

## SUSY dark matter, axions, LHC and ILC

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Guided by the twin pillars of simplicity and naturalness, and upon clarification of the notion of naturalness, we find regions of highly natural SUSY with light higgsinos and highly mixed TeV-scale top squarks and gluinos with mass less than 4 TeV. The gauge hierarchy problem is solved by SUSY and the strong CP and SUSY mu problems are solved by SUSY DFSZ axions. Then the Little Hierarchy with  $\mu \ll m_{3/2}$  is natural. Ultimately, we expect detection of both a DFSZ axion and a Higgsino-like WIMP. LHC14 can probably eke out SUSY signals in the new same-sign diboson and jet+soft dilepton channels. The ILC with  $\sqrt{s} > 2m(\text{higgsino})$  would likely be a Higgsino factory and usher in the era of the superworld.

*The 11th International Workshop Dark Side of the Universe 2015  
14-18 December 2015  
Kyoto, Japan*

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## 1. Introduction

The LHC experiments Atlas and CMS have discovered a very Standard Model (SM)-like Higgs boson[1, 2] with mass  $m_h \simeq 125$  GeV while, at present, no sign of physics from beyond the SM occurs (aside from a slew of anomalies– especially the 750 GeV diphoton mass bump– awaiting confirmation or rejection from larger data sets). This situation is especially puzzling since the Higgs boson seems to be an elementary spin-0 excitation which ought to receive enormous, quadratically divergent, quantum corrections to its mass. Since the SM is renormalizable, there is always the possibility of fine-tuning parameters to whatever accuracy is required so as to maintain the measured value of  $m_h$ . Such extraordinary fine-tunings are generally thought to be indicative of some sort of pathology with, or missing link within, the theory under question. To put it bluntly, the Higgs mass should be what it is because the disparate contributions to its squared mass, some positive and some negative, are less than or of order  $m_h$ : such a situation is dubbed as *natural*.

As a guide to physics beyond the SM, we will here be guided by two principles: simplicity and naturalness. Simplicity guides our search for new physics in that the further one strays from the SM, especially with poorly motivated extensions, the more likely one is to be wrong. Thus, the advice from Einstein is

Everything should be made as simple as possible, but not simpler

To this we add some direction from Weinberg

The appearance of fine-tuning in a scientific theory is like a cry of distress from nature, complaining that something needs to be better explained

There are two fine-tuning problems in the SM. One already mentioned occurs in the Higgs sector of the model and involves quadratic corrections to  $m_h$ . The electroweak fine-tuning problem is solved once-and-for-all by the introduction of supersymmetry (SUSY) into the SM. Softly broken SUSY guarantees cancellation of quadratic divergences to all orders in perturbation theory and is the maximal extension of the set of spacetime symmetries that undergirds quantum field theory. The other fine-tuning problem occurs in the QCD sector where the  $\mathcal{L}_{QCD} \ni \frac{\bar{\theta}}{32\pi^2} G_{\mu\nu A} \tilde{G}_A^{\mu\nu}$  gluon field strength term seems required by 'tHooft's theta-vacuum solution to the  $U(1)_A$  problem while measurements of the neutron EDM tell us it is tiny:  $\bar{\theta} \lesssim 10^{-10}$ . This *strong CP problem* is elegantly solved via the introduction of Peccei-Quinn symmetry and its concomitant axion[3]. While SUSY solves the gauge hierarchy problem and the PQ axion solves the strong CP problem, each of these solutions also gives rise to dark matter candidates: the SUSY WIMP and the axion. We would expect *both* to be present in nature.

While SUSY tames the quadratic divergence problem, log divergent contributions to  $m_h$  remain. The rather large value of  $m_h \simeq 125$  GeV in the context of SUSY requires highly mixed TeV-scale top-squarks to bolster its mass[4]. In addition, recent limits on sparticle masses from LHC searches require  $m_{\tilde{g}} \gtrsim 1.8$  TeV and  $m_{\tilde{q}} \gtrsim 2$  TeV[5, 6] (within the context of certain simplified models). Comparing these lower bounds with upper bounds from Barbieri-Giudice-Dimopoulos[7, 8, 9] naturalness with better than 3% fine-tuning–  $m_{\tilde{g}} \lesssim 350$  GeV and  $m_{\tilde{u}_R} \lesssim 700$  GeV– we see the fine-tuning problem potentially re-emerging. This has led some authors to proclaim a *crisis* in physics, and has spurred some movement away from SUSY as a solution to the hierarchy problem. In this

talk, we emphasize that this supposed crisis arises only due to *over-estimates* of SUSY fine-tuning which arise from not cancelling *dependent* contributions to observables such as  $m_h^2$ . To guard against this, we have articulated a Fine-tuning Rule[10]:

*When evaluating fine-tuning of contributions to some observable, it is not permissible to claim fine-tuning of dependent quantities one against another.*

As an example, in the SM, the tree-level Higgs squared mass  $m_h^2(\text{tree}) = 2\mu^2$  is independent of the leading quadratic divergences so that  $\mu^2$  can be freely tuned to maintain  $m_h \sim 125$  GeV. But this then means the SM is likely only valid as an effective theory up to energy scales  $\Lambda \sim 1$  TeV. In contrast, analogous reasoning applied to the case of  $m_h^2$  in the MSSM leads to violation of the Fine-tuning Rule. Proper evaluation of fine-tuning in the MSSM then guides the way to where SUSY might be hiding!

## 2. Naturalness clarified

Here we cover three common measures of naturalness found in the literature.

### 2.1 Weak scale fine-tuning in the MSSM

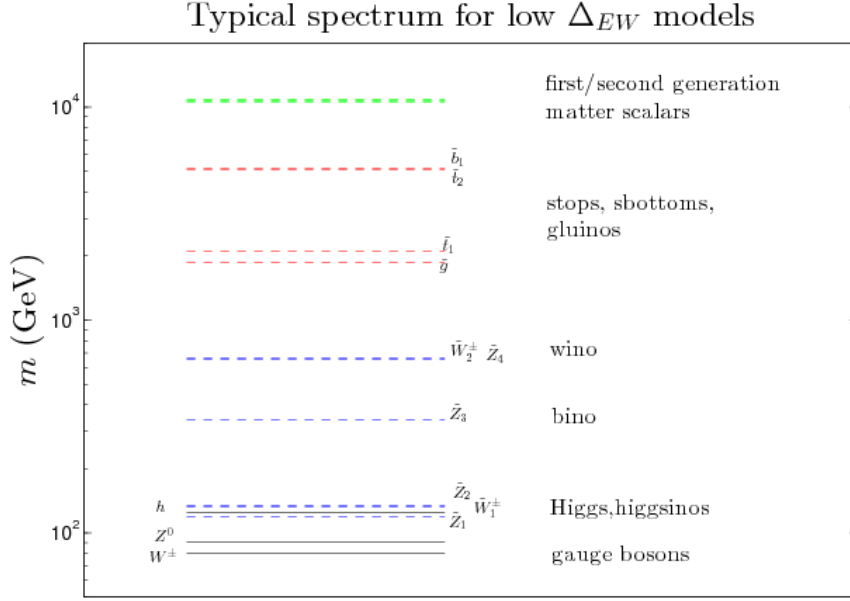
In supersymmetric models, minimization of the scalar potential to find the Higgs field vevs leads to the well-known relation between the Z-boson mass and the weak scale SUSY Lagrangian parameters:

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \quad (2.1)$$

$$\simeq -m_{H_u}^2 - \Sigma_u^u(k) - \mu^2. \quad (2.2)$$

Here,  $m_{H_u}^2$  and  $m_{H_d}^2$  are squared soft SUSY breaking Higgs mass terms,  $\mu$  is the superpotential Higgsino mass parameter,  $\tan \beta = v_u/v_d$  is the ratio of Higgs field vacuum-expectation-values and the  $\Sigma_u^u(k)$  and  $\Sigma_d^d(j)$  contain an assortment of radiative corrections, the largest of which typically arise from the top squarks. Expressions for the  $\Sigma_u^u$  and  $\Sigma_d^d$  are given in the Appendix of Ref. [11]. The value of  $\Delta_{EW}$  compares the largest independent contribution on the right-hand-side (RHS) of Eq. (2.2) to the left-hand-side  $m_Z^2/2$ . If the RHS terms in Eq. (2.2) are individually comparable to  $m_Z^2/2$ , then no unnatural fine-tunings are required to generate  $m_Z = 91.2$  GeV. The onset of fine-tuning is visually displayed in Ref. [12] and occurs for  $\Delta_{EW} \gtrsim 30$ . The main requirements for low fine-tuning ( $\Delta_{EW} \lesssim 30$ ) are easy to read off.

- $|\mu| \sim 100 - 300$  GeV, the closer to  $m_Z$  the better.
- $m_{H_u}^2$  is driven radiatively to small negative values  $\sim -(100 \text{ GeV})^2$  at the weak scale [11]. (This part is called radiatively-driven naturalness.)
- The top squark contributions to the radiative corrections  $\Sigma_u^u(\tilde{t}_{1,2})$  are minimized for TeV-scale highly mixed top squarks[11]. This latter condition also lifts the Higgs mass to  $m_h \sim 125$  GeV. For  $\Delta_{EW} \lesssim 30$ , the lighter top squarks are bounded by  $m_{\tilde{t}_1} \lesssim 3$  TeV.



**Figure 1:** Typical particle mass spectrum from SUSY models with low  $\Delta_{EW}$ , *i.e.* radiatively-driven naturalness.

- The gluino mass which feeds into the  $\Sigma_u^{\prime\prime}(\tilde{t}_{1,2})$  via RG contributions to the stop masses is required to be  $m_{\tilde{g}} \lesssim 3 - 4$  TeV, possibly beyond the reach of LHC.
- First and second generation squark and slepton masses may range as high as 5-20 TeV with little cost to naturalness[11, 13, 12].

SUSY models with these properties have been dubbed radiatively-driven natural SUSY or RNS. The presence of a high degree of fine-tuning generally indicates a pathology or missing element in a physical theory.

A typical sparticle mass spectrum with radiatively-driven naturalness is shown in Fig. 1.

## 2.2 Large log fine-tuning

It is common to hear that low fine-tuning in SUSY requires (several) light third generation squarks with mass  $m_{\tilde{t}_{1,2}, \tilde{b}_1} \lesssim 600$  GeV. This arises from requiring[14]

$$\delta m_{H_u}^2 \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln(\Lambda^2/m_{SUSY}^2). \quad (2.3)$$

not too big. The problem here is that a variety of intertwined large logs contribute to  $\delta m_{H_u}^2$  and their combined effect must be evaluated using the  $dm_{H_u}^2/dt$  renormalization group equation (RGE). The RGE actually contains dependence on  $m_{H_u}^2$  itself: in fact, the larger the boundary condition  $m_{H_u}^2(\Lambda)$  then the larger is the cancelling correction  $\delta m_{H_u}^2$ . This is different from the case of the SM.

Properly combining dependent contributions according to the Fine-tuning Rule leads to

$$m_h^2 \sim \mu^2 + (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2) \quad (2.4)$$

where  $(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2) \equiv m_{H_u}^2(\text{weak})$ . By combining the dependent bracketed terms, and since  $\mu$  hardly evolves, then  $\mu(\text{weak})$  and  $m_{H_u}^2(\text{weak})$  are again both required to have weak scale values (as in  $\Delta_{EW}$ ). Heavier stops are allowed when  $m_{H_u}^2(\Lambda)$  and  $\delta m_{H_u}^2$  nearly cancel. This condition occurs in the hyperbolic branch/focus-point region of the mSUGRA model[15] but can occur more generally in models with non-universal soft terms[16].

### 2.3 BG fine-tuning

The BG measure  $\Delta_{BG} \equiv \max_i |\partial \log m_Z^2 / \partial \log p_i|$  measures the sensitivity of  $m_Z^2$  to fundamental model parameters  $p_i$ . For the case of pMSSM, the derivatives show that  $\Delta_{BG} \sim \Delta_{EW}$ . For models valid at some high scale  $\Lambda$  (where  $\Lambda$  may range as high as  $m_{GUT}$  or  $m_P$ ), then one must evaluate  $m_{H_u}^2$  in terms of corresponding high scale  $p_i$ . In most cases,  $\Delta_{BG}$  has been evaluated for the multi-parameter effective theories where the multiple parameters, assumed to be independent, parametrize our ignorance of SUSY breaking. However, when the SUSY breaking sector is well-specified, then the soft parameters are all *dependent* quantities, and are calculated in terms of more fundamental entities. For example, for gravity mediation they are all calculated as multiples of the gravitino mass  $m_{3/2}$ . In this case, the soft term contributions to  $m_Z^2$  combine while  $\mu$  hardly evolves so that again  $\Delta_{BG} \simeq \Delta_{EW}$ .

### 3. The mu parameter and DFSZ axions

A crucial aspect of Eq. 2.2 is that the soft terms such as  $m_{H_u}^2$  and the mu parameter  $\mu^2$  can arise from very different sectors of the theory:  $\mu$  from the superpotential and soft terms from the SUSY breaking sector. This brings to bear the origin of  $\mu$  and the so-called SUSY mu problem: one expects superpotential mass terms of order the Planck mass  $m_P$  instead of where phenomenology requires it: at the weak scale. To solve the SUSY mu problem, one must first forbid mu from appearing, then regenerate it via some mechanism. In the original Kim-Nilles mechanism[17], one invokes the SUSY DFSZ axion solution to the strong CP problem. The MSSM Higgs multiplets both carry PQ charge  $-1$  so that the mu term is indeed forbidden. But the Higgs doublets couple to PQ scalars via

$$\lambda_\mu X^2 H_u H_d / m_P \quad (3.1)$$

where the PQ field  $X$  carries PQ charge  $+1$ . When PQ symmetry is spontaneously broken, then  $X$  develops a vev of order the PQ scale  $v_{PQ} \sim 10^{11}$  GeV. A value of  $\mu \sim \lambda_\mu v_{PQ}^2 / m_P$  is generated. This is in contrast to the SUSY particle mass scale  $m_{SUSY} \sim m_{3/2} \sim m_{hidden}^2 / m_P$ . A Little Hierarchy  $v_{PQ} < m_{hidden}$  then generates a visible sector Little Hierarchy of  $\mu \ll m_{SUSY}$ . Since the axion mass is also determined by  $f_a \sim v_{PQ}$ , then the PQ scale sets the scale for the axion mass, the  $\mu$  parameter and by naturalness the Higgs mass!

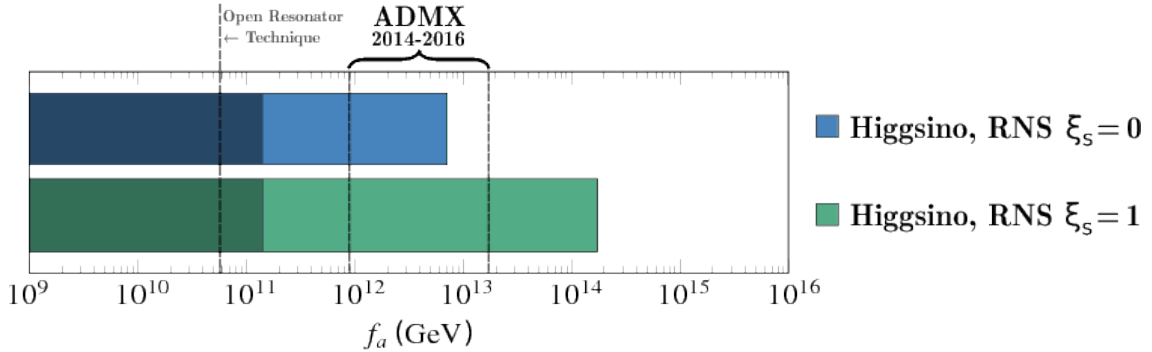
A class of models exists with radiatively induced PQ symmetry breaking[18, 19]. Large soft terms  $m_{SUSY} \sim 5 - 20$  TeV produce a small value of  $\mu \sim 100$  GeV[20]. The Majorana neutrino mass scale is also generated and is comparable to PQ scale  $f_a$ . Such models contain solutions to the

gauge hierarchy problem, the strong CP problem, the SUSY mu problem and the Little Hierarchy or naturalness problem. Awesome!

#### 4. Consequences for axion and WIMP searches

The dark matter is then expected to be an admixture of a higgsino-like WIMP and a DFSZ axion. While the Higgsino-like WIMPs are thermally *underproduced*, they can also be non-thermally produced from axino and/or saxion decay in the early universe. If too many WIMPs are produced from late axino/saxion decays, then they may re-annihilate resulting in a larger WIMP abundance[21]. Axinos are produced thermally while saxions are produced thermally or via coherent saxion field oscillations. Saxion decay to SM particles dilute any abundance already present while  $s \rightarrow aa$  decays produce dark radiation for which there are strong limits. One must also account for gravitino production and decay. The complete calculation requires solution of eight coupled Boltzmann equations[22, 23]. For low  $f_a \lesssim 10^{10}$  GeV, then dark matter tends to be axion-dominated[22], where axions are mainly produced via coherent axion field oscillations. For higher  $f_a$  values, then axinos and perhaps saxions decay after WIMP freeze-out and augment the WIMP abundance. If too many WIMPs or dark radiation are produced, then the model is excluded.

The ADMX experiment will soon initiate a search over a wide range of  $f_a$  values and expect to reach sensitivity to the DFSZ axion[24].

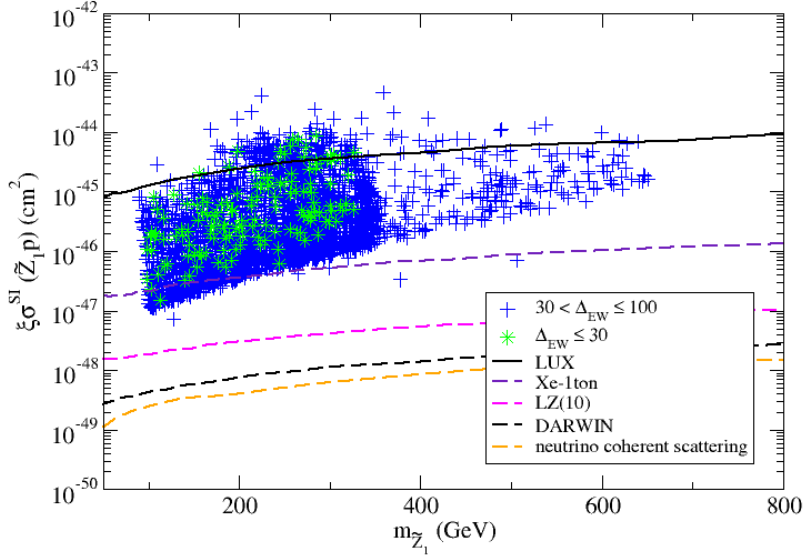


**Figure 2:** Range of  $f_a$  which is allowed in each PQMSSM scenario for the RNS benchmark models (from Ref. [24]). Shaded regions indicate the unnatural range of  $f_a$  where  $\theta_i > 3$ .

In addition, ton-scale noble liquid WIMP detectors should probe the entire parameter space of natural SUSY models[25]. The reason is that, in spite of a possible reduced local abundance of WIMPs, their coupling via Higgs exchange is never small: it is a product of gaugino times Higgsino components and naturalness requires both to be significant so the WIMP-Higgs coupling is never small.

#### 5. Consequences for LHC searches

SUSY models with small  $\mu$  and highly mixed TeV-scale top squarks and  $m_{\tilde{g}} \lesssim 3 - 4$  TeV are completely natural. However, the expected signatures at LHC can be quite different from previous expectations such as in the CMSSM/mSUGRA model where  $\mu$  is large.



**Figure 3:** Plot of rescaled higgsino-like WIMP spin-independent direct detection rate  $\xi \sigma^{\text{SI}}(\tilde{Z}_1 p)$  versus  $m_{\tilde{Z}_1}$  from a scan over NUHM2 parameter space with  $\Delta_{\text{EW}} < 30$  (green) and  $30 < \Delta_{\text{EW}} < 100$  (blue). We also show the current reach from the LUX experiment and projected reaches of Xe-1-ton, LZ(10) and Darwin.

For RNS SUSY, gluino pair production is still an important discovery channel where it is expected that  $\tilde{g} \rightarrow t\tilde{t}\tilde{Z}_i$  or  $t\tilde{b}\tilde{W}_j$ . Thus, gluino pair events are expected to be rich in  $b$ -jets and  $t$ -jets. An important difference is that the  $\tilde{Z}_2$ , which is produced in cascade decays, can decay  $\tilde{Z}_2 \rightarrow \tilde{Z}_1 \ell\ell$  where  $m(\ell^+\ell^-)$  is bounded kinematically by  $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \lesssim 10 - 20$  GeV[26, 27]. Cascade decay events containing an OS/SF dilepton pair with a bump from  $0 \rightarrow 10 - 20$  GeV would signal the presence of light Higgsinos within the events. The  $5\sigma$  reach of LHC14 with  $1000 \text{ fb}^{-1}$  of integrated luminosity extends to  $m_{\tilde{g}} \sim 2$  TeV so that LHC14 can probe only a portion of natural SUSY parameter space via gluino pair production.

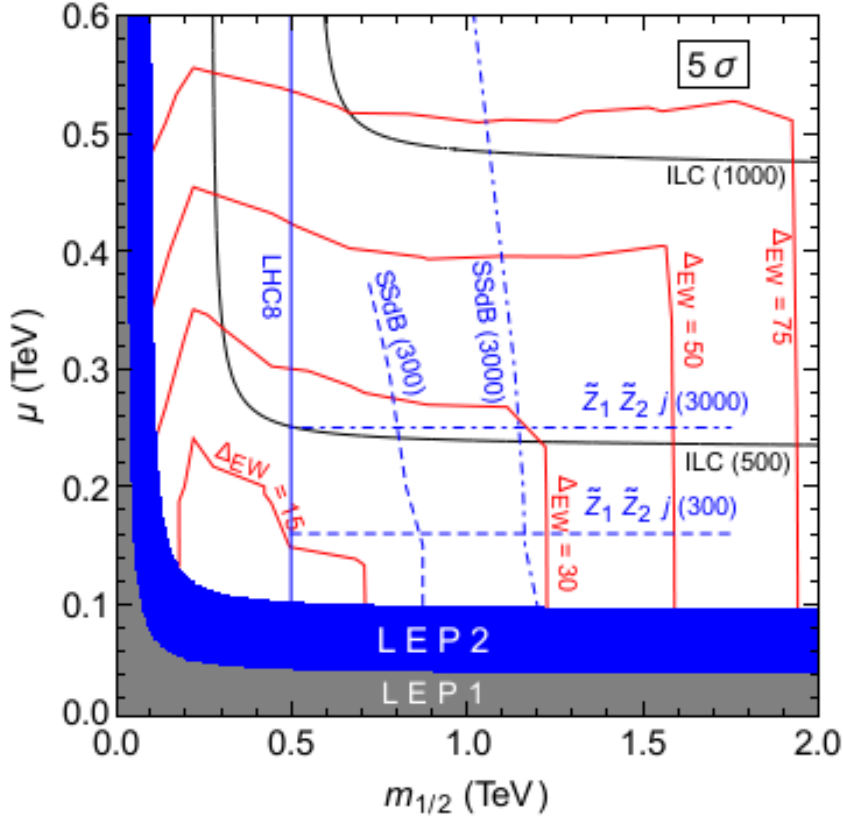
A qualitatively new signature arises from  $pp \rightarrow \tilde{W}_2 \tilde{Z}_4$  (wino pair production) followed by  $\tilde{W}_2^+ \rightarrow W^+ \tilde{Z}_{1,2}$  and  $\tilde{Z}_4 \rightarrow W^+ \tilde{W}_1^-$ . The Higgsinos yield only soft tracks so these events look like same-sign boson production:  $W^+ W^+ + \cancel{E}_T$  or  $W^- W^- + \cancel{E}_T$ . This SSdB signature has very small backgrounds. The  $3000 \text{ fb}^{-1}$  reach of LHC14 is to  $m_{1/2} \sim 1.2$  TeV or  $m_{\tilde{g}} \sim 3$  TeV[29].

In addition, it appears  $pp \rightarrow \tilde{Z}_1 \tilde{Z}_2 j$  where  $\tilde{Z}_2 \rightarrow \ell^+ \ell^- \tilde{Z}_1$  should be visible above BG for high luminosity LHC14[28]. The  $j$  comes from initial state radiation and is in addition to a soft OS/SF dilepton pair plus  $\cancel{E}_T$ . The combined SSdB and  $\tilde{Z}_1 \tilde{Z}_2 j$  signals appear to allow LHC14 to probe virtually the entire natural SUSY parameter space with  $\Delta_{\text{EW}} < 30$ ![29]

## 6. How natural SUSY cries out for ILC

Also shown on Fig. 4 is the reach of ILC for natural SUSY with  $\sqrt{s} = 500$  or  $1000$  GeV. Since





**Figure 4:** Plot of  $\Delta_{EW}$  contours (red) in the  $m_{1/2}$  vs.  $\mu$  plane of NUHM2 model for  $A_0 = -1.6m_0$  and  $m_0 = 5$  TeV and  $\tan\beta = 15$ . We also show the region excluded by LHC8 gluino pair searches (left of solid blue contour), and the projected region accessible to LHC14 searches via the SSdB channel with 300/3000  $\text{fb}^{-1}$  of integrated luminosity (dashed/dot-dashed contours). The LHC14 reach via the  $\tilde{Z}_1\tilde{Z}_2j$  channel is also shown, assuming it is insensitive to the choice of  $m_{1/2}$  in the low  $\Delta_{EW}$  region of interest. We also show the reach of various ILC machines for higgsino pair production (black contours). The blue (gray) shaded region is excluded by LEP2 (LEP1) searches for chargino pair production. To aid the reader, we note that  $m_{\tilde{g}} \simeq 2.5m_{1/2}$ .

in RNS SUSY we have  $\mu \sim 100 - 300$  GeV and  $\mu$  sets the mass scale for  $m_{\tilde{W}_1}$  and  $m_{\tilde{Z}_{1,2}}$ , then the reactions  $e^+e^- \rightarrow \tilde{W}_1^+\tilde{W}_1^-$  and  $\tilde{Z}_1\tilde{Z}_2$  should be open for  $\sqrt{s} \gtrsim 2|\mu|$ . In this case, the ILC would be a *Higgsino factory*.<sup>[30]</sup> The light Higgsinos—so difficult to see at LHC—are easy to see above SM background at ILC. In addition, a variety of precision measurements could be made to pin down the underlying SUSY model and parameters. Natural SUSY cries out for the ILC to be built!

## 7. Conclusions

Guided by the twin pillars of simplicity (the further one strays from the SM, the more one is likely to be wrong) and naturalness (unnatural models are likely wrong models), we have investigated the consequences of naturalness for future dark matter and collider experiments. Clarification of naturalness finds that some SUSY models remain which are highly natural: those with light Hig-



ginos, TeV-scale highly mixed stops and gluinos below 3-4 TeV. Dark matter is expected to be a DFSZ axion, Higgsino-like WIMP admixture. Ultimately, direct detection of both WIMPs and axions is to be expected. Ton-scale noble liquid WIMP detectors can see the entire natural SUSY parameter space. LHC14 should be able to eke out a signal for SUSY but it may take the full 3000 fb<sup>-1</sup> of HL-LHC. Finally, natural SUSY cried out for construction of ILC since an  $e^+e^-$  collider with  $\sqrt{s} > 2m(\text{higgsino})$  would usher in the era of SUSY discovery and precision measurements within the superworld!

## 8. Acknowledgements

I thank my most excellent collaborators for doing all the work and I thank the DSU organizers for an excellent conference! This work is supported in part by the US Department of Energy, Office of High Energy Physics.

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