Study of wave packet treatment of neutrino oscillations at Daya Bay

Maria Dolgareva (Joint Institute for Nuclear Researches)
on behalf of the Daya Bay collaboration, ICHEP 2016, August 3-10

Plane-wave approach

In the plane-wave approximation neutrino flavor state is a coherent superposition of mass eigenstates $\nu_i(p)$ with the same momentum p: $|\nu_\alpha(p)\rangle = \sum_{k=1}^{3} V^*_{\alpha k}|\nu_k(p)\rangle$. This leads to the oscillatory behavior of the probability to detect a neutrino originally of flavor α as having flavor β:

$$P_{\alpha\beta}(L) = \sum_{k,j=1}^{3} V^*_{\alpha k} V^*_{\beta j} V_{\beta k} V_{\alpha j} e^{-iL/L^{osc}_{kj}},$$

where $L^{osc}_{kj} = 4\pi p/\Delta m^2_{kj}$ is the oscillation length due to the non-zero differences $\Delta m^2_{kj} = m^2_k - m^2_j$.

Assumptions:

- Neutrino mass eigenstates are **plane waves** (p.w.).
- Neutrino mass eigenstates are produced and detected coherently.
- Neutrino mass eigenstates have definite and identical momenta.
- Neutrino speed is the speed of light.

Contradictions:

- The distance L cannot be rigorously defined with fully delocalized p.w.
- The assumption about coherent production/detection should be justified.
- Assumption of same momentum is not Lorentz invariant.
- Accounting for the fact that $v_i \neq c$ leads to spurious factor of two change in the oscillation phase.

The plane-wave model of neutrino oscillation is not self-consistent, and leads to a number of paradoxes. A **wave packet** treatment of neutrino oscillation is necessary.

Wave packet approach

A wave packet is a coherent superposition of mass eigenstates with different momenta

$$|\nu_i(k)\rangle \to \int \frac{dk}{2\pi} f(k,p,\sigma^2)|\nu_i(k)\rangle,$$

where $f(k,p,\sigma^2)$ is assumed be a Gaussian. The oscillation probability reads **[arXiv:1608.01661]**

$$P_{\alpha\beta}(L) = \sum_{k,j=1}^{3} \frac{V_{k\beta}V^*_{\alpha k}V_{j\alpha}V^*_{\beta j}}{\sqrt[4]{1+\left(L/L^{d}_{kj}\right)^2}} \exp\left[-\frac{\left(L/L^{coh}_{kj}\right)^2}{1+\left(L/L^{d}_{kj}\right)^2} - \mathrm{D}^2_{kj}\right] e^{-i(\varphi_{kj}+\varphi^d_{kj})},$$

where
$$\varphi^d_{kj} = -\frac{L/L^d_{kj}}{1+\left(L/L^d_{kj}\right)^2}\left(\frac{L}{L^{coh}_{kj}}\right)^2 + \frac{1}{2}\arctan\frac{L}{L^d_{kj}}, \quad \varphi_{kl} = \frac{L}{L^{osc}_{kj}}.$$

$$L^{coh}_{kj} = \frac{L^{osc}_{kj}}{\sqrt{2}\pi\sigma_{rel}}, \quad L^{d}_{kj} = \frac{L^{coh}_{kj}}{2\sqrt{2}\sigma_{rel}}, \quad \mathrm{D}^2_{kj} = \frac{1}{2}\left(\frac{\Delta m^2_{kj}}{4p^2\sigma_{rel}}\right)^2$$

- $\boldsymbol{\sigma_{rel} = \sigma_p/p}$ is the relative wave packet momentum dispersion.
  $\boldsymbol{\sigma_p}$ is the intrinsic momentum dispersion of the neutrino wave packet that depends on the kinematics of neutrino production and detection in general. In QM formalism $\sigma_{rel} = \mathrm{const}$.
  $\boldsymbol{\sigma_x = 1/2\sigma_p}$ is a spatial width of wave packet.
- L^{osc}_{kj} is the oscillation length.
- L^{coh}_{kj} is the coherence length, i.e. a distance at which the interference of neutrino mass eigenstates vanishes due to spatial separation of $|\nu_k\rangle$ and $|\nu_j\rangle$.
- L^{d}_{kj} is the dispersion length, i.e. a distance at which the wave packet is doubled in its spatial dimension due to the dispersion of waves moving with different velocities.
- The factor D^2_{kj} suppresses the production and detection of massive neutrino states $|\nu_k\rangle$ and $|\nu_j\rangle$ in the coherent flavour state.

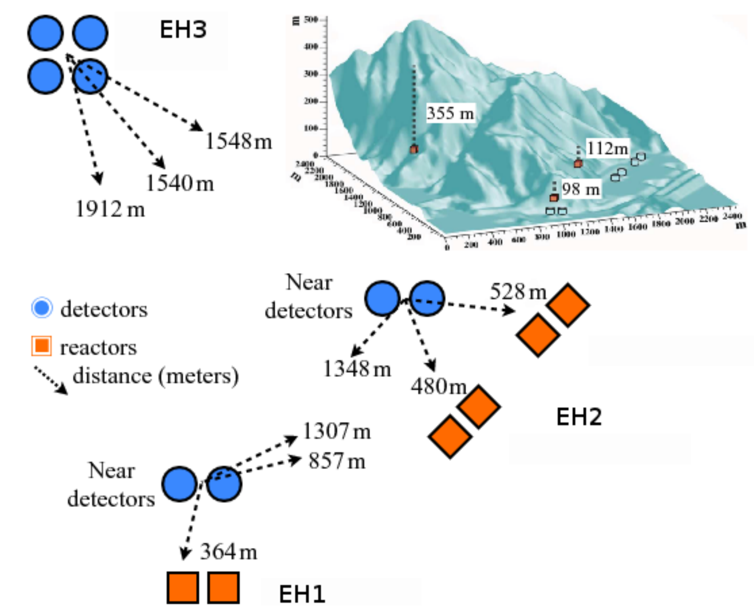
Current knowledge about σ_p

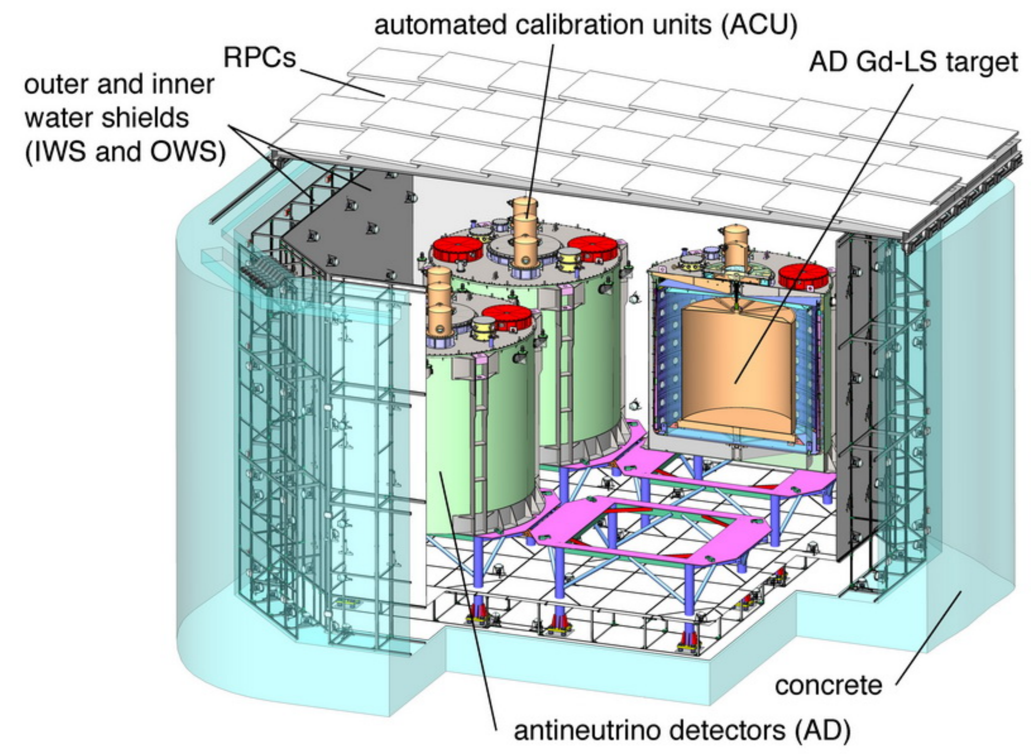
- Theoretical estimates vary by orders of magnitude, associating the dispersion of the neutrino wave packet with various scales; for example, uranium nucleus size ($\sigma_x \simeq 10^{-11}$ cm, $\sigma_p \simeq 1$ MeV), atomic or inter-atomic size ($\sigma_x \simeq (10^{-8} - 10^{-7})$ cm, $\sigma_p \simeq (10^3 - 10^2)$ eV), pressure broadening ($\sigma_x \simeq 10^{-4}$ cm, $\sigma_p \simeq 0.1$ eV), etc.
- **No experimental studies.**

Daya Bay experiment

The Daya Bay detectors measure the **electron antineutrino flux** from nuclear reactors in China.

- $\overline{\nu}_e$ source: 6 nuclear reactors.
- Detector: 8 Gd-loaded liquid scintillator detectors, 20 ton each.
- Baselines: from 0.5 to 1.9 km.
- Statistics: $\sim 10^6$ $\overline{\nu}_e$ events collected between December 2011 and November 2013.



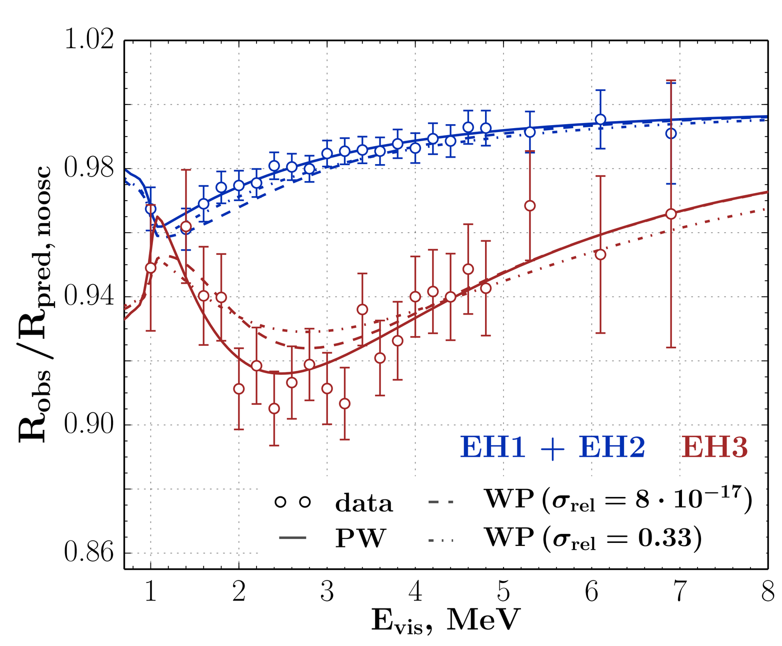


$\overline{\nu}_e$ are registered via the inverse β-decay in liquid scintillator:

$$\overline{\nu}_e + p \to n + e^+$$
$$e^+ + e^- \to \gamma + \gamma$$
$$n + X \to X^* \to X + n\gamma$$

The coincidence of the prompt (e^+ ionization and annihilation) and delayed (n capture on Gd) signals is used.

Data analysis



△ Predicted and observed $\overline{\nu}_e$ spectra as ratios to spectra without oscillations.

As the goodness-of-fit measure we use

$$\chi^2(\boldsymbol{\eta}) = (\mathbf{d} - \mathbf{t}(\boldsymbol{\eta}))^T V^{-1}(\mathbf{d} - \mathbf{t}(\boldsymbol{\eta})),$$

where $\mathbf{d}$ is a data vector, $\mathbf{t}(\boldsymbol{\eta})$ is the theoretical model vector, $\boldsymbol{\eta}$ is the vector of constrained and unconstrained parameters. All uncertainties of the model are considered in the covariance matrix V.

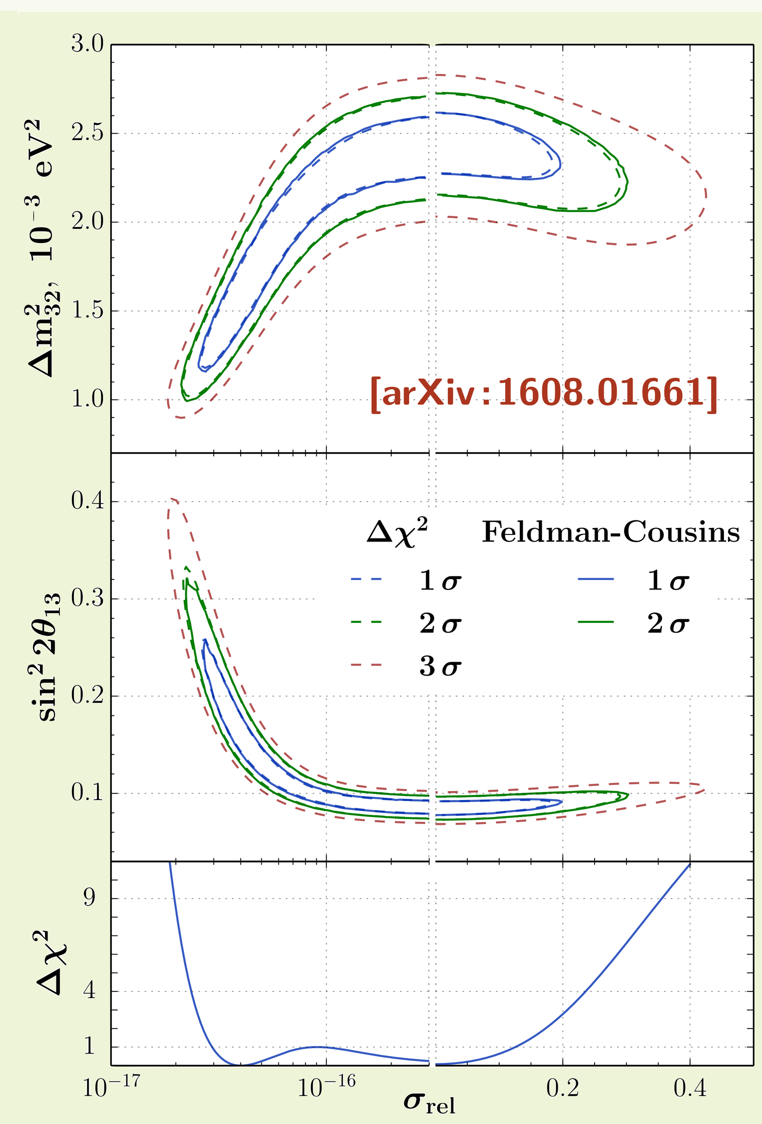
The analysis was done with parameters $\boldsymbol{\sigma_{rel}}$, $\boldsymbol{\Delta m^2_{32}}$, $\boldsymbol{\sin^2 2\theta_{13}}$, flux normalization **N** taken into account.

The marginalized $\Delta\chi^2$ statistic:

$$\Delta\chi^2(\xi') = \min_{\xi \text{ but } \xi'} \chi^2(\xi) - \min_{\xi} \chi^2(\xi),$$

where $\xi = (\sigma_{rel}, \Delta m^2_{32}, \sin^2 2\theta_{13}, N)$ and ξ' is used to construct the confidence intervals with help of the fixed-level $\Delta\chi^2$ and Feldman-Cousins methods.

Results and Discussion



△ Allowed regions in $(\Delta m^2_{32}, \sigma_{rel})$ and $(\sin^2 2\theta_{13}, \sigma_{rel})$ obtained with fixed-level $\Delta\chi^2$ and Feldman-Cousins methods.

- The region of $10^{-16} < \sigma_{rel} < 0.1$ is where the impact of wave packet on neutrino oscillation is negligible for the Daya Bay experiment.
- In the region $\sigma_{rel} < 10^{-16}$ localization term D^2_{kj} suppresses the oscillations.
- In the region $\sigma_{rel} > 0.1$ spatial separation (L^{coh}_{kj}) and dispersion (L^{d}_{kj}) are important.

- Allowed region for σ_{rel} at 95% C.L.:
  $$2.38 \cdot 10^{-17} < \sigma_{rel} < 0.232.$$

The lower limit is much weaker than the constraint which takes into account the dimensions of the reactor core and the detector: $\sigma_x \lesssim 2$m or $\sigma_{rel} \gtrsim 10^{-14}$ for $p = 4$ MeV.

- Thus the obtained limits can be read:
  $$10^{-11}\,\text{cm} \lesssim \sigma_x \lesssim 2\text{m}$$

Such a σ_{rel} corresponds to the regime where the localization term can be safely neglected.

- This allows us to put an upper limit:
  $$\sigma_{rel} < 0.20 \text{ at } 95\% \text{ C.L.}$$

The first limits on the neutrino wave packet momentum dispersion are obtained.
The allowed decoherence effect due to the wave packet nature of neutrino oscillation is found to be insignificant for reactor antineutrinos detected by the Daya Bay experiment. That ensures an unbiased measurement of the oscillation parameters $\sin^2 2\theta_{13}$ and Δm^2_{32} within the plane-wave model.