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Attacking One-loop Multi-leg Feynman Integrals with the Loop-Tree Duality

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We discuss briefly the first numerical implementation of the Loop-Tree Duality (LTD) method. We apply the LTD method in order to calculate ultraviolet and infrared finite multi-leg one-loop Feynman integrals. We attack scalar and tensor integrals with up to six legs (hexagons). The LTD method shows an excellent performance independently of the number of external legs.

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1. Introduction

The progress in calculating hadronic processes to higher orders of the perturbative expansion and in particular, calculating LHC observables to next-to-leading order (NLO) accuracy was really impressive in recent years. Naturally, the complexity for any observable to be computed at fixed order is related to the number of initial and final states present in the process and the specific order of the radiative corrections one takes into account.

Typically, in order to finally have a theoretical estimate that can be compared to experimental data, a number of steps needs to be completed before, each with its own difficulties: setting up the virtual and real parts of the scattering amplitude, integrating over any loop momenta, dealing with the cancelation of the infrared divergencies [1], integrating over phase-space, etc. One path to follow toward automated NLO calculations is the purely numerical approach which has seen considerable development in the past years [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Computations based on a solid algorithmic setup at NLO are now standard, either purely numerical [16, 17, 18] or a mixture of analytical and numerical techniques [19, 20, 21]. Substantial progress has also been made at higher orders [22, 23, 24].

The loop–tree duality (LTD) method [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38] states that loop integrals and amplitudes with loop diagrams with n external legs may be decomposed into a sum of tree-level-like graphs which still would need to be integrated over a measure that resembles the usual $(n+1)-$body three-dimensional phase-space [25, 26]. This brings forward the intriguing possibility that loop- and tree-like radiative corrections can be treated on equal footing under a common integral using Monte Carlo integration through a convenient mapping of the external momenta entering the virtual and real scattering amplitudes [36, 37].

Here we focus on the use of the LDT framework in computing one-loop Feynman diagrams. We discuss briefly the principle behind the LTD method, we describe our numerical implementation [35] and we give some details on the performance of our code in computing both scalar and tensor multi-leg integrals.

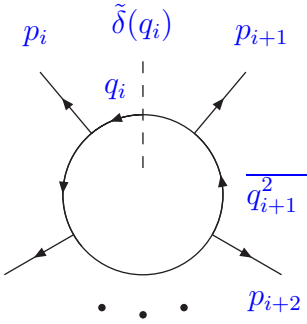
2. Numerical Implementation of the LDT

A generic one-loop scalar integral can be written in dimensional regularisation as

$$L^{(1)}(p_1,p_2,\ldots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i)\ , \tag{2.1}$$

where $G_F(q_i) = 1/(q_i^2 - m_i^2 + i0)$ are Feynman propagators, q_i are internal momenta depending on the loop momentum ℓ and $\int_\ell = -i\int d^d\ell/(2\pi)^d$ is used as a shorthand notation. To apply the LTD we integrate over the energy coordinate of the loop four-momentum by taking residues (Cauchy's theorem) after choosing an integration contour which encloses the poles with positive energy and negative imaginary part. This action decomposes the initial diagram from an integral with loop four-momentum to a sum of integrals over three-momentum:

$$L^{(1)}(p_1,p_2,\ldots,p_N) = -\sum_{i=1}^{N}$$



$$\frac{1}{q_{i+1}^2 - m_{i+1}^2 - i0\,\eta p_{i+1}}$$

We defined $\tilde{\delta}(q_i) = 2\pi i \delta_+(q_i^2 - m_i^2)$ where the delta function "+" subscript indicates that we take the positive-energy solution whereas, η is a future-like vector. To summarise, LTD in its essence tells us that by using Cauchy's theorem, me may rewrite a one-loop amplitude as a sum of single-cut phase-space integrals (which we call "dual contributions") over the loop three-momentum. To integrate the dual contributions over the latter requires a proper contour deformation due to the presence of the so-called *ellipsoid* and *hyperboloid* singularities [35].

Our implementation of the LTD method was done in `C++` and to perform the momenta integration we employ the Cuba library [39]. The user needs only to specify the external four-momenta, the internal propagator masses and has also the freedom to change the parameters of the contour deformation. One may also choose between Cuhre [40, 41] and VEGAS [42], and specify the desired number of evaluations. At run time, the code performs the following steps:

1. Reads in and assigns masses and external momenta.
2. Makes an analysis of the ellipsoid and hyperboloid singularities to properly group the dual contributions and sets up the contour deformation.
3. Sends the integrand to the chosen routine (either Cuhre or VEGAS) for the integration to be done and returns the result.

To generate a very wide range of random momenta and masses so that we could test our code in various regions of the phase-space we used `MATHEMATICA 10.0` [43]. For most of the numerical results in [35], we used the routine Cuhre, VEGAS was a slower choice in general. In order to produce reference values to compare our results to, `LoopTools 2.10` [44] and `SecDec 3.0` [45] were used.

We have run our code for a large number of scalar and up to rank three tensor integrals. We did an exhaustive comparison of our results against the reference values from `LoopTools 2.10` and `SecDec 3.0` for triangles (3-external legs), boxes (4-external legs) pentagons (5-external legs) and hexagons (6-external legs). The execution time for scalar integrals with a wanted precision of 4-digits, on a typical Desktop machine (Intel i7 @ 3.4 GHz processor, 4-cores 8-threads), varied from below a second to around 30 seconds, the latter for pentagons. Tensor integrals were generally slower to compute but not by much.

As a non-trivial example, we display in Table 1 the results we obtained for tensor hexagons and for three phase-space points (P22, P23, P24) along with the execution times. The momenta and the masses of the these phase-space points can be found in the Appendix of Ref. [35]. The running times of SecDec are shown only for completeness, we do not imply that our code compares better or worse to SecDec.

	Rank	Tensor Hexagon	Real Part	Imaginary Part	Time[s]
P22	1	SecDec	$1.01359(23) \times 10^{-15}$	$+i\, 2.68657(26) \times 10^{-15}$	33
		LTD	$1.01345(130) \times 10^{-15}$	$+i\, 2.68633(130) \times 10^{-15}$	72
P23	2	SecDec	$2.45315(24) \times 10^{-12}$	$-i\, 2.06087(20) \times 10^{-12}$	337
		LTD	$2.45273(727) \times 10^{-12}$	$-i\, 2.06202(727) \times 10^{-12}$	75
P24	3	SecDec	$-2.07531(19) \times 10^{-6}$	$+i\, 6.97158(56) \times 10^{-7}$	14280
		LTD	$-2.07526(8) \times 10^{-6}$	$+i\, 6.97192(8) \times 10^{-7}$	85

Table 1: Tensor hexagons involving numerators of rank one to three.

3. Conclusions

The Loop-Tree Duality is a method that incorporates many appealing theoretical features when considering processes with many external legs. The first numerical implementation of the LTD and its application to a very large number of phase-space test points proved to be very successful. The code excels in diagrams with many external legs as it shows only a moderate increase in execution times in comparison to diagrams with smaller number of external particles.

From this first study, and without any serious attempt toward the optimization of our code for faster execution times, we conclude that our implementation of the LTD method offers a promising alternative for computing multi-leg scalar and tensor one-loop integrals with an arbitrary number of scales. Finally, we are looking forward to testing our LTD numerical implementation onto physical processes.

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