SIDIS cross sections: perturbative and non-perturbative aspects

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I will address the study of Semi-Inclusive Deep Inelastic Scattering (SIDIS) cross sections over a wide range of transverse momenta, and in particular in the region of low to moderate q_T, where the interplay between perturbative and non-perturbative effects become more relevant. Some realistic SIDIS kinematic will be considered where the commonly used prescription of "matching" through the so-called Y-factor cannot be applied.

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1. Introduction

Describing a hadronic process at high resolution scale Q, over the whole range of transverse momenta q_T, is a highly non-trivial task. Collinear perturbative QCD calculations allow us to compute its cross section at large q_T, where $q_T > Q$, but fail at low q_T.

Let's consider, for example, the q_T differential cross section typical of a Drell Yan (DY) process. Although fixed order calculations correctly reproduce its large q_T behaviour, they cannot describe DY data at small q_T: in fact, at Born level the cross section is vanishing while at order α_s it is badly divergent. Low energy DY data, instead, show that low q_T cross sections are not divergent. Its M^2 dependence is remarkably well described by a Gaussian distribution, due to a successful interplay among the $1/M^2$ behaviour of the Born cross section, the DGLAP evolution in Q^2 and kinematics. Considering the same DY processes at different (but still low) energies, one easily realizes that data keep being consistent with a Gaussian behaviour, but require larger and larger Gaussian widths with increasing $\sqrt{s}$. This is shown in Fig. 1 where the q_T distributions from the E288 experiment [1] at three different c.m. energies are reported: from left to right, $\sqrt{s} = 19.4$ GeV, $\sqrt{s} = 23.8$ GeV and $\sqrt{s} = 27.4$ GeV. In each panel, different ranges in M can be successfully reproduced by the same Gaussian width, while increasing values of $\sqrt{s}$ indicate the need of growing Gaussian widths: from left to right $\langle k_T^2 \rangle = 0.47$ GeV$^2/c^2$, $\langle k_T^2 \rangle = 0.57$ GeV$^2/c^2$, $\langle k_T^2 \rangle = 0.71$ GeV$^2/c^2$ respectively. On the other hand, as soon as q_T grows larger, DY cross sections do not follow a Gaussian distribution anymore, and present the expected pertubative tail, well described by collinear QCD computations. This is shown in Fig. 2, where DY data from the CDF Collaboration [2], corresponding to $\sqrt{s} = 1.96$ TeV, are compared to a typical Gaussian distribution.

Diverging contributions in the range of lower q_T are due to large logarithms arising from the emission of soft and collinear gluons; they need to be resummed in order to obtain the physical cross section. Here α_s cannot be used as the effective expansion parameter of the perturbative series; instead, in this region, a perturbative expansion in terms of logarithms is performed, and this perturbative series is then resummed into the so-called Sudakov exponential form factor. This is
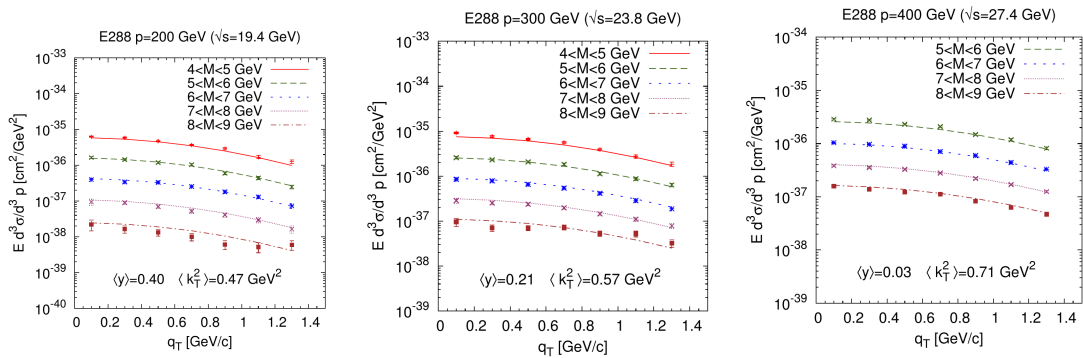
Figure 1: The q_T distributions from the E288 experiment [1] at three different c.m. energies are reported: left panel $\sqrt{s} = 19.4$ GeV, central panel $\sqrt{s} = 23.8$ GeV, right panel $\sqrt{s} = 27.4$ GeV. At each fixed value of $\sqrt{s}$, i.e. in each panel, different ranges in M can be successfully reproduced by the same Gaussian width, while increasing values of $\sqrt{s}$ indicate the need of growing Gaussian widths: left panel $\langle k_T^2 \rangle = 0.47$ GeV$^2/c^2$, central panel $\langle k_T^2 \rangle = 0.57$ GeV$^2/c^2$, right panel $\langle k_T^2 \rangle = 0.71$ GeV$^2/c^2$.
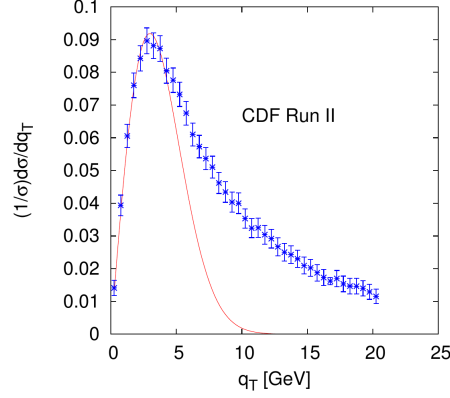
Figure 2: The q_T distributions from the CDF Collaboration at Fermilab (Run II), at $\sqrt{s} = 1.96$ TeV. Data are from Ref. [2].

usually achieved by applying the Collins-Soper-Sterman (CSS) factorization scheme [3] or, equivalently, the improved Transverse Momentum Dependent (TMD) framework, developed more recently [4, 5]. In fact, these two formalisms differ only at higher orders in α_s, provided the auxiliary scales ζ_F and ζ_D are appropriately fixed so that $\zeta_F = \zeta_D = Q^2$. This equivalence was explicitly shown, for example, in Appendix B of Ref. [6].

Resummation is performed in the impact parameter space, b_T, the Fourier conjugate of the transverse momentum space, where momentum conservation can easily be applied. The cross section is then separated into two parts: the resummed term, W, which contains the whole essence of resummation itself, and the Y term, which is regular at small q_T (i.e. less singular than $1/q_T^2$). The CSS resummation scheme was originally developed for DY processes, but can be applied to Semi-Inclusive Deep Inelastic Scattering (SIDIS) reactions as well. For unpolarized SIDIS processes, $\ell N \rightarrow \ell h X$, the following CSS expression [7, 8] holds

$$\frac{d\sigma^{total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{DIS} \int \frac{d^2\boldsymbol{b}_T\, e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} W^{SIDIS}(x,z,b_T,Q) + Y^{SIDIS}(x,z,q_T,Q), \tag{1.1}$$

where q_T is the virtual photon momentum in the frame where the incident nucleon N and the produced hadron h are head to head, and σ_0^{DIS} is the LO elementary DIS cross section.

$$\sigma_0^{DIS} = \frac{4\pi\alpha_{em}^2}{sxy^2}\left(1 - y + \frac{y^2}{2}\right), \tag{1.2}$$

with the usual DIS kinematical variables $x = Q^2/(2P\cdot q)$, $y = P\cdot q/P\cdot l$. Notice that, for SIDIS, we most commonly refer to the transverse momentum $\boldsymbol{P}_T$ of the final detected hadron, h, in the $\gamma^* N$ c.m. frame, rather than to the virtual photon momentum $\boldsymbol{q}_T$, in the Nh c.m. frame. They are simply related by the hadronic momentum fraction z through the expression $\boldsymbol{P}_T = -z\boldsymbol{q}_T$, so that

$$\frac{d\sigma}{dx\,dy\,dz\,dP_T^2} = \frac{d\sigma}{dx\,dy\,dz\,dq_T^2}\frac{1}{z^2}. \tag{1.3}$$

We will come back on this distinction below. At this stage, it is worth stressing that working in b_T space makes phenomenological analyses more difficult, as we lose intuition and direct connection with "real world experience". In fact, experimental data are usually provided in momentum space.
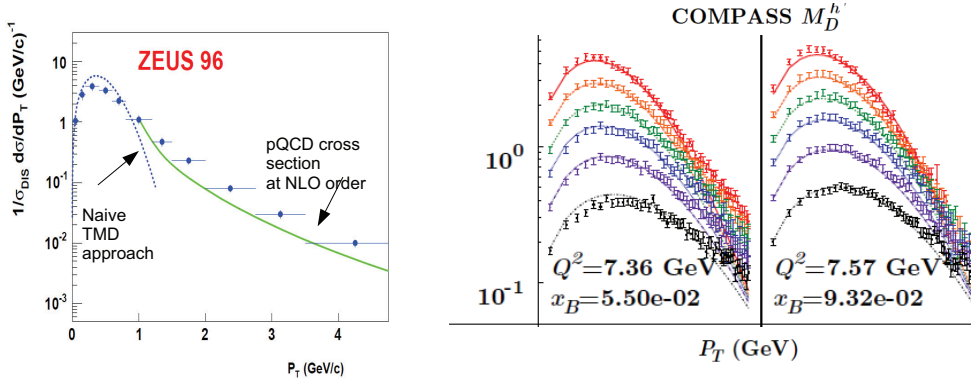
Figure 3: SIDIS cross sections show a Gaussian behaviour at very small q_T, where fixed order calculations diverge and cannot describe them correctly. However, they do not show a Gaussian tail at larger q_T, where perturbative QCD works well. The left panel is from Ref. [11], and shows the differential charged particle rates as a function of the transverse momentum of the produced hadron as measured by ZEUS detector at HERA [12], compared to the results presented in Ref. [11]. The right panel is from Ref. [9] and shows the COMPASS multiplicities distributions [13], in two bins of relatively large Q^2, compared to the results of the gaussian fit presented in Ref. [9].

A successful resummation scheme should take care of matching the fixed order hadronic cross section, computed in perturbative QCD at large q_T, with the so-called resummed cross section, valid at low $q_T \ll Q$, where large logarithms are properly treated. This matching should happen, roughly, at $q_T \sim Q$ where logarithms are small [3]. The regular Y-term, appropriately defined, should ensure a continuous and smooth matching of the cross section over the entire q_T range.

Nevertheless, the perturbative resummed series does not converge at extremely low values of q_T, where we expect the transverse momentum to be "intrinsic" rather than generated by gluon radiation. Phenomenological analyses show that both Drell-Yan (DY) and SIDIS cross sections are consistent with a Gaussian behaviour at very small values of q_T. Recent analyses based on naive Gaussian models and extensive discussions on this subject can be found, for example, in Refs. [9, 10]. However, neither DY nor SIDIS cross sections show a Gaussian tail at larger q_T where, instead, perturbative QCD works well. This is illustrated, for SIDIS processes, in Fig. 3.

For the full description of the cross section, one should therefore be able to incorporate in the resummation scheme the non-perturbative behaviour as well. It is common to define W^{NLL} the NLL resummed cross section which includes the non-perturbative Sudakov factor

$$W^{NLL} = \pi \sigma_0^{DIS} \int\limits_0^\infty \frac{db_T \, b_T}{(2\pi)} J_0(q_T b_T) W^{SIDIS}(x,z,b_*,Q) \exp\left[S_{NP}(x,z,b_T,Q)\right], \qquad (1.4)$$

with $W^{SIDIS}(x,z,b_*,Q)$ calculated at NLL order. The non-pertubative part of the cross section is subject to phenomenological prescriptions and needs to be modeled. A commonly used parameterization is

$$S_{NP} = \left(-\frac{g_1}{2} - \frac{g_{1f}}{2z^2} - g_2 \ln\left(\frac{Q}{Q_0}\right) \right) b_T^2. \qquad (1.5)$$

Different values of the parameters g_1, g_{1f} and g_2, should, in principle, affect the hadronic cross section only in the range where $q_T \to 0$. Instead, it turns out that, for SIDIS processes like those

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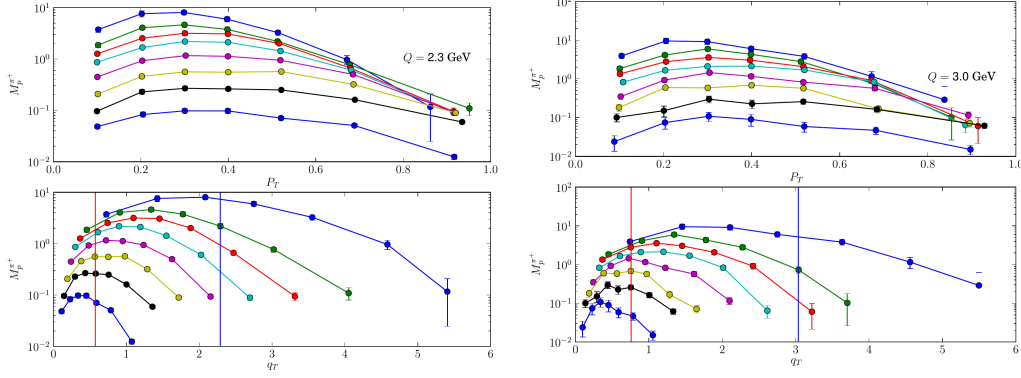
Figure 4: Two examples of data selection, according to P_T (upper panels) and according to q_T (lower panels), for $Q = 2.3$ GeV (left panels) and $Q = 3.0$ GeV (right panel).

measured by COMPASS [13] and HERMES [14] Collaborations, where Q is only a few GeV's, the modeled non-perturbative contributions dominate over the entire range of measured q_T's. Therefore any resummation scheme would be inadequate in this case, and hardly applicable. This was shown and thoroughly discussed in Ref. [6].

Another controversial issue, which plays an important role in SIDIS phenomenological analyses, concerns the criteria used for data selection. The original CSS factorization scheme was derived for DY processes, where the relevant scales are Q and q_T. Instead, as mentioned above, SIDIS differential cross sections are usually provided in terms of P_T, the transverse momentum of the measured final hadron h. Therefore, previous analyses of HERMES and COMPASS multiplicities were performed by selecting data, bin by bin, according to the value of P_T as compared to their relative value of Q. However, it is important to point out that, as P_T and q_T are related by $\boldsymbol{P}_T = -z\boldsymbol{q}_T$, very different results are obtained by using q_T rather than P_T, especially at low z. This effect is shown in Fig. 4, where the results obtained by cutting at $P_T < 0.9$ are shown in the upper panels (for $Q = 2.3$ GeV on the left and for $Q = 3.0$ GeV on the right). The lower panels show the same data, plotted as a function of q_T instead of P_T; here the vertical red lines correspond to $q_T = Q/4$ while the blue vertical lines mark $q_T = Q$. From this plot one could conclude that the region corresponding to $q_T \ll Q$ should be roughly limited to the data points falling to the left of the red vertical lines, which, in turn, would lead to excluding most of the experimental information on the low q_T behaviour of the SIDIS cross section. Moreover, in this case, one should expect the matching between the fixed order perturbative cross section and the resummed term to fall at (or around) the vertical blue line, where $q_T \sim Q$, i.e. just one or two GeVs above the "very small" q_T region.

A pictorial representation of this situation, more in general, is given in Fig. 5 and can be summarized as follows. The TMD factorization scheme holds and can be applied when four different ranges of q_T values can clearly be defined, and are neatly separated:

1. $q_T \sim \lambda_{QCD}$, where the transverse momentum is expected to be "intrinsic";

2. $q_T \ll Q$, where TMD evolution is expected to be at work;
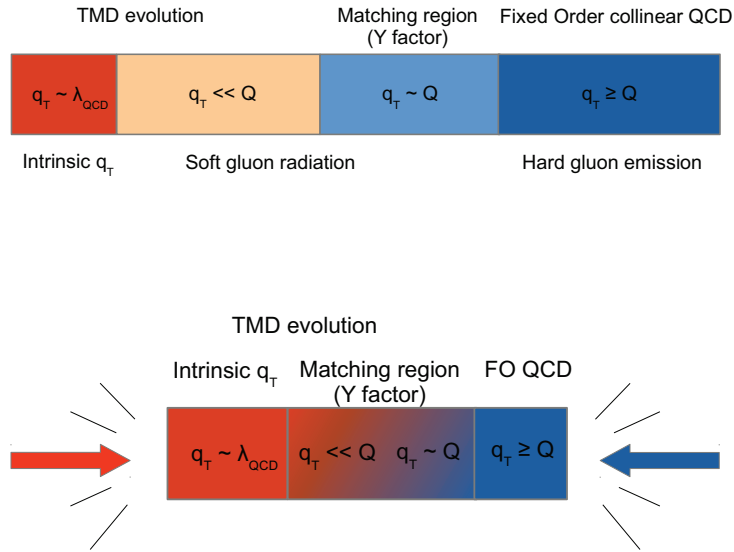
Figure 5: Different configurations of the q_T ranges in a TMD factorization framework.

3. $q_T \sim Q$, where the matching between the fixed order perturbative cross section and the re-summed term should take place;

4. $q_T > Q$, where the cross section can be computed perturbatively, at fixed order.

According to our more recent studies [6, 15, 16], while in DY processes this is most usually the case, for SIDIS processes as measured at COMPASS and HERMES, where Q is limited to a few GeVs, the q_T range is extremely small, and all four regions are "compressed" and tend to overlap. Far from being neatly separated, they become very difficult to be identified: so much so that there is no room for resummation, nor space enough to perform a conventional matching. In this cases, non-perturbative effects completely dominate the cross section over the whole q_T range.

Issues related to the identification of the current fragmentation region – the kinematical regime where a factorization picture with fragmentation functions is appropriate – has recently been addressed in Ref. [17], where general criteria for estimating the proximity to the current region at large Q are presented.

Acknowledgments

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