Lorentz invariance relations for twist-3 functions and frame-independence of twist-3 cross sections

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In order to derive twist-3 cross sections in the collinear factorization, it is convenient to introduce three types (i.e., *intrinsic*, *kinematical* and *dynamical*) of twist-3 distribution and fragmentation functions. We derive a complete set of the constraint relations among those twist-3 functions, using the QCD equation-of-motion (EOM) and the Lorentz invariance properties of the correlation functions. The twist-3 cross sections, in general, depend on the auxiliary light-like vectors which are introduced to define the twist-3 functions from the correlation functions. We argue that the twist-3 cross sections acquire a frame-independent (Lorentz invariant) form thanks to those constraint relations.

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1. Collinear twist-3 distribution and fragmentation functions

We first summarize the distribution and fragmentation functions which are necessary for the calculation of twist-3 cross sections. The collinear quark distribution functions in the nucleon are defined from the lightcone correlator

$$\Phi_{ij}(x) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \, e^{i\lambda x} \langle PS| \, \bar{q}_j(0) \, [0; \lambda n] \, q_i(\lambda n) \, |PS\rangle, \qquad (1.1)$$

where q_i is a quark field with the spinor index i, and $|PS\rangle$ is the nucleon state with momentum P ($P^2 = M_N^2$) and the spin vector S ($P \cdot S = 0$). For $P^\mu = (E, \vec{P})$ we define two lightlike vectors p^μ and n^μ as $p^\mu = (P^+/\sqrt{2})(1, \vec{P}/|\vec{P}|)$ and $n^\mu = (1, -\vec{P}/|\vec{P}|)/(\sqrt{2}P^+)$ with $P^+ = (E+|\vec{P}|)/\sqrt{2}$, which satisfy $P^\mu = p^\mu + (M_N^2/2)n^\mu$ and $p \cdot n = 1$. Decomposition of $\Phi(x)$ defines the quark distributions listed in Table 1. $[0; \lambda n]$ is a gauge link connecting 0 and λn. Likewise collinear quark fragmentation functions can be defined from the fragmentation correlator

$$\Delta_{ij}(z) = \frac{1}{N_c} \sum_X \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \, e^{-i\frac{\lambda}{z}} \langle 0| \, [\pm\infty m; 0] \, q_i(0) \, |P_h S_h; X\rangle \langle P_h S_h; X| \, \bar{q}_j(\lambda m) \, [\lambda m; \pm\infty m] \, |0\rangle. \quad (1.2)$$

The hadron momentum P_h ($P_h^2 = M_h^2$) can also be decomposed as $P_h^\mu = p_h^\mu + (M_h^2/2)m^\mu$ by introducing two lightlike vectors p_h^μ and m^μ which are defined similarly to p^μ and n^μ. Fragmentation functions defined from $\Delta(z)$ are summarized in Table 1. For precise definition of each distribution and fragmentation function, we refer the readers to [1]. We name the twist-3 functions defined from $\Phi(x)$ and $\Delta(z)$ as *intrinsic* twist-3 distribution/fragmentation functions.

N \ q	Ave.	$S_\parallel$	$S_\perp$
Ave.	$f_1(x), e(x)$		
$S_\parallel$		$g_1(x)$	$h_L(x)$
$S_\perp$		$g_T(x)$	$h_1(x)$

h \ q	Ave.	$S_\parallel$	$S_\perp$
Ave.	$D_1(z), E(z), H(z)$		
$S_\parallel$	$E_L(z)$	$G_1(z)$	$H_L(z)$
$S_\perp$	$D_T(z)$	$G_T(z)$	$H_1(z)$

Table 1: Collinear quark distribution functions in the nucleon (left) and quark fragmentation functions for spin-1/2 hadron (right) which are classified by the spin of the hadron and a quark. In the left table e, h_L and g_T are *intrinsic* twist-3 distributions while others are twist-2 distributions. In the right table, E, H_L and G_T are naively T-even *intrinsic* twist-3 functions, while H, E_L and D_T are naively T-odd *intrinsic* twist-3 functions. Others are twist-2.

The second type of twist-3 functions are called *kinematical* ones and are defined from the transverse-momentum-dependent correlators as

$$\Phi^\rho_{\partial,ij}(x) = \int d^2 k_T \, k_T^\rho \, \Phi_{ij}(x, k_T), \qquad\qquad \Delta^\rho_{\partial,ij}(z) = \int d^2 p_\perp \, p_\perp^\rho \, \Delta_{ij}(z, p_\perp), \qquad (1.3)$$

where $\Phi(x, k_T)$ and $\Delta(z, p_\perp)$ are given as

$$\Phi_{ij}(x, k_T) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} \, e^{ix\lambda + ik_T \cdot z_T} \langle P, S| \, \bar{q}_j(0) \, [0; \infty n][\infty n; \infty n + z_T]$$
$$\times [\infty n + z_T; \lambda n + z_T] \, q_i(\lambda n + z_T) \, |P, S\rangle, \qquad (1.4)$$

$$\Delta_{ij}(z, p_\perp) = \frac{1}{N_c} \sum_X \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int \frac{d^2 z_\perp}{(2\pi)^2} \, e^{-i\frac{\lambda}{z} - ip_\perp \cdot z_\perp} \langle 0| \, [\pm\infty m + \infty z_\perp; \pm\infty m] \, [\pm\infty m; 0] \, q_i(0) \, |P_h S_h; X\rangle$$
$$\times \langle P_h S_h; X| \, \bar{q}_j(\lambda m + z_\perp) \, [\lambda m + z_\perp; \pm\infty m + z_\perp] \, [\pm\infty m + z_\perp; \pm\infty m + \infty z_\perp] \, |0\rangle. \quad (1.5)$$

N \ q	Ave.	$S_\parallel$	$S_\perp$
Ave.			$h_1^{\perp(1)}(x)$
$S_\parallel$			$h_{1L}^{\perp(1)}(x)$
$S_\perp$	$f_{1T}^{\perp(1)}(x)$	$g_{1T}^{(1)}(x)$	

h \ q	Ave.	$S_\parallel$	$S_\perp$
Ave.			$H_1^{\perp(1)}(z)$
$S_\parallel$			$H_{1L}^{\perp(1)}(z)$
$S_\perp$	$D_{1T}^{\perp(1)}(z)$	$G_{1T}^{(1)}(z)$	

Table 2: *Kinematical* twist-3 distribution functions (left) and fragmentation functions (right) classified by the spin of the hadron and a quark.

Decomposition of $\Phi_{\partial,ij}^\rho(x)$ and $\Delta_{\partial,ij}^\rho(z)$ defines kinematical twist-3 functions as listed in Table 2.

The third type of the twist-3 functions are called *dynamical* twist-3 functions and defined from the quark-gluon correlator:

$$\Phi_{F,ij}^\rho(x,x') = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \int_{-\infty}^{\infty} \frac{d\mu}{2\pi} e^{ix'\lambda + i(x-x')\mu} \langle P,S|\bar{q}_j(0)\,[0;\mu n]\,ign_\eta F^{\eta\rho}(\mu n)\,[\mu n;\lambda n]\,q_i(\lambda n)|P,S\rangle, \tag{1.6}$$

$$\begin{aligned}
\Delta_{F,ij}^\rho(z,z') = \frac{1}{N_c}\sum_X \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}\int_{-\infty}^{\infty}\frac{d\mu}{2\pi} e^{i\frac{\lambda}{z} + i(\frac{1}{z}-\frac{1}{z'})\mu} \\
\times \langle 0|[\pm\infty m;\mu m]\,igm_\eta F^{\eta\rho}(\mu m)[\mu m;\lambda m]q_i(\lambda m)|P_h S_h;X\rangle \\
\times \langle P_h S_h;X|\bar{q}_j(0)\,[0;\pm\infty m]|0\rangle.
\end{aligned} \tag{1.7}$$

Decomposition of $\Phi_{F,ij}^\rho(x,x')$ and $\Delta_{F,ij}^\rho(z,z')$, respectively, defines the dynamical twist-3 functions as listed in Table 3. The dynamical twist-3 distribution functions are real, and have definite symmetry under $x_1 \leftrightarrow x_2$: H_{FU} and F_{FT} are symmetric, while H_{FL} and G_{FT} are anti-symmetric. The dynamical twist-3 fragmentation functions are complex functions: The real part is naively T-even, while imaginary part is naively T-odd. Neither real nor imaginary parts have definite symmetry properties. The support of dynamical twist-3 functions is $-1 < |x_{1,2}| < 1$, $-1 < |x_2 - x_1| < 1$ for distribution functions and $0 < z_1 < 1$, $z_1 < z_2 < \infty$ for the fragmentation functions.

Ave.	$H_{FU}(x_1,x_2)$
$S_\parallel$	$H_{FL}(x_1,x_2)$
$S_\perp$	$F_{FT}(x_1,x_2),\ G_{FT}(x_1,x_2)$

Ave.	$\widehat{H}_{FU}(z_1,z_2)$
$S_\parallel$	$\widehat{H}_{FL}(z_1,z_2)$
$S_\perp$	$\widehat{D}_{FT}(z_1,z_2),\ \widehat{G}_{FT}(z_1,z_2)$

Table 3: *Dynamical* twist-3 distribution functions (left) and fragmentation functions (right) classified by the hadron spin configuration.

2. Operator constraints for the twist-3 functions

The twist-3 functions given in Tables 1-3 are not independent but have to obey constraint relations which follow from the QCD equation of motion (EOM) and the Lorentz invariance property of the correlator for each distribution and fragmentation function. The latter is a nonlocal version of the operator product expansion (OPE). In [1], we derived the complete set of those relations, in particular, those for the twist-3 fragmentation functions are new. Here we list the results.

2.1 EOM relations

2.1.1 EOM relations for the twist-3 distribution functions

(i) Unpolarized nucleon:

$$x\,e(x) = \frac{m_q}{M}\,f_1(x) - 2\,\mathscr{P}\int_{-1}^{1}dx'\,\frac{H_{FU}(x,x')}{x-x'}, \tag{2.1}$$

where $\mathscr{P}$ denotes the principal value prescription.
(ii)Transversely polarized nucleon:

$$g_{1T}^{(1)}(x) = x\,g_T(x) - \frac{m_q}{M}\,h_1(x) + \mathscr{P}\int_{-1}^{1}dx'\,\frac{F_{FT}(x,x') - G_{FT}(x,x')}{x-x'}. \tag{2.2}$$

(iii) Longitudinally polarized nucleon:

$$h_{1L}^{\perp(1)}(x) = -\frac{x}{2}\,h_L(x) + \frac{m_q}{2M}\,g_1(x) - \mathscr{P}\int_{-1}^{1}dx'\,\frac{H_{FL}(x,x')}{x-x'}, \tag{2.3}$$

2.1.2 EOM relations for the twist-3 fragmentation functions

(i) Unpolarized hadron:

$$H_1^{\perp(1)}(z) = -\frac{H(z)}{2z} + \int_z^{\infty}\frac{dz'}{z'^2}\,\frac{\Im[\hat{H}_{FU}(z,z')]}{\frac{1}{z}-\frac{1}{z'}}, \tag{2.4}$$

$$-\frac{E(z)}{2z} = \int_z^{\infty}\frac{dz'}{z'^2}\,\frac{\Re[\hat{H}_{FU}(z,z')]}{\frac{1}{z}-\frac{1}{z'}} - \frac{m_q}{2M_h}D_1(z), \tag{2.5}$$

where $\Im$ and $\Re$ represent imaginary and real parts, respectively.
(ii) Transversely polarized spin-1/2 hadron:

$$D_{1T}^{\perp(1)}(z) = -\frac{D_T(z)}{z} + \int_z^{\infty}\frac{dz'}{z'^2}\,\frac{\Im[\hat{D}_{FT}(z,z')] - \Im[\hat{G}_{FT}(z,z')]}{\frac{1}{z}-\frac{1}{z'}}, \tag{2.6}$$

$$G_{1T}^{(1)}(z) = \frac{G_T(z)}{z} - \frac{m_q}{M_h}H_1(z) + \int_z^{\infty}\frac{dz'}{z'^2}\,\frac{\Re[\hat{D}_{FT}(z,z')] - \Re[\hat{G}_{FT}(z,z')]}{\frac{1}{z}-\frac{1}{z'}}. \tag{2.7}$$

(iii) Longitudinally polarized spin-1/2 hadron:

$$\frac{E_L(z)}{2z} = -\int_z^{\infty}\frac{dz'}{z'^2}\,\frac{\Im[\hat{H}_{FL}(z,z')]}{\frac{1}{z}-\frac{1}{z'}}, \tag{2.8}$$

$$H_{1L}^{\perp(1)}(z) = -\frac{H_L(z)}{2z} + \frac{m_q}{2M_h}G_1(z) - \int_z^{\infty}\frac{dz'}{z'^2}\,\frac{\Re[\hat{H}_{FL}(z,z')]}{\frac{1}{z}-\frac{1}{z'}}. \tag{2.9}$$

2.2 Lorentz invariance relations

In this subsection, we list the relations which follows from the Lorentz invariance of the correlators. We present the relations in the form of the so-called Lorentz invariance relations (LIR) which are obtained by combining the OPE relations with the EOM relations.

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2.2.1 LIRs for twist-3 distribtuion functions

(i) Transversely polarized nucleon:

$$g_T(x) = g_1(x) + \frac{d}{dx}g_{1T}^{(1)}(x) - 2\,\mathscr{P}\int_{-1}^{1}dx'\,\frac{G_{FT}(x,x')}{(x-x')^2}. \tag{2.10}$$

(ii) Longitudinally polarized nucleon:

$$h_L(x) = h_1(x) - \frac{d}{dx}h_{1L}^{\perp(1)}(x) + 2\,\mathscr{P}\int_{-1}^{1}dx'\,\frac{H_{FL}(x,x')}{(x-x')^2}. \tag{2.11}$$

2.2.2 LIRs for twist-3 fragmentation functions

(i) Transversely polarized spin-1/2 hadron:

$$\frac{G_T(z)}{z} = \frac{G_1(z)}{z} + \left(1 - z\frac{d}{dz}\right)G_{1T}^{(1)}(z) - \frac{2}{z}\int_z^\infty\frac{dz'}{z'^2}\frac{\Re[\hat{G}_{FT}(z,z')]}{(1/z-1/z')^2}, \tag{2.12}$$

$$\frac{D_T(z)}{z} = -\left(1 - z\frac{d}{dz}\right)D_{1T}^{\perp(1)}(z) - \frac{2}{z}\int_z^\infty\frac{dz'}{z'^2}\frac{\Im[\hat{D}_{FT}(z,z')]}{(1/z-1/z')^2}. \tag{2.13}$$

(ii) Longitudinally polarized spin-1/2 hadron:

$$\frac{H_L(z)}{z} = \frac{H_1(z)}{z} - \left(1 - z\frac{d}{dz}\right)H_{1L}^{\perp(1)}(z) + \frac{2}{z}\int_z^\infty\frac{dz'}{z'^2}\frac{\Re[\hat{H}_{FL}(z,z')]}{(1/z-1/z')^2}, \tag{2.14}$$

$$\frac{H(z)}{z} = -\left(1 - z\frac{d}{dz}\right)H_1^{\perp(1)}(z) - \frac{2}{z}\int_z^\infty\frac{dz'}{z'^2}\frac{\Im[\hat{H}_{FU}(z,z')]}{(1/z-1/z')^2}. \tag{2.15}$$

Using the EOM relations and LIRs, one can eliminate the intrinsic and kinematical twist-3 functions in favor of the dynamical twist-3 functions and the twist-2 functions. In this sense the intrinsic and kinematical twist-3 functions may be regarded auxiliary functions. On the other hand, in the leading-order calculation, simple structure of partonic hard cross sections sometimes leads to a compact formula for the twist-3 cross sections in terms of the intrinsic and kinematical twist-3 functions [2, 3].

3. Lorentz invariance of the twist-3 cross sections

Twist-3 cross sections can be obtained as a convolution of the hadronic correlators introduced in Sec. 1 and the corresponding hard part. The resulting twist-3 cross sections involve intrinsic, kinematical and dynamical twist-3 distribution and fragmentation functions together with twist-2 functions. In the decomposition of the hadronic correlators, the coefficients of the twist-3 functions contain lightlike vectors n and m, and thus the expression for the cross sections also contain these vectors. On the other hand, the cross section should have a Lorentz invariant expression in terms of only the physical vectors such as momenta of the particles participating in the process and the spin vectors. Though n and m are uniquely determined from P and P_h, their expression in terms of

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the physical vectors depends on a frame we choose, which makes the frame-independence of the cross section unclear.

In our recent paper [1] (see also [4]), we showed that the EOM relations and LIRs lead to frame-independent expression for the twist-3 cross sections, using, as an example, the inclusive hadron production in the lepton-nucleon collision, $e(l) + P(P) \rightarrow h(P_h) + X$. We define Mandelstam variables for this process as $S = (P+l)^2$, $T = (P - P_h)^2$ and $U = (l - P_h)^2$. In the cross section, l, P and P_h can be regarded as lightlike in the twist-3 accuracy, and therefore P and P_h can be identified as p and p_h, respectively. To get a cross section in terms of the physical momenta, we note that n^μ and m^μ can be expanded in terms of P_h^μ, P^μ, l^μ and $\varepsilon^{\mu l P P_h}$. Taking into account the conditions, $n^2 = m^2 = 0$, $P \cdot n = 1$ and $P_h \cdot m = 1$, one may write

$$n^\mu = \chi P_h^\mu + \frac{2 + \chi T}{S} l^\mu + \left[\frac{\chi U (2 + \chi T)}{2S} + \frac{STU}{8} \chi_\varepsilon^2 \right] P^\mu + \chi_\varepsilon \varepsilon^{\mu l P P_h}, \qquad (3.1)$$

$$m^\mu = \eta P^\mu - \frac{2 + \eta T}{U} l^\mu + \left[\frac{\eta S (2 + \eta T)}{2U} + \frac{STU}{8} \eta_\varepsilon^2 \right] P_h^\mu + \eta_\varepsilon \varepsilon^{\mu l P P_h}. \qquad (3.2)$$

In the eN center-of-mass frame, $n^\mu = 2l^\mu / S$, which corresponds to $\chi = \chi_\varepsilon = 0$. In the hP center-of-mass frame, $n^\mu = -2P_h^\mu / T$ and $m^\mu = -2P^\mu / T$, which corresponds to $\chi = \eta = -2/T$ and $\chi_\varepsilon = \eta_\varepsilon = 0$. After obtaining the twist-3 cross sections in terms of n and m, one can eliminate n and m by (3.1) and (3.2). For $eN^\uparrow \rightarrow hX$, $\vec{e}N^\uparrow \rightarrow hX$, $eN \rightarrow \Lambda^\uparrow X$ and $\vec{e}N \rightarrow \Lambda^\uparrow X$, we have shown that the EOM relations and LIRs, and

$$\pi F_{FT}(x, x) = f_{1T}^{\perp (1)}(x), \qquad \pi H_{FU}(x, x) = h_1^{\perp (1)}(x), \qquad (3.3)$$

lead to Lorentz invariant expression for the twist-3 cross sections which are independent of the parameters χ, χ_ε, η and η_ε. Therefore the frame-independence of the twist-3 cross sections is guaranteed.

Acknowledgments

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