PROCEEDINGS OF SCIENCE

Quark and Gluon collinear and TMD parton distributions from HERA DIS data

Aleksandra Lelek*
DESY
E-mail: aleksandra.lelek@desy.de

Francesco Hautmann
University of Oxford
E-mail: Francesco.Hautmann@physics.ox.ac.uk

Hannes Jung
DESY
E-mail: hannes.jung@desy.de

We describe a new approach to solve coupled quark and gluon DGLAP evolution equations with a Monte Carlo method. We show that this method is equivalent to other semi-analytical methods. We apply this method to extract quark and gluon parton densities, both collinear and transverse momentum dependent (TMD), using the precision HERA DIS data.

XXIV International Workshop on Deep-Inelastic Scattering and Related Subjects
11-15 April, 2016
DESY Hamburg, Germany

*Speaker.

1. Motivation

The goal of the project [1] is to provide for the first time quark and gluon collinear and transverse momentum dependent (TMD [2]) parton distribution functions (PDFs) from a Monte Carlo solution of the QCD DGLAP evolution equations, by performing fits to the precision DIS data [3].

In Ref. [4] the TMD gluon density was determined from DIS fits based on high-energy factorization [5] and CCFM evolution equations. This article is based on a different approach, in which we start from fully coupled quark and gluon DGLAP equations, solve these by Monte Carlo method [6], and determine from this both collinear and TMD PDFs. It is close in spirit to the works of Refs. [7] and [8]. To perform the fit to DIS data we use an updated version of the program [9] within the `xFitter` open-source QCD platform [10].

2. Introduction to the method

The starting point of the discussion is the DGLAP evolution equation for momentum weighted parton density $xf(x,\mu^2) = \widetilde{f}(x,\mu^2)$

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 \mathrm{dz}\, P_{ab}\left(\alpha_s(\mu^2),z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) \tag{2.1}$$

where a,b are quark ($2N_f$ flavours) or gluon, x- longitudinal momentum fraction of the proton carried by a parton a, $z = x_i/x_{i-1}$ is the splitting variable and μ is the evolution mass scale. The splitting functions P_{ab} have the following structure

$$P_{ab}\left(\alpha_s(\mu^2),z\right) = D_{ab}\left(\alpha_s(\mu^2)\right)\delta(1-z) + K_{ab}\left(\alpha_s(\mu^2)\right)\frac{1}{(1-z)_+} + R_{ab}\left(\alpha_s(\mu^2),z\right) \tag{2.2}$$

where $D_{ab}\left(\alpha_s(\mu^2)\right) = \delta_{ab}d_a\left(\alpha_s(\mu^2)\right)$, $K_{ab}\left(\alpha_s(\mu^2)\right) = \delta_{ab}k_a\left(\alpha_s(\mu^2)\right)$ and $R_{ab}\left(\alpha_s(\mu^2),z\right)$ contains logarithmic terms in $\ln(1-z)$ and has no power divergences $(1-z)^{-n}$ for $z \to 1$. With that eq. (2.1) can be written in a form

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 \mathrm{dz}\, P^R_{ab}\left(\alpha_s(\mu^2),z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) - \widetilde{f}_a\left(x,\mu^2\right)\int_0^1 \mathrm{dz}\, P^V_a\left(\alpha_s(\mu^2),z\right) \tag{2.3}$$

where the *real* part of the splitting function is $P^R_{ab}\left(\alpha_s(\mu^2),z\right) = R_{ab}\left(\alpha_s(\mu^2),z\right) + K_{ab}\left(\alpha_s(\mu^2)\right)1/(1-z)$ and the *virtual* part is $P^V_a\left(\alpha_s(\mu^2),z\right) = k_a\left(\alpha_s(\mu^2)\right)1/(1-z) - d_a\left(\alpha_s(\mu^2)\right)\delta(1-z)$. The integrals in eq.(2.3) are divergent near $z \to 1$ and a method to regularize them is needed. It will be described in section (3). Thanks to the *momentum sum rule* $\sum_c \int_0^1 \mathrm{dz} z P_{ca}\left(\alpha_s(\mu^2),z\right) = 0$, eq.(2.3) can be written with $P^R_{ab}\left(\alpha_s(\mu^2),z\right)$ also in the second term on the right hand side. This can be obtained by subtracting from eq.(2.3) the term proportional to momentum sum rule

$$\begin{aligned}\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} &= \sum_b \int_x^1 \mathrm{dz} P^R_{ab}\left(\alpha_s(\mu^2),z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) - \widetilde{f}_a\left(x,\mu^2\right)\int_0^1 \mathrm{dz} P^V_a\left(\alpha_s(\mu^2),z\right) + \\ &-\widetilde{f}_a\left(x,\mu^2\right)\sum_c \int_0^1 \mathrm{dz} z P_{ca}\left(\alpha_s(\mu^2),z\right) = \sum_b \int_x^1 \mathrm{dz} P^R_{ab}\left(\alpha_s(\mu^2),z\right) \widetilde{f}_b\left(\frac{x}{z},\mu^2\right) + \\ &-\widetilde{f}_a\left(x,\mu^2\right)\sum_c \int_0^1 \mathrm{dz} z P^R_{ca}\left(\alpha_s(\mu^2),z\right).\end{aligned} \tag{2.4}$$

This trick is possible only with momentum weighted parton densities. An additional advantage of using momentum weighted parton densities is that the convergence of the integrals is improved by removing $1/z$ terms in P_{gg} and P_{gq}. Defining now the *Sudakov form factor* as

$$\Delta_a(\mu^2) = \exp\left(- \int_{\ln\mu_0^2}^{\ln\mu^2} d\left(\ln\mu'^2\right) \sum_b \int_0^1 \mathrm{d}z z P^R_{ba}\left(\alpha_s(\mu'^2), z\right) \right) \tag{2.5}$$

eq.(2.4) can be rewritten

$$\frac{d\widetilde{f}_a(x,\mu^2)}{d\ln\mu^2} = \sum_b \int_x^1 \mathrm{d}z P^R_{ab}\left(\alpha_s(\mu^2), z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu^2\right) + \widetilde{f}_a\left(x, \mu^2\right) \frac{1}{\Delta_a(\mu^2)} \frac{d\Delta_a(\mu^2)}{d\ln\mu^2}. \tag{2.6}$$

This equation has an iterative solution which can be easily implemented in a MC method. The details of the MC solution can be found in [1].

Technically, two different evolution grids are defined: for processes initiated by a quark, quark grid is filled, and for processes initiated by a gluon, gluon grid is filled. Kernels for the evolution initiated by gluons and by quarks are calculated separately only once per run and combined at the end. Thanks to that the fitting procedure in `xFitter` is fast. To get the final pdf, the evolution kernel is folded with starting distribution as $xf(x,t)_g = x\int \mathrm{d}x_0 \int \mathrm{d}z \left(f_{0g}(x_0)K_{gg} + f_{0q}(x_0)K_{gq}\right)\delta(zx_0 - x)$.

3. Integrated PDFs from TMD evolution using MC method

Some of the splitting functions are divergent for $z \to 1$. To avoid divergences, a cut off must be introduced and the upper limit in the integral over z in eq. (2.6) as well as in the *Sudakov form factor* eq.(2.5) is put to z_{max} instead of 1. It can be shown that the net effect of the skipped terms $\int_{z_{max}}^1$ is of order $\mathscr{O}(1 - z_{max})$. There are different choices of z_{max} possible: z_{max} can be fixed or can change dynamically with the scale, for example like in angular ordering: $z_{max} = 1 - (Q_0/Q)$. In this paper we present results for fixed z_{max}.

In the fig.(1) we show results for integrated distributions for sea-quarks and gluon coming from the MC solution and from QCDNUM [11]. The initial distributions are taken from QCDNUM. We obtain a very good agreement between these two methods.

The differences between MC and QCDNUM at large x are an artefact of the histogram binning.

4. Results for TMDs

In the presented MC method Q is generated for every branching so the information about k_T is available for every branching. k_T contains the whole history of the evolution. When we want to calculate $k_{T,n}$ at a given step n then we have to perform a vectorial sum of $k_{T,n-1}$ and Q: $\vec{k}_{T,n} = \vec{k}_{T,n-1} + \vec{Q}_{T,n-1}$. In this method k_T is treated properly from the beginning of the evolution- no extra reshuffling at the end is required.

In the fig.(2) we present results for TMDs for sea-quarks and gluon. We observe k_T tails, which can be larger than the evolution scale.

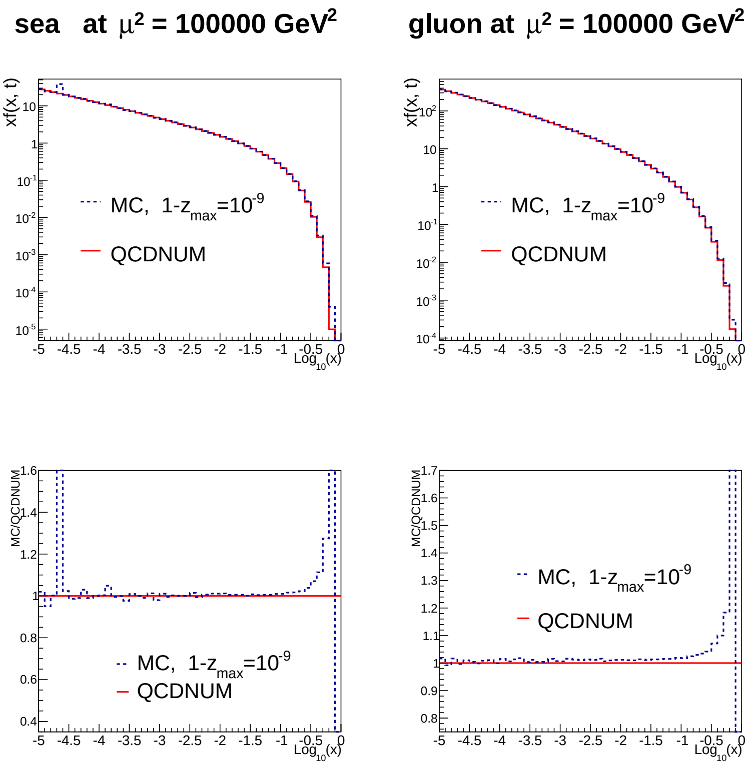

Figure 1: Left: sea quarks, right: gluon. First row: $xf(x,t)$ vs $\mathrm{Log}_{10}(x)$ after evolution up to 100000 GeV^2, second row: ratios MC/QCDNUM vs $\mathrm{Log}_{10}(x)$ at 100000 GeV^2. Red: QCDNUM, blue dotted: $1-z_{max} = 10^{-9}$

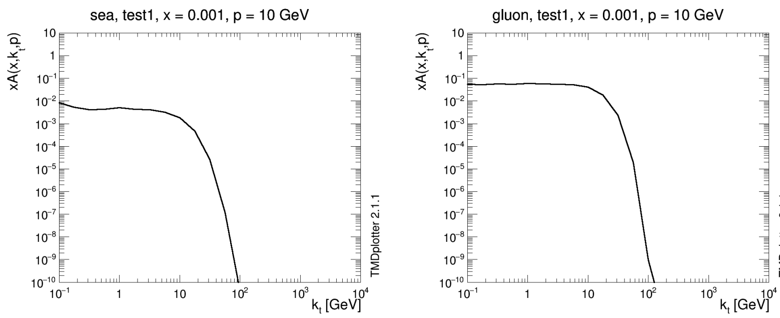

Figure 2: Left: sea TMD after evolution up to 100 GeV^2 for $x = 0.001$, right: gluon TMD after evolution up to 100 GeV^2 for $x = 0.001$. $1 - z_{max} = 10^{-9}$

5. First fit of full integrated TMDs to HERA DIS data with xFitter

As a consistency check, we performed the fit of integrated TMDs from MC solution to F_2. This is also a check of the flavour decomposition with MC method.

In the past a gluon TMD pdf was fitted within `xFitter` [10] to F_2 from H1/Zeus data for $Q^2 > 5\mathrm{GeV}^2$ and $x < 0.01$ [4]. Now we present the first fit of full (gluon, valence and sea) integrated TMDs to HERA H1 and Zeus data using `xFitter`. Results are presented in the fig.(3).

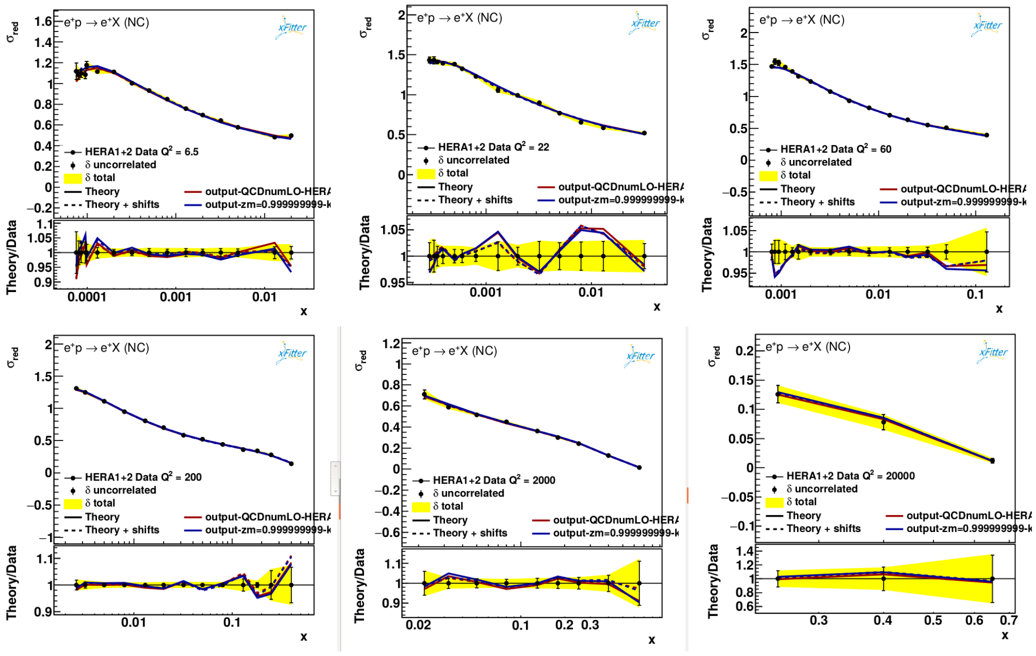

Figure 3: Results of the first fit of the full (gluon, valence and sea) integrated TMDs to HERA H1 and Zeus data.

The fit works reasonably well for the whole x range and $Q^2 > 5\mathrm{GeV}^2(\chi^2/ndf \approx 1)$.

6. Summary

A new approach to solve the coupled gluon and quark DGLAP evolution equation with a MC method was presented. The full TMD pdf evolution including gluon, sea and valence quarks over the full range in x and Q^2 with the k_T dependence in the whole kinematically available range (not limited to the small k_T) is obtained. Results coming from the MC solution reproduce semi-analytical results (QCDNUM) and they can be used in PS matched calculation.

Moreover, TMDs are implemented in the preliminary version of `xFitter`. New results of fitting integrated TMD pdfs to F_2 with `xFitter` were shown: gluon and quark are fitted for $Q^2 > 5\mathrm{GeV}^2$ for all x with $\chi^2/ndf \approx 1$.

References

[1] F. Hautmann, H. Jung and A. Lelek, preprint DESY 16-174.

[2] R. Angeles-Martinez *et al.*, Acta Phys. Polon. B **46** (2015) 2501.

[3] F. Aaron *et al.*, JHEP **1001** (2010) 109.

[4] F. Hautmann and H. Jung, Nucl. Phys. B**883** (2014) 1.

[5] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B**242** (1990) 97.

[6] B. R. Webber, Ann. Rev. Nucl. Part. Sci. **36** (1986) 253.

[7] H. Tanaka, PTEP **2015** (2015) 043B03.

[8] S. Jadach *et al.*, Comput. Phys. Commun. **181** (2010) 393.

[9] F. Hautmann, H. Jung and S. Taheri Monfared, Eur. Phys. J. C **74** (2014) 3082.

[10] S. Alekhin *et al.*, Eur. Phys. J. C **75** (2015) 304.

[11] M. Botje *QCDNUM* Comput. Phys. Commun. 182 (2011) 490