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V_{us} from tau decay data

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We revisit the conventional implementation of the determination of V_{us} via flavor-breaking (FB) finite-energy sum rule (FESR) analyses of inclusive hadronic τ decay data, which is known to produce results $> 3\sigma$ low compared to determinations from kaon physics and the expectations of three-family unitarity. We show that this implementation fails self-consistency tests, and that the source of this problem is a breakdown of assumptions concerning the treatment of higher dimension OPE contributions. We then provide an alternate implementation of the FB FESR approach which cures these problems. Lattice data for the relevant flavor-breaking correlator combination is also employed to clarify the treatment of the slowly-converging dimension 2 OPE contribution to the relevant sum rules and quantify the associated truncation uncertainty. We implement this new approach using ALEPH non-strange data, and a combination of ALEPH, BaBar and Belle strange τ decay data. Normalizing the exclusive $\tau \to K^-\pi^0\nu_\tau$ mode component of the inclusive strange decay distribution using the recent preliminary BaBar result for the corresponding branching fraction we find a result, $V_{us} = 0.2228(23)_{exp}(6)_{th}$, in excellent agreement with the results of $K_{\ell 3}$-based analyses, and in agreement within errors with three-family-unitarity expectations, thus resolving the long-standing inclusive τ V_{us} puzzle.

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1. Introduction

The super-allowed $0^+ \to 0^+$ nuclear β decay result $|V_{ud}| = 0.97417(21)$ [1], together with three-family unitarity, leads to the expectation $|V_{us}| = 0.2258(9)$. Direct determinations from $K_{\ell 3}$ and $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$, using the recent 2014 FlaviaNet experimental results $f_+(0)|V_{us}| = 0.2165(4)$ and $|f_K V_{us}|/|f_\pi V_{ud}| = 0.2760(4)$ [2], and employing the 2016 FLAG $n_f = 2+1+1$ lattice results, $f_+(0) = 0.9704(33)$ and $f_K/f_\pi = 1.193(3)$ [3], as input, yield results $|V_{us}| = 0.2231(9)$ and $0.2253(7)$, respectively, both compatible within errors with the three-family-unitarity expectation.

In contrast, much lower values are obtained from conventional implementations of the determination based on FB FESR analyses of inclusive non-strange and strange hadronic τ decay distributions [4]. The most recent update of this approach [5] produces a result

$$|V_{us}| = 0.2176(21)\,, \tag{1.1}$$

which is 3.6σ lower than the three-family-unitarity expectation. It is this inclusive τ V_{us} puzzle which we address (and resolve) in this paper.

In the Standard Model (SM), with $R_{V/A;ij} \equiv \Gamma[\tau^- \to \nu_\tau \, \mathrm{hadrons}_{V/A;ij}\,(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$, the differential distributions, $dR_{V/A;ij}/ds$, for flavor $ij = ud, us$, vector (V) or axial vector (A) current mediated decays are related to the spectral functions, $\rho^{(J)}_{V/A;ij}$, of the spin $J = 0,1$ scalar polarizations, $\Pi^{(J)}_{V/A;ij}$, of the flavor ij, V or A current-current two-point function [6]. Explicitly,

$$\frac{dR_{V/A;ij}}{ds} = \frac{12\pi^2 |V_{ij}|^2 S_{EW}}{m_\tau^2} \left[w_\tau(y_\tau) \rho^{(0+1)}_{V/A;ij}(s) - w_L(y_\tau) \rho^{(0)}_{V/A;ij}(s) \right] \,, \tag{1.2}$$

with $y_\tau = s/m_\tau^2$, $w_\tau(y) = (1-y)^2(1+2y)$, $w_L(y) = 2y(1-y)^2$, S_{EW} a known short-distance electroweak correction, and V_{ij} the flavor ij CKM matrix element. $\rho^{(0)}_{A;ij}(s)$ is dominated by the accurately known, non-chirally-suppressed π or K pole contribution. The remaining, continuum V and A $J = 0$ contributions are $\propto (m_i \mp m_j)^2$, and hence negligible for $ij = ud$. With mildly model-dependent determinations of the small, but not entirely negligible, $ij = us$ continuum $J = 0$ contributions via analyses of the associated $ij = us$ scalar and pseudoscalar sum rules [7, 8], the experimental $dR_{V/A;ij}/ds$ distributions then provide a direct determination of $\rho^{(0+1)}_{V/A;ud,us}(s)$.

The inclusive FB τ decay approach to $|V_{us}|$ [4] is based on FESRs involving the FB polarization difference, $\Delta\Pi_\tau \equiv \Pi^{(0+1)}_{V+A;ud} - \Pi^{(0+1)}_{V+A;us}$, and associated spectral function, $\Delta\rho_\tau \equiv \rho^{(0+1)}_{V+A;ud} - \rho^{(0+1)}_{V+A;us}$. Explicitly, with $w(s)$ analytic in the region of the contour, one has, for any $s_0 > 0$,

$$\int_0^{s_0} w(s) \Delta\rho_\tau(s)\, ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s) \Delta\Pi_\tau(s)\, ds \,. \tag{1.3}$$

Experimental data is to be used on the LHS, the OPE (for sufficiently large s_0) on the RHS.

This relation is used to determine $|V_{us}|$ as follows. $J = 0$ contributions are first subtracted from $dR_{V/A;ij}/ds$, yielding the $J = 0+1$ analogue $dR^{(0+1)}_{V/A;ij}/ds$. Re-weighted versions

$$R^w_{V+A;ij}(s_0) \equiv \int_0^{s_0} ds\, \frac{w(s)}{w_\tau(s)} \frac{dR^{(0+1)}_{V+A;ij}(s)}{ds} \tag{1.4}$$

are then constructable for any w and any $s_0 \le m_\tau^2$. Forming the FB difference

$$\delta R^w_{V+A}(s_0) \equiv \frac{R^w_{V+A;ud}(s_0)}{|V_{ud}|^2} - \frac{R^w_{V+A;us}(s_0)}{|V_{us}|^2}, \tag{1.5}$$

using the OPE representation, Eq. (1.3), to replace the LHS, and solving for $|V_{us}|$, one finds [4],

$$|V_{us}| = \sqrt{R^w_{V+A;us}(s_0) / \left[\frac{R^w_{V+A;ud}(s_0)}{|V_{ud}|^2} - \delta R^{w,OPE}_{V+A}(s_0)\right]}. \tag{1.6}$$

The results for $|V_{us}|$ should, of course, be independent of s_0 and w if experimental and OPE input is reliable. Varying s_0 and/or w thus provides a means of testing such input for self-consistency.

The conventional implementation of Eq. (1.6) [4] responsible for the low values of $|V_{us}|$ noted above employs a single s_0, $s_0 = m_\tau^2$ and single weight $w = w_\tau$. This allows the associated spectral integrals to be fixed using inclusive non-strange and strange branching fractions alone, but has the disadvantage of making variable s_0 and w self-consistency tests impossible. This is potentially problematic since w_τ has degree 3, and hence produces unsuppressed OPE contributions with dimension up to $D = 8$ in $\delta R^{w_\tau,OPE}_{V+A}(s_0)$. While the leading $D = 2$ and sub-leading $D = 4$ contributions (fixed by α_s, $m_{u,d}$, m_s, $\langle\bar{u}u\rangle$ and $\langle\bar{s}s\rangle$ [9]) can be taken as external input [3, 10, 11], the $D = 6$ and 8 condensates are not known experimentally. The former have usually been estimated using the vacuum saturation approximation (VSA) [4, 12] and the latter neglected. These "approximations" are potentially dangerous given the very strong double cancellation (by a factor of ~ 20) present in the $D = 6$ VSA estimate, and the sizeable (as much as a factor of $4-5$), channel-dependent VSA violations seen in the ud sector[13]. We investigate this issue further in the next section.

2. Problems with the conventional implementation and an alternate strategy

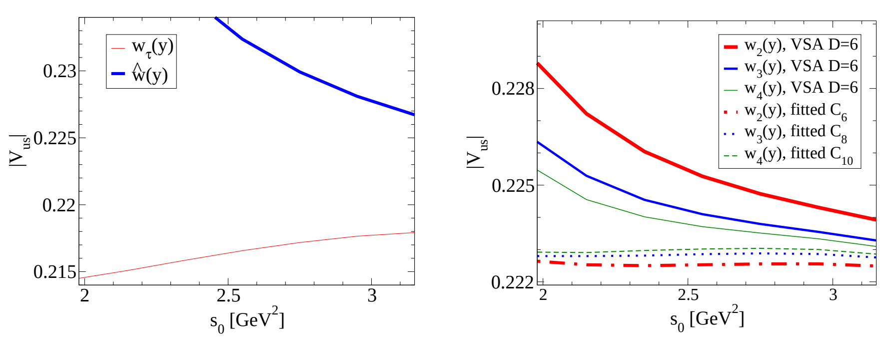

Figure 1: Left panel: $|V_{us}|$ from the w_τ and $\hat{w}$ FESRs with conventional OPE input (including contour improved perturbation theory for the $D = 2$ series). Right panel: Comparison of conventional implementation results (solid lines) with those obtained using central fitted $C_{6,8,10}$ and the fixed order perturbation theory $D = 2$ prescription favored by lattice results, for the weights $w_{2,3,4}$ (dashed lines).

The reliability of the conventional implementation treatment of $D = 6$ and 8 contributions can be tested by comparing the results for $|V_{us}|$, as a function of s_0, obtained from the $w_\tau(y) = 1 - 3y^2 +$

$2y^2$ and $\hat{w}(y) = 1 - 3y + 3y^2 - y^3$ FESRs, where $y = s/s_0$. These weights are such that the integrated $D = 6$ OPE contributions to the two FESRs are identical in magnitude but opposite in sign. The conventional implementation takes $D = 6$ contributions to be small, and $D = 8$ contributions to be negligible, for w_τ. If these assumptions are valid for w_τ, they are thus also valid for $\hat{w}$. $|V_{us}|$ results obtained from the w_τ and $\hat{w}$ FESRs should thus display good individual s_0 stability and be in good agreement if the conventional implementation assumptions are valid. If not, one should find s_0-instabilities of opposite signs in the two cases. Since integrated $D = 6$ and 8 contributions scale as $1/s_0^2$ and $1/s_0^3$ the differences between the two sets of s_0-dependent results should then decrease with increasing s_0. The left panel of Figure 1 shows it is the second scenario which is realized, and hence that the assumptions of the conventional implementation are not valid. Further evidence of the existence of problems with the conventional implementation is provided by the solid lines of the right panel of Figure 1, which display the significant s_0-instability of conventional implementation results obtained from additional FESRs, with weights $w_N(y)$, $N = 2, 3, 4$, where

$$w_N(y) \,=\, 1 - \frac{N}{N-1} y + \frac{1}{N-1} y^N \,. \tag{2.1}$$

The dashed lines in this panel show the much more stable results obtained using the new implementation discussed below, in which $D > 4$ OPE condensates, denoted C_D below, are fitted using experimental data.

The demonstrated unreliability of conventional implementation assumptions for the $C_{D>4}$ suggests considering an alternate implementation of the FB FESR approach in which the $C_{D>4}$ are fit to data. We will see that, having done so, the new $C_{D>4}$ so obtained also naturally solve the problem of the observed s_0- and w-instabilities noted above.

Before proceeding, let us deal with another potential problem for the FB FESR approach: the slow convergence of the relevant $D = 2$ OPE series. To four loops, neglecting $O(m_{u,d}^2/m_s^2)$ corrections, one has [9]

$$\left[\Delta\Pi_\tau(Q^2)\right]_{D=2}^{OPE} \,=\, \frac{3}{2\pi^2} \frac{m_s(Q^2)}{Q^2} \left[1 + \frac{7}{3}\bar{a} + 19.93\bar{a}^2 + 208.75\bar{a}^3 + \cdots \right] , \tag{2.2}$$

with $\bar{a} = \alpha_s(Q^2)/\pi$, and $m_s(Q^2)$ and $\alpha_s(Q^2)$ the running $\overline{MS}$ strange mass and coupling. With $\bar{a}(m_\tau^2) \simeq 0.1$, one thus has, at the spacelike point on $|s| = s_0$, an $O(\bar{a}^3)$ term larger than the $O(\bar{a}^2)$ term for all s_0 accessible in τ decays. This complicates the task of deciding on an appropriate $D = 2$ truncation order and providing a reliable estimate of the associated uncertainty. We have investigated this issue by comparing OPE expectations to $n_f = 2+1$ RBC/UKQCD lattice data [14] for $\Delta\Pi_\tau(Q^2)$ over a range of Euclidean Q^2. We find an excellent match of lattice results and the $D = 2+4$ OPE sum over a broad high-Q^2 interval from $Q^2 \sim 10$ GeV^2 down to ~ 4 GeV^2, provided the $D = 2$ series is evaluated with 3-loop truncation and fixed (rather than local) scale treatment of logarithmic contributions [15].[1] The high-Q^2 comparison also shows conventional $D = 2+4$ OPE error estimates to be extremely conservative [15]. Below $Q^2 \sim 4$ GeV^2, clear deviations of the $D = 2+4$ OPE sum from the lattice data much larger in size than those expected from

[1] The fixed-/local-scale treatment is the analogue of the "fixed-order" (FOPT)/"contour-improved" (CIPT) FESR $D = 2$ series treatment.

conventional implementation $D > 4$ assumptions are also seen [15], confirming the conclusions of the w_τ-$\hat{w}$ comparison discussed above.

An alternate implementation of the FB FESR approach is now obvious. On the theory side, we use the 3-loop-truncated, FOPT version of the $D = 2$ OPE series favored by lattice data, and, by varying s_0, fit the effective $D > 4$ OPE condensates, $C_{D>4}$, rather than making assumptions about their values. On the spectral integral side, we use $\pi_{\mu 2}$, $K_{\mu 2}$ and SM expectations for the π and K pole contributions, the ALEPH continuum ud V+A distribution [16], $\bar{K}^0\pi^-$ and $K^-\pi^0$ distributions from Belle [17] and BaBar [18, 19], $K^-\pi^+\pi^-$ and $\bar{K}^0\pi^-\pi^0$ distributions from BaBar [20] and Belle [21], and 1999 ALEPH results [22] for the combined distribution of the remaining exclusive strange modes not remeasured by the B-factory experiments. For the $K^-\pi^0$ branching fraction, which normalizes the corresponding exclusive distribution, two versions were used: the 2014 HFAG summer fit result, $0.00433(15)$ [23], and the preliminary BaBar thesis result $0.00500(14)$ [19] favored by the BaBar collaboration, whose earlier result dominates the HFAG average. Central results reported below thus correspond to the latter choice.

Results of this analysis, employing the weights $w_N(y)$, were reported in Ref. [15]. These weights have the advantage that, apart from known $D = 2$ and 4 OPE contributions, the w_N FESR involves only a single $C_{D>4}$ with $D = 2N + 2$. The w_N FESR ($N = 2, 3, 4$) was then employed to determine $|V_{us}|$ and C_{2N+2}, using the fit window $2.15\ GeV^2 \leq s_0 \leq 3.15\ GeV^2$, and the $|V_{us}|$ results from the different FESRs checked for consistency. Excellent consistency was observed [15]. The dashed lines in Figure 1 show the results for $|V_{us}|$ as a function of s_0 obtained from an altered version of the conventional implementations of the $w_{2,3,4}$ FESRs in which the central fitted value of the relevant ($D = 6, 8, 10$) condensate is used as input in place of the value usually assumed in the conventional implementation. Use of the fitted versions of the $C_{D>4}$ evidently completely cures the s_0- and w-instabilities seen above.

Given the excellent consistency of the results for $|V_{us}|$ obtained from the w_2, w_3 and w_4 FESRs, we take our final result from a combined fit to all three FESRs. For the version of the strange inclusive distribution obtained using the preliminary updated BaBar $\tau^- \to K^-\pi^0\nu_\tau$ branching fraction as normalization for the exclusive $K^-\pi^0$ contribution, we find [15]

$$|V_{us}| = 0.2228(5)_{th}(23)_{exp}\,. \tag{2.3}$$

The theory error is dominated by the uncertainty in $\langle m_s\bar{s}s\rangle$, the experimental error by the errors in the strange exclusive distributions [15]. The result of Eq. (2.3) agrees well with that obtained from $K_{\ell 3}$, and, within errors, with 3-family unitarity expectations. Using instead the strange inclusive distribution obtained employing the non-updated HFAG 2014 $\tau^- \to K^-\pi^0\nu_\tau$ branching fraction to normalize the exclusive $K^-\pi^0$ contribution yields $|V_{us}| = 0.2200(5)_{th}(23)_{exp}$, 0.0024 higher than the result of the conventional implementation using the same input. Further work on the branching fraction of this mode is of obvious interest.

In conclusion, the long-standing puzzle of the low $|V_{us}|$ obtained from FB hadronic tau decay data based FESRs has been resolved. Current results for $|V_{us}|$ agree well with those obtained from other sources. Roughly half of this improved agreement is attributable to the new, data-based treatment of higher dimension OPE contribution, while the other half results from the use of the new preliminary BaBar normalization for the $K^-\pi^0$ exclusive distribution. Improvements to the low-multiplicity strange exclusive branching fractions would allow for significant reductions in the error on $|V_{us}|$ obtained from the new implementation of the FB FESR approach.

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