PROCEEDINGS OF SCIENCE

New pole contribution to transverse-momentum-weighted single-transverse spin asymmetry in semi-inclusive deep inelastic scattering

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We discuss a new hard pole contribution to the transverse-momentum-weighted single-transverse spin asymmetry in semi-inclusive deep inelastic scattering. We perform a complete next-to-leading order calculation of the $P_{h\perp}$-weighted cross section and show that the new hard pole contribution is required in order to obtain the complete evolution equation for the twist-3 Qiu-Sterman function.

XXIV International Workshop on Deep-Inelastic Scattering and Related Subjects
11-15 April, 2016
DESY Hamburg, Germany

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PoS(DIS2016)209

1. Introduction

In a perturbative QCD calculation, higher-order corrections bring the logarithmic energy-scale dependence of nonperturbative function which is described by the scale evolution equation. Systematic treatment of the scale dependence of the twist-3 functions is important for a quantitative description of the single-transverse spin asymmetry(SSA). The twist-3 distribution effect of the transversely polarized proton is embodied as the so-called Qiu-Sterman (QS) function $G_F(x_1,x_2)(T_F(x_1,x_2))$ in the spin-dependent cross section formula. The scale evolution equation of the QS function was discussed by using several different approaches so far [1, 2, 3, 4, 5, 6, 7, 8]. One of the approaches is the next-to-leading-order (NLO) calculation of the transverse momentum $P_{h\perp}$-weighted cross section. We present a complete NLO cross section for the twist-3 P_h-weighted cross section for SSA in semi-inclusive deep inelastic scattering.

We consider the SSA for light-hadron production in SIDIS,

$$e(\ell)+p(p,S_\perp)\to e(\ell')+h(P_h)+X. \tag{1.1}$$

Within the collinear factorization framework, the SSA can be described by the twist-3 effects. In this process, the SSA receives two types of twist-3 contributions, the distribution effect of the transversely polarized proton and the fragmentation effect of the light-hadron. We focus on the former contribution in this paper to derive the evolution equation of $G_F(x,x)$. In the case of SIDIS, the cross section formula can be expressed in terms of the following Lorentz invariant variables,

$$S_{ep}=(p+\ell)^2,\qquad Q^2=-q^2,\qquad x_B=\frac{Q^2}{2p\cdot q},\qquad z_h=\frac{p\cdot P_h}{p\cdot q}, \tag{1.2}$$

where $q=(\ell-\ell')$ is the momentum of the virtual photon. In this paper, we discuss the NLO $P_{h\perp}$-weighted polarized cross section defined as

$$\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle}{dx_BdQ^2dz_hd\phi}\equiv\int d^2P_{h\perp}\epsilon^{\alpha\beta+-}S_{\perp\alpha}P_{h\perp\beta}\left(\frac{d^6\Delta\sigma}{dx_BdQ^2dz_hdP_{h\perp}^2d\phi d\chi}\right). \tag{1.3}$$

2. Leading-order cross section and next-to-leading-order virtual correction cotribution

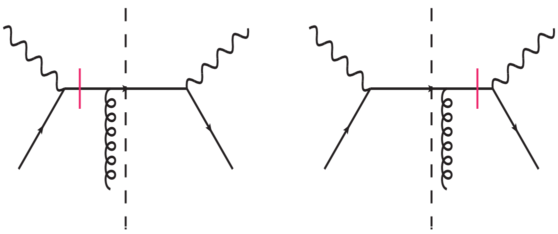

Figure 1: Leading order diagram for the $P_{h\perp}$-weighted cross section. The red barred propagators give pole contributions.

We discuss a leading-order(LO) $P_{h\perp}$-weighted cross section. LO diagrams are shown in Fig. 1. In the collinear factorization approach, the SSA requires a complex phase given by a pole of an internal propagator. The red barred propagators in Fig.1 give pole contributions. The LO cross section can be easily calculated as

$$\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle^{\rm LO}}{dx_BdQ^2dz_hd\phi}=-\frac{z_h\pi M_N\alpha_{em}^2}{4x_B^2S_{ep}^2Q^2}\sum_q e_q^2G^q(x_B,x_B)D^q(z_h), \tag{2.1}$$

where M_N is the nucleon mass and α_{em} is the QED coupling constant. The next-to-leading-order(NLO) virtual correction contribution is given by one-loop correction to the LO diagrams. This has been calculated in [8] as

$$\begin{aligned}\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle^{\rm virtual}}{dx_BdQ^2dz_hd\phi}&=-\frac{z_h\pi M_N\alpha_{em}^2}{4x_B^2S_{ep}^2Q^2}\frac{\alpha_s}{2\pi}\sum_q e_q^2G^q(x_B,x_B)D^q(z_h)\\&\times\Big[C_F\Big(\frac{4\pi\mu^2}{Q^2}\Big)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\Big(-\frac{2}{\varepsilon^2}-\frac{3}{\varepsilon}\Big)\Big]+\cdots.\end{aligned} \tag{2.2}$$

Here we adopted the dimensional regularization scheme and $\varepsilon=2-D/2$. We neglect $O(1)$-contributions throughout.

3. Next-leading-order real emission contribution

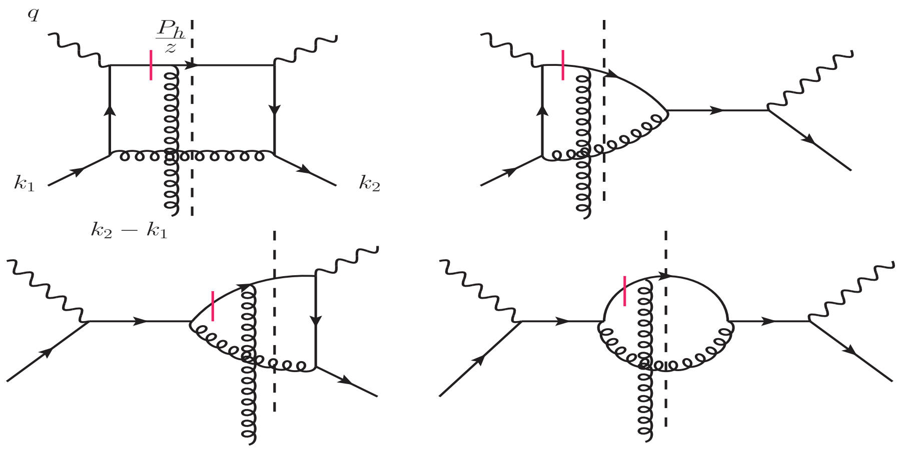

Figure 2: Diagrammatic description for SGP contribution.

In this section, we discuss NLO real emission contributions. In SIDIS case, the pole contributions can be classified into four types as soft-gluon-pole(SGP), soft-fermion-pole(SFP), hard-pole(HP) and another hard-pole(HP2) [9, 10]. It was found in [11] that the SFP contribution is completely cancelled in the SIDIS case. We discuss other 3 pole contributions below.

First we discuss the SGP contribution. This is given by the diagrams in Fig. 2. The SGP contribution has been calculated in [8] as

$$\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle^{\rm SGP}}{dx_BdQ^2dz_hd\phi}$$

$$= -\frac{\pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\int dz D(z)\int\frac{dx}{x}G_F(x,x)$$
$$\times\frac{1}{2N}\left[-\frac{2}{\varepsilon^2}\delta(1-\hat{x})\delta(1-\hat{z})+\frac{1}{\varepsilon}\frac{1+\hat{z}^2}{(1-\hat{z})_+}\delta(1-\hat{x})-\frac{1}{\varepsilon}\frac{2\hat{x}^3-3\hat{x}^2-1}{(1-\hat{x})_+}\delta(1-\hat{z})\right]+\cdots, \quad (3.1)$$

where α_s is the QCD coupling constant, $\hat{x} = x_B/x$ and $\hat{z} = z_h/z$.

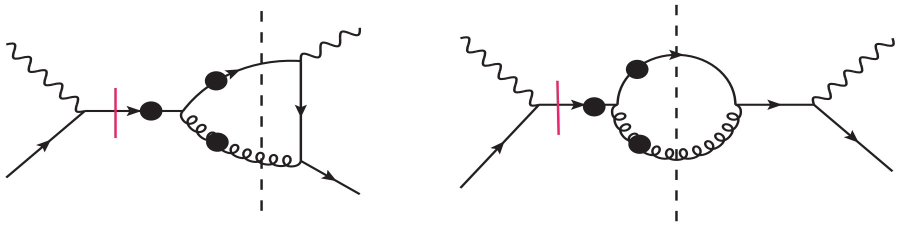

Figure 3: Diagrammatic description for HP contribution.

Next we discuss the HP contribution shown in Fig. 3. This contribution can be calculated as

$$\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle^{\rm HP}}{dx_B dQ^2 dz_h d\phi}$$
$$= -\frac{\pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\int dz D(z)\int\frac{dx}{x}\left(\hat{z}C_F+\frac{1}{2N}\right)$$
$$\times\left\{G_F(x,x_B)\left[\frac{2}{\varepsilon^2}\delta(1-\hat{x})\delta(1-\hat{z})+\frac{1}{\varepsilon}\left(2\delta(1-\hat{x})\delta(1-\hat{z})-\frac{1+\hat{z}^2}{(1-\hat{z})_+}\delta(1-\hat{x})\right.\right.\right.$$
$$\left.\left.\left.-\frac{1+\hat{x}}{(1-\hat{x})_+}\delta(1-\hat{z})\right)\right]+\tilde{G}_F(x,x_B)\frac{1}{\varepsilon}\delta(1-\hat{z})\right\}+\cdots \quad (3.2)$$

The last term associated with another twist-3 function $\tilde{G}_F(x,x_B)$ was found in [12] and other part was calculated in [8].

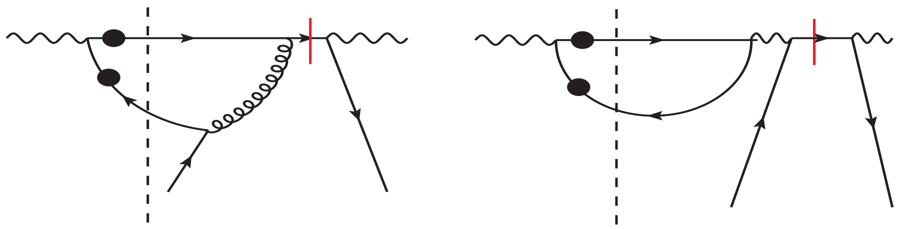

Figure 4: Diagrammatic description for HP2 contribution.

Finally we discuss the HP2 contribution shown in Fig.4. This was first calculated in [12] as

$$\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle^{\rm HP2}}{dx_B dQ^2 dz_h d\phi}$$
$$= -\frac{\pi M_N \alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\int dz D(z)\int\frac{dx}{x}$$
$$\times\frac{1}{2N}\left\{-G_F(x_B,x_B-x)\frac{1}{\varepsilon}(1-2\hat{x})\delta(1-\hat{z})-\tilde{G}_F(x_B,x_B-x)\frac{1}{\varepsilon}\delta(1-\hat{z})\right\}+\cdots \quad (3.3)$$

4. Scale evolution equation of $G_F(x,x)$

After combining all contributions in previous sections, we can obtain the NLO $P_{h\perp}$-weighted cross section as

$$
\begin{aligned}
\frac{d^4\langle P_{h\perp}\Delta\sigma\rangle^{\rm LO+NLO}}{dx_B dQ^2 dz_h d\phi} &= -\frac{z_h\pi M_N\alpha_{em}^2}{4x_B^2 S_{ep}^2 Q^2}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\sum_q e_q^2 \\
&\times\Bigg[\left(-\frac{1}{\varepsilon}\right)\Bigg\{D^q(z_h)\Big\{\int_{x_B}^1\frac{dx}{x}\Big[C_F P_{qq}(\hat{x})G_F^q(x,x)+\frac{N}{2}\Big(\frac{(1+\hat{x})G_F^q(x_B,x)-(1+\hat{x}^2)G_F^q(x,x)}{(1-\hat{x})_+} \\
&+\tilde{G}_F^q(x_B,x)\Big)\Big]-NG_F^q(x_B,x_B)+\frac{1}{2N}\int_{x_B}^1\frac{dx}{x}\Big((1-2\hat{x})G_F^q(x_B,x_B-x)+\tilde{G}_F^q(x_B,x_B-x)\Big)\Big\} \\
&+G_F^q(x_B,x_B)C_F\int_{z_h}^1\frac{dz}{z}P_{qq}(\hat{z})D^q(z)\Bigg\}\Bigg]+\cdots,
\end{aligned}
\tag{4.1}
$$

where $P_{qq}(x)$ is the splitting function

$$
P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right]. \tag{4.2}
$$

From the structure of the collinear divergence $1/\varepsilon$, we can derive the scale evolution equation of $G_F(x,x)$ as

$$
\begin{aligned}
\frac{\partial}{\partial\ln\mu^2}G_F(x_B,x_B,\mu^2) &= \frac{\alpha_s}{2\pi}\Big\{\int_{x_B}^1\frac{dx}{x}\Big[P_{qq}(\hat{x})G_F(x,x,\mu^2) \\
&+\frac{N}{2}\Big(\frac{(1+\hat{x})G_F(x_B,x,\mu^2)-(1+\hat{x}^2)G_F(x,x,\mu^2)}{(1-\hat{x})_+}+\tilde{G}_F(x_B,x,\mu^2)\Big)\Big]-NG_F(x_B,x_B,\mu^2) \\
&+\frac{1}{2N}\int_{x_B}^1\frac{dx}{x}\Big((1-2\hat{x})G_F(x_B,x_B-x,\mu^2)+\tilde{G}_F(x_B,x_B-x,\mu^2)\Big)\Big\},
\end{aligned}
\tag{4.3}
$$

which completely agrees with the results in other approaches [4, 6, 7].

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