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Current status of ε_K in lattice QCD

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We present updated results of ε_K evaluated directly from the standard model with lattice QCD inputs. Here, we use the lattice QCD inputs for $\hat{B}_K$, $|V_{cb}|$, ξ_0, ξ_2, $|V_{us}|$, and $m_c(m_c)$. Recently, FLAG has updated $\hat{B}_K$. RBC-UKQCD has also updated ξ_0 and ξ_2. Exclusive $|V_{cb}|$ has been updated with new lattice data in the $\bar{B} \to D\ell\bar{\nu}$ decay mode, too. We find that the theoretical value of ε_K with exclusive $|V_{cb}|$ (lattice QCD inputs) evaluated directly from the standard model is 3.2σ lower than the experimental value, while that with inclusive $|V_{cb}|$ (heavy quark expansion) has no tension.

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1. Introduction

We have been monitoring ε_K since 2012, which is the indirect CP violation parameter in neutral kaons calculated directly from the standard model (SM) using lattice QCD inputs. The parameter ε_K is very precisely measured in experiment. From the theoretical point of view, it comes from the FCNC loop effects of box diagrams in the SM, and so provide a direct probe of CP violation in the neutral kaon system. Hence, naturally it is sensitive to physics models beyond the standard model (BSM). In this paper, we present results of ε_K evaluated directly from the SM with lattice QCD inputs. We also compare them with the experimental results. This paper is an update of our previous paper [1, 2].

2. Input parameters

The master formula for ε_K in the SM is

$$\varepsilon_K = e^{i\theta}\sqrt{2}\sin\theta\left(C_\varepsilon X_{\rm SD}\hat{B}_K + \frac{\xi_0}{\sqrt{2}} + \xi_{\rm LD}\right) + \mathcal{O}(\omega\varepsilon') + \mathcal{O}(\xi_0\Gamma_2/\Gamma_1)\,. \tag{2.1}$$

Here, the short distance contribution proportional to $\hat{B}_K$ gives a contribution of about 105% of ε_K. The long distance effect, ξ_0 from the absorptive part gives about -5% correction. The long distance effect, $\xi_{\rm LD}$ from the dispersive part gives about $\pm 1.6\%$ correction. Details on remaining input parameters such as C_ε, $X_{\rm SD}$, ξ_0, and $\xi_{\rm LD}$ are given in Ref. [1]. We need 18 input parameters to determine ε_K in the SM. Six of them can, in principle, be obtained from lattice QCD: $\hat{B}_K$, V_{cb}, V_{us}, ξ_0, $\xi_{\rm LD}$, and $m_c(m_c)$. Here, we address recent progress on determining those input parameters.

Decay mode	$\lvert V_{ub}\rvert$	Ref.
$\bar{B}\to\pi\ell\bar{\nu}$	3.72(16)	[3]
$\bar{B}\to\pi\ell\bar{\nu}$	3.61(32)	[4]
ex-combined	3.70(14)	this paper
$\bar{B}\to X_u\ell\bar{\nu}$	4.45(16)(22)	[5]

Table 1: Results for $\lvert V_{ub}\rvert$

Decay mode	$\lvert V_{cb}\rvert$	Ref.
$\bar{B}\to D^*\ell\bar{\nu}$	39.04(49)(53)(19)	[6]
$\bar{B}\to D\ell\bar{\nu}$	40.7(10)(2)	[7]
ex-combined	39.62(60)	this paper
$\bar{B}\to X_c\ell\bar{\nu}$	42.00(64)	[8]

Table 2: Results for $\lvert V_{cb}\rvert$

Recent results for $\lvert V_{ub}\rvert$ and $\lvert V_{cb}\rvert$ are presented in Tables 1 and 2, respectively. Recently, DeTar has collected the lattice QCD results of FNAL/MILC [9] and HPQCD [10], and the experimental results of Babar [11] and Belle [12] for the $\bar{B}\to D\ell\bar{\nu}$ decay mode. He has made combined fit of all of them simultaneously to determine $\lvert V_{cb}\rvert$ [7]. The "ex-combined" result in Table 2 corresponds to a weighted average of the V_{cb} results from the $\bar{B}\to D^*\ell\bar{\nu}$ and $\bar{B}\to D\ell\bar{\nu}$ decay channels. Similarly, the "ex-combined" result in Table 1 corresponds to a weighted average of the two V_{ub} results from $\bar{B}\to\pi\ell\bar{\nu}$ decay. In Fig. 1, we show all the results simultaneously.[1] We find that the inclusive results show about 3σ tension with those from exclusive B meson decays respectively as well as from the LHCb results for $\lvert V_{ub}/V_{cb}\rvert$, which corresponds to the magenta band in Fig. 1.

We have two independent methods to determine ξ_0 in lattice QCD: the indirect and direct methods. In the indirect method, we determine ξ_0 from the experimental values of ${\rm Re}(\varepsilon'/\varepsilon)$, ω,

[1] The plot is based on that by Andreas Kronfeld in Ref. [7].

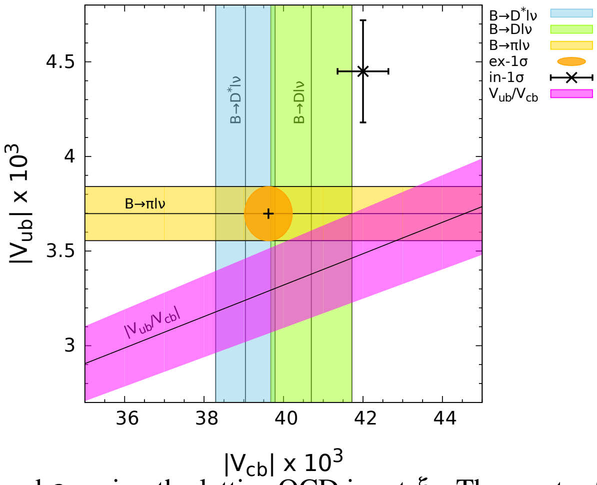

Figure 1: $|V_{ub}|$ versus $|V_{cb}|$. The sky-blue band represents $|V_{cb}|$ determined from the $\bar{B} \to D^*\ell\bar{\nu}$ decay, and the yellow-green band $|V_{cb}|$ determined from the $\bar{B} \to D\ell\bar{\nu}$ decay. The yellow band represents $|V_{ub}|$ determined from the $\bar{B} \to \pi\ell\bar{\nu}$ decay, and the magenta band $|V_{ub}/V_{cb}|$ determined from the LHCb data of the $\Lambda_b \to \Lambda_c\ell\bar{\nu}$ and $\Lambda_b \to p\ell\bar{\nu}$ decays. The orange circle represents the combined results for exclusive $|V_{cb}|$ and $|V_{ub}|$ from the B meson decays, and the black cross × the inclusive $|V_{cb}|$ and $|V_{ub}|$ (heavy quark expansion).

and ε_K using the lattice QCD input ξ_2. The master formulas are

$$\xi_0 = \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}, \qquad \xi_2 = \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}, \qquad \mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{\omega}{\sqrt{2}|\varepsilon_K|}(\xi_2 - \xi_0)\,. \tag{2.2}$$

Recently, RBC-UKQCD reported updated results for ξ_2 [13]. The results for ξ_0 from the indirect method are presented in Table 3.

Input	Method	Value	Ref.
ξ_0	indirect	$-1.63(19) \times 10^{-4}$	[13]
ξ_0	direct	$-0.57(49) \times 10^{-4}$	[14]
ξ_{LD}	—	$(0 \pm 1.6)\,\%$	[15]

Table 3: Long distance effects: ξ_0 and ξ_{LD}.

Collaboration	δ_0	Ref.
RBC-UK-2016	$23.8(49)(12)^\circ$	[14]
KPY-2011	$39.1(6)^\circ$	[16]
CGL-2001	$39.2(15)^\circ$	[17, 18]

Table 4: $\pi - \pi$ scattering phase shift: δ_0

Recently, RBC-UKQCD has reported new lattice QCD results for $\mathrm{Im}A_0$ [14]. Combining them with the experimental value of $\mathrm{Re}A_0$, we can determine ξ_0 directly from the lattice input $\mathrm{Im}A_0$ using the master formula in Eq. (2.2). This is the direct method. In Ref. [14], RBC-UKQCD has also reported the S-wave $\pi - \pi$ scattering phase shift with isospin $I = 0$: $\delta_0 = 23.8(49)(12)$. This value is 3.0σ lower than the conventional value of δ_0 in Refs. [16] (KPY-2011) and [17, 18] (CGL-2001). KPY-2011 used a singly subtracted Roy-like equation and CGL-2001 used a doubly subtracted Roy equation (CGL-2001) to do the interpolation around $\sqrt{s} = m_K \approx 500\,\mathrm{MeV}$. The values for δ_0 are summarized in Table 4. The KPY-2011 fits to the experimental data work well from the $\pi - \pi$ threshold ($\approx 280\,\mathrm{MeV}$) to $\sqrt{s} = 800\,\mathrm{MeV}$. In addition, KPY-2011 is highly consistent with CGL-2001 in the interpolating region around $\sqrt{s} = m_K \approx 500\,\mathrm{MeV}$.

For δ_0 (S-wave, I=0), we plot the results of RBC-UKQCD together with those of KPY-2011 and CGL-2001 in Fig. 2. We find that there is essentially no difference between KPY-2011 and CGL-2001 in the region near $\sqrt{s} = m_K \approx 500\,\mathrm{MeV}$. Here, we observe the 3.0σ gap between RBC-UKQCD and KPY-2011. In contrast, for δ_2 (S-wave, I=2), we observe no tension between RBC-UKQCD and KPY-2011, as one can see in Fig. 3.

Therefore, we conclude that the results of the indirect method are more reliable than those of the direct method for ξ_0, since the direct calculation of $\mathrm{Im}A_0$ by RBC-UKQCD might have unresolved issues. Hence, we use the indirect method to determine ξ_0 in this paper.

ξ_{LD} represents the long distance effect in the dispersive part. Its master formula in the continuum is given in Ref. [1]. A theoretical framework for calculating it on the lattice is well established

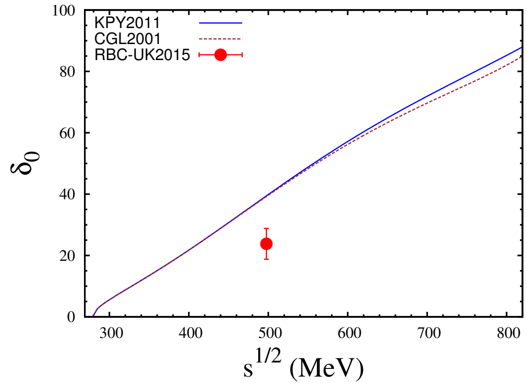

Figure 2: Comparison of δ_0.

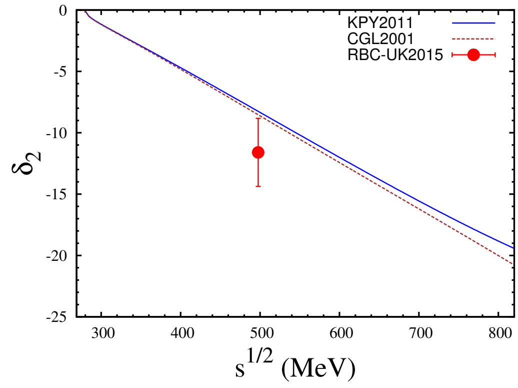

Figure 3: Comparison of δ_2.

in Ref. [15]. An on-going efforts to calculate it on the lattice can be found in [19]. However, this attempt [20], at present, is in a exploratory stage yet. Hence, we use the rough estimate of $\xi_{\rm LD}$ given in Ref. [15].

Recent results for $\hat{B}_K$ in lattice QCD market with $N_f = 2+1$ flavors are summarized in Table 5. Here, FLAG-2016 represents the global average of the results of BMW-2011 [21], Laiho-2011 [22], RBC-UK-2016 [23], and SWME-2016 [24]. For more details, refer to Ref. [25]. SWME-2014 and RBC-UK-2016 represent the $\hat{B}_K$ results reported in Refs. [26] and [23], respectively. Here we use the FLAG-2016 result for $\hat{B}_K$.

Collaboration	Value	Ref.
FLAG-2016	0.7625(97)	[25]
SWME-2014	0.7379(47)(365)	[26]
RBC-UK-2016	0.7499(24)(150)	[23]

Table 5: $\hat{B}_K$

	CKMfitter	UTfit	AOF
λ	0.22548(68)/[27]	0.22497(69)/[28]	0.2253(8)/[29]
$\bar{\rho}$	0.145(13)/[27]	0.153(13)/[28]	0.139(29)/[30]
$\bar{\eta}$	0.343(12)/[27]	0.343(11)/[28]	0.337(16)/[30]

Table 6: Wolfenstein parameters

For the Wolfenstein parameters λ, $\bar{\rho}$, and $\bar{\eta}$, both CKMfitter and UTfit updated their results in Refs. [27, 28]. However, the angle-only-fit has not been updated since Lattice 2015. The global unitarity triangle (UT) fits of both CKMfitter and UTfit use ε_K and $|V_{cb}|$ as input parameters to determine Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$. Hence, using them to evaluate ε_K leads to unwanted correlations through ε_K and $|V_{cb}|$. In contrast, the angle-only-fit (AOF) results have no correlation with ε_K and $|V_{cb}|$. Hence, we use the AOF results in this paper.

For the QCD corrections η_{cc}, η_{ct}, and η_{tt}, we use the same values as in Ref. [1]. They are collected in Table 7. In particular, we use the SWME value of η_{cc} reported in Ref. [1] instead of that in Ref. [31]. This issue is well explained in Ref. [1]. One reason is that the size of the NNLO correction is already a conservative estimate for the truncation error of the NNNLO level in perturbation theory. Another reason is that the SWME result is consistent with that of Ref. [32].

In Table 8, we summarize remaining input parameters. They are the same as those in Ref. [1] except for the charm quark mass $m_c(m_c)$. For the charm quark mass, we use the HPQCD result reported in Ref. [35].

3. Current status of ε_K

Here, we present the results for ε_K evaluated directly from the SM with the lattice QCD inputs

Input	Value	Ref.
η_{cc}	1.72(27)	[1]
η_{tt}	0.5765(65)	[33]
η_{ct}	0.496(47)	[34]

Table 7: QCD corrections.

Input	Value	Ref.
G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[29]
M_W	80.385(15) GeV	[29]
$m_c(m_c)$	1.2733(76) GeV	[35]
$m_t(m_t)$	163.3(2.7) GeV	[36]
θ	$43.52(5)^\circ$	[29]
m_{K^0}	497.614(24) MeV	[29]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	[29]
F_K	156.2(7) MeV	[29]

Table 8: Other input parameters.

described in the previous section. Our preliminary results are, in units of 1.0×10^{-3},

$$|\varepsilon_K| = 1.69 \pm 0.17 \quad \text{for exclusive } V_{cb} \text{ (lattice QCD)} \tag{3.1}$$

$$|\varepsilon_K| = 2.10 \pm 0.21 \quad \text{for inclusive } V_{cb} \text{ (heavy quark expansion)} \tag{3.2}$$

$$|\varepsilon_K| = 2.228 \pm 0.011 \quad \text{(experimental value)} \tag{3.3}$$

Here, exclusive V_{cb} represents the theoretical evaluation of ε_K with the FLAG-2016 $\hat{B}_K$, AOF for the Wolfenstein parameters, and exclusive $|V_{cb}|$ that corresponds to ex-combined in Table 2. We observe 3.2σ tension in the exclusive V_{cb} channel (lattice QCD), and no tension in the inclusive V_{cb} channel (heavy quark expansion; QCD sum rules).

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