gluons in Model-I and Model-II respectively [18].

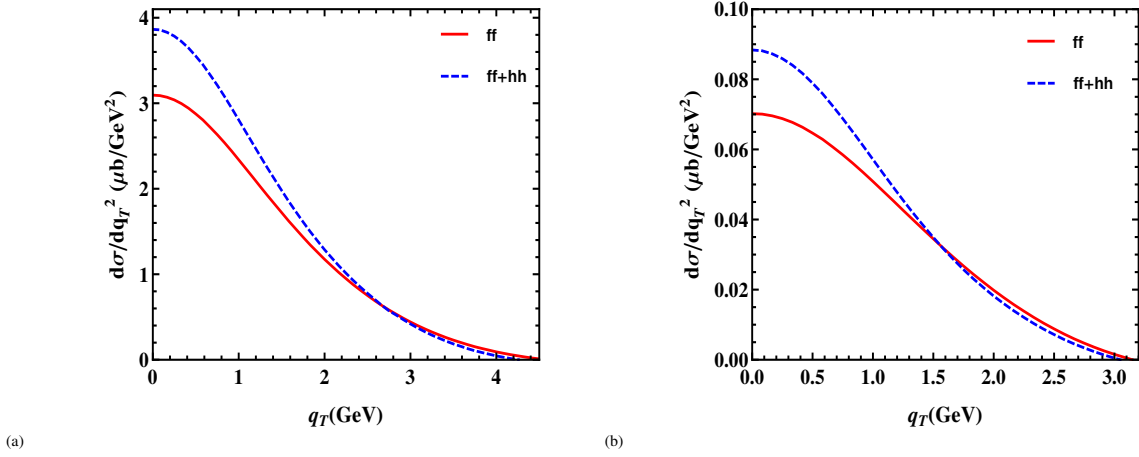
have used the  $b_*$  prescription, with  $b_*(b_{\perp}) = \frac{b_{\perp}}{\sqrt{1 + \left(\frac{b_{\perp}}{b_{\max}}\right)^2}} \approx b_{\max}$ . We have used the leading order (LO) anomalous dimensions in  $R_{pert}$  and for  $R_{NP}$ , the parametrization from [23]. No experimental data is yet available for the extraction of  $h_1^{\perp g}$ , and we use the same  $R_{NP}$  for it as for the unpolarized distribution.

#### 4. NUMERICAL RESULTS

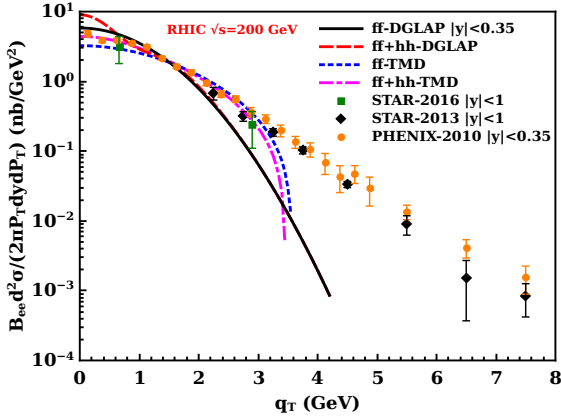
We calculate the transverse momentum distribution for  $J/\psi$  and  $\Upsilon$  production. For  $J/\psi$  production, we took the charm quark mass ( $m_c = 1.275$  GeV) for  $m_Q$  and lightest D meson mass ( $m_D = 1.863$  GeV) for  $m_{Q\bar{q}}$ . For the  $\Upsilon$ , bottom quark mass ( $m_b = 4.18$  GeV) for  $m_Q$  and lightest B meson mass ( $m_B = 5.279$  GeV) for  $m_{Q\bar{q}}$  were used. The ranges of rapidity integration are :  $y \in [2.0, 4.5]$ ,  $y \in [-3.0, 3.0]$  and  $y \in [-0.5, 0.5]$  for LHCb, RHIC and AFTER respectively, to obtain the differential cross section as a function of  $q_T$ .

The  $q_T$  distributions for  $J/\Psi$  and  $\Upsilon$  at the center-of-mass energies of different experiments are shown in Fig. 1. We have normalized the results by the total cross section. In this plot, we have not incorporated the TMD evolution, instead only the DGLAP evolution of the unpolarized pdf is used. The normalized results overlap for the different kinematics of different experiments. The results are larger in magnitude in model II compared to model I. In particular for lower values of  $q_T$ , the effect of linearly polarized gluons are seen in the cross section. Above  $q_T \approx 1$  GeV this effect is not seen any more. We have shown the results for two values of  $\langle k_{\perp}^2 \rangle$ , namely 0.25 and 1 GeV<sup>2</sup> respectively. For small value of the Gaussian width, the magnitude is higher. We have chosen  $r = 2/3$ .

In Fig. 2, we have shown the  $q_T$  distributions for  $J/\Psi$  production for the kinematics of LHCb and AFTER at LHC respectively. In these plots we have incorporated the TMD evolution. The



**Figure 2:** Differential cross section of  $J/\psi$  as function of  $q_T$  at LHCb ( $\sqrt{s} = 7$  TeV) (left panel) and AFTER ( $\sqrt{s} = 115$  GeV) (right panel) energies using TMD evolution approach. The solid (ff) and dashed (ff+hh) lines are obtained by considering unpolarized gluons only and unpolarized plus linearly polarized gluons respectively [18].



**Figure 3:** Differential cross section of  $J/\psi$  production calculated in CEM model as function of transverse momentum in the dielectron decay channel. Center-of-mass energy is 200 GeV. DGLAP denotes results calculated using DGLAP evolution for the unpolarized pdf, TMD indicates the results are calculated in the TMD evolution approach. Theoretical results are compared with experimental data from the STAR [24, 25] and PHENIX [26] experiment at RHIC.

results are not normalized here by the total cross section. We see again that at low  $q_T$  values the cross section is modified when contribution from linearly polarized gluons are taken into account.

In Fig. 3, we have compared our results with the experimental data from STAR [24, 25] and PHENIX experiments [26] at RHIC. Here we use the dielectron decay channel of  $J/\Psi$ .  $B_{ee}$  is the branching ratio for this channel. In this plot we have used the overall normalization to be  $\rho = 0.9$ . It is seen that the data is described well by the theoretical plot, especially for low values of  $q_T$ . The TMD evolved plots match the data upto  $q_T \approx 3$  GeV, and the plots using DGLAP evolution fall faster and match the data only upto  $q_T \approx 2$  GeV. The effect of the linearly polarized gluons in the cross section is not that much visible, due to the log scale of the y-axis. It has been

shown that CEM explains the data quite well till about  $q_T = 10$  GeV [24] when higher order corrections are incorporated. Further work in this direction in the TMD approach would include the process dependent gauge links, and also the so called  $Y$ -term, which we did not include in our phenomenological study here.

## 5. Conclusion

We presented a recent calculation of heavy quarkonium production in unpolarized  $pp$  collision in CEM using TMD formalism. At leading order the gluon-gluon channel dominates. We have shown that the cross section has substantial effect from the linearly polarized gluons at low  $q_T$  of the heavy quarkonium. We predicted the results for the kinematics of different experiments and compared with data from RHIC. We found that the TMD evolution formalism gives a better agreement with the data. Thus, heavy quarkonium production in  $pp$  collision is an important tool to probe the unpolarized gluon TMDs and linearly polarized gluon TMDs.

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